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Ancient Discussion on Determinism from the Perspective of a Branching Time

Abstract: The paper is devoted to the problem of the Master Argument – a historical argument of the Greek philosopher Diodorus Cronus. The objective of Diodorus' argument was to evidence the correctness of the temporarily defined modalities: necessity and probability. Since the original argument was lost, many contemporary logicians have tried to reformulate it, using modern tools of logic. We present the philosophical significance of one of such reconstructions, relying on the monograph (*On the Sea Battle Tomorrow That May Not Happen. A Logical and Philosophical Analysis of the Master Argument*), by showing that some reconstructions of the Master Argument allowed to be interpreted in the branching time structures. This is the main argument to prove that despite the acceptance of Diodorus' notions, the Aristotelian Sea-Battle Tomorrow may still happen or not happen. On the one hand, we focus on problems of logical tools of temporal logic and logical structures of time involved in the reconstructions of Diodorus' argument. We mainly compare Jarmużek's proposal with another reconstruction, also founded upon positional logic. On the other hand, we show what kind of structures must be assumed if the inferences are valid. This enables us to discuss branching and non–branching time structures that correspond to philosophical deterministic and indeterministic views on the nature of time. We conclude that logical analysis of the Master Argument raises some methodological hints for the joint use of historical and logical analysis.

Keywords: Diodorus Cronus, Master Argument, branching time, positional logic, determinism, indeterminism

Introduction

The field of perennial philosophical inquiries into time and determinism stands out as a remarkable intersection of logic, philosophy, and other branches of human knowledge. Within the Western tradition, these inquiries have been the subject of discussions since the time of Aristotle, who provided a vivid example in his work.

Written around 350 BC, the work known as *De Interpretatione* introduced a problem that has exerted a profound and enduring influence on Western philosophy for millennia. Referred to as the Sea-Battle or Sea-Fight Tomorrow problem, it revolves around the truth values of statements concerning future states of affairs, the *futura contingentia*. The issue has been widely discussed and referenced by many authors, demonstrating its long-lasting influence on philosophical discourse.

While modern logic in the late 19th century did not primarily address the formal exploration of time and determinism, significant advancements emerged in the 1920s and beyond. Notably, Jan Łukasiewicz's engagement with Aristotle's logic, alongside a renewed focus on philosophical arguments concerning time and modality, played a crucial role in this development[1]. In hindsight, these contributions have had a profound impact on discussions surrounding determinism, time, and modalities, leading to the creation of temporal logic and the emergence of many-valued logic[2].

This paper provides an overview of a logico–philosophical problem known as the Master Argument, historically attributed to the Greek philosopher Diodorus Cronus. Diodorus aimed to demonstrate the validity of the temporally defined modalities of necessity and possibility through his argument, which adopted a deterministic form, in contrast to Aristotle's indeterministic approach. As the original argument has been lost, numerous contemporary logicians have attempted to reconstruct it using modern logical tools. In this paper, we will explore two formal reconstructions of the argument in greater detail.

In the first section, we explain the fundamental philosophical disparities between Aristotle's and Diodorus' perspectives by highlighting their core assumptions. Moving forward, the second section introduces essential concepts and basic definitions that are necessary for formal frameworks for reconstructions of the Master Argument. The third section presents the core aspects of positional logic. Subsequently, in the fourth section, we present Rescher's formal reconstructions of the argument. The fifth section presents Jarmużek's novel interpretation of the Master Argument.

[1] Łukasiewicz Jan, "On Determinism", in: *Selected Works*, ed. L. Borkowski (Amsterdam: North-Holland, 1979).

[2] Müller Thomas, "Time and Determinism", *J Philos Logic* 44 (2015).

Our particular focus lies on Jarmużek's proposition, aiming to demonstrate that certain reconstructions of the Master Argument embrace branching time structures. These structures align with the core concept of indeterminism by considering alternative future possibilities. This line of reasoning establishes that, despite endorsing Diodorus' ideas, the Aristotelian Sea-Battle Tomorrow may both occur or not occur. In the sixth section, we contend that the issues raised by the Master Argument remain relevant in the contemporary determinism/indeterminism debate. Lastly, we conclude with a brief reflection on the joint use of historical and logical analysis that can benefit philosophical inquiries.

1. Debate between indeterminism and determinism: ancient roots

The Sea-Battle or Sea-Fight Tomorrow problem traces its origins back to the ninth chapter of *De Interpretatione*, where it is articulated in the following passage[3]:

> For example, it would be necessary that a sea-fight should neither take place nor fail to take place on the next day. These awkward results and others of the same kind follow if it is an irrefragable law that of every pair of contradictory propositions [...] one must be true and the other false, and that there are no real alternatives, but that all that is or takes place is the outcome of necessity.

If we consider a future event such as sea-battle tomorrow, taking into account the above quotation clearly shows that according to Aristotle, the law of an excluded middle holds. Consequently, the sentences concerning the sea-battle tomorrow can only take the following forms:

A1 Tomorrow there will be a sea-battle.

A2 Tomorrow there will not be a sea-battle.

The problem with the sea-battle is centred on the question of whether statements about future events can be definitively classified as either true or false in the present. The solution of this problem given by Aristotle represents one of the main sides of the discussion. According to him, the fact that the sea-battle tomorrow will or will not happen is not determined before that specified day. Thus, it becomes apparent that

[3] Aristotle, "On Interpretation", trans. E. M. Edghill, in: *The Works of Aristotle*, ed. W. D. Ross, J. A. Smith. Vol 1 (London: Oxford University Press, 1995), 56.

the underlying problem extends far beyond a mere logical inquiry. It investigates a deeper ontological consideration, exploring whether a sentence pertaining to a future event is predetermined or remains indeterminate until the event unfolds.

Diodorus Cronus (ca. 340–280 BC), a prominent figure from the Megarian school of logic, offered a distinctly contrasting perspective on the matter. While Aristotle advocated for an open and indeterminate future, Diodorus tackled the issue from a different angle, seemingly attempting to justify the notion that future events are predetermined and inevitable. He discussed the same philosophical problem as Aristotle, explicitly referring to three aspects: the truth value, time and modality.

In the so–called Master Argument, Diodorus Cronus introduced a set of three theorems that were commonly accepted at the time. The combination of those three theorems, in Diodorean argumentation, demonstrated that one of them has to be false. The outcome of his argument was the formulation of the following conclusion:

DC There is no such proposition that is possible, but which is neither true now nor it will be true in some future time.

Due to the above result, Diodorus' Master Argument has become well-known as a key argument for logical determinism or fatalism.

Tomasz Jarmużek's work offers a conceptual framework for discussing both Aristotle's and Diodorus' views on the sea-battle problem[4]. He identified crucial similarities between the two perspectives and analyzed their key philosophical problems, ultimately presenting a new interpretation of the Master Argument. In what follows, we deeply rely on Jarmużek's insights on the debate between Aristotle and Diodorus.

Let us start by evoking one of the Aristotelian premises adopted in the reasoning. In the opening lines of the ninth chapter of *De Interpretatione*, Aristotle declared that in the case of something which is or has taken place, propositions must be true or false[5]. In this way, Aristotle highlighted two very important facts. Firstly, he emphasized the idea that every proposition in a logical sense has a specified value from the only two possible – truth and falsehood. This principle, known as the principle of bivalence, serves as the foundation for modern propositional logic. Secondly, Aristotle asserted that this principle can be universally applied, without any limitations, to statements concerning both the past and the present. However, what deserves attention is that, according to Aristotle, the principle does not apply to statements about the future.

[4] Jarmużek Tomasz, *On the Sea Battle Tomorrow That May Not Happen. A Logical and Philosophical Analysis of the Master Argument* (Berlin–Warsaw: Peter Lang Publishing, 2018).

[5] Aristotle, "On Interpretation", 54.

Besides the mentioned principle, the huge role in the formulation of Aristotle's and Diodorus' arguments had definitions of modalities. In Aristotle's work, we can find one of the earliest definitions of necessity in the following passage[6]:

> If a thing is white now, it was true before to say that it would be white, so that of anything that has taken place it was always true to say 'it is' or 'it will be'. But if it was always true to say that a thing is or will be, it is not possible that it should not be or not be about to be. For when a thing cannot not come to be, it is impossible that it should not come to be, and when it is impossible that it should not come to be, it must come to be.

Considering the aforementioned quotation, it becomes apparent that the concept of necessity, as discussed by Aristotle, is interpreted as a temporal necessity. By examining this temporal necessity, we can understand Aristotle's claim that the principle of bivalence cannot hold as an absolute truth when applied to statements concerning the future. In other words, it is not valid to claim that affirmations or denials must be either true or false[7].

The concept of *necessity* employed in the cited passages carries a more profound significance beyond its purely logical connotation. According to Aristotle, this notion can encompass both the state of affairs or truth-making objects, as well as the sentences or propositions themselves. Aristotle did not consider the problem only from the logical standpoint, but also from an ontological perspective. Justification for such a claim could be the following citation:

> Further, it makes no difference whether people have or have not actually made the contradictory statements. For it is manifest that the circumstances are not influenced by the fact of an affirmation or denial on the part of anyone.

Accepting this premise would result in an ontological state in which all events are predetermined. This metaphysical perspective is commonly referred to as *determinism*, as exemplified in the following passage by Aristotle[8]:

> Now if this be so, nothing is or takes place fortuitously, either in the present or in the future, and there are no real alternatives; everything takes place of necessity and is fixed.

[6] Ibidem, 55.
[7] Ibidem.
[8] Ibidem.

Aristotle was aware of these conclusions. On the other hand, removing the principle of bivalence from the axiomatic system would allow some sentences to be neither true nor false at the same time. Which, from Aristotle's perspective, would be a position impossible to defend. In his attempt to avoid the deterministic implications while upholding the principle of bivalence, Aristotle made modifications to the latter. This can be observed in the following passage, where he devised a solution to address the challenge posed by the *futura contingentia*[9]:

> A sea battle must either take place tomorrow or not, but it is not necessary that it should take place tomorrow, neither is it necessary that it should not take place, yet it is necessary that it either should or should not take place tomorrow.

Aristotle claimed that the principle of bivalence is in fact universally true, implying that one of two contradictory statements about the future must be true. However, he emphasized that the determination of which statement is true remains undetermined until the future unfolds. Thus, Aristotle acknowledged that we must suspend judgment on the truth value of these contradictory statements until the specific moment arrives. The position of Aristotle could be interpreted as an argument for ontological or logical indeterminism.

In summary, Aristotle's resolution to the Sea-Battle Tomorrow problem can be encapsulated in the three sentences presented below and recreated using the formalism of modern logic. By $\vdash$, we mean that the formula immediately following the symbol can be proven based on a given logical system.

1. There exists a genuine alternative regarding the fact that the sea–battle will or will not happen. $\vdash Ar1 \vee Ar2$
2. The sea-battle realization tomorrow is not necessary, nor is its non–occurrence tomorrow necessary. $\nvdash \Box Ar1 \vee \Box Ar2$
3. However, it is necessary that the sea-battle either occurs or does not occur tomorrow. $\vdash \Box(Ar1 \vee Ar2)$

A reflection on Aristotle's view raises questions about the historical connection between Aristotle's reasoning and that of Diodorus. While opinions differ on the direct influence Aristotle had on Diodorus, it is undeniable that a theoretical and substantial relationship existed between the two positions in ancient times.

Indeed, from a philosophical standpoint, both perspectives bear a profound connection to one another as they research the same subject matter, the interplay between

[9] Ibidem, 57.

logical values, time, and modalities. Moreover, these viewpoints are often regarded as integral components within the larger discourse of the indeterminism (Aristotle) versus fatalism (Diodorus) debate, further highlighting their shared context and relevance.

While limited details are available regarding Diodorus' Master Argument, we possess knowledge of its premises and conclusion. Epictetus, in his work, provides insights into the three premises that were formulated[10]:

D1 Everything past must of necessity be true.

D2 An impossibility does not follow a possibility.

D3 A thing is possible which neither is nor will be true.

According to Diodorus, the three above propositions are contradictory. This resulted in proving the falsity of the third sentence and accepting the plausibility of the first two premises.

The conclusion of the Master Argument arises from the negation of the third premise. In addition to the conclusion, the argument and its premises yield specific definitions of temporal possibility. According to these definitions, a sentence is only considered possible if it is true or will be true in the future. Conversely, the concept of necessity pertains to what, being true, cannot be false. Diodorus perceived possibility and necessity as interdefinable concepts when combined with negation. Based on the above definitions, we can neutrally establish that something is necessary if and only if it cannot be otherwise, and something is possible if and only if it is not necessary for it to be otherwise.

The relationship between necessity and truth value, acknowledged by both Aristotle and Diodorus, can be referred to as past determinism and present determinism. These positions imply the non–reversibility of history, meaning that if a sentence pertains to the past or the present, its logical value remains unchanged over time. However, unlike Aristotle, Diodorus incorporated in his argument the shared belief of the Megarians and Stoics that not only sentences about the past or the present have determined logical values, but also sentences about the future.

2. Formal representation of time

In the latter half of the 19th century, two prominent philosophers, Jan Łukasiewicz and Arthur Prior, questioned the general applicability of the principle of bivalence.

[10] Epictetus, *The Discourses*, trans. G. Long (London: George Bell and Sons, 1903), 173–174.

Łukasiewicz developed systems of multivalued logics that introduced additional truth values to account for undetermined propositions[11]. On the other hand, Prior adopted a different approach, constructing a tense logic focused on temporal propositions. By distinguishing atemporal from temporal truth values, he allowed for cases where a sentence could be true at one point in time and false at another. He built upon the modal systems formulated by Lewis, representing basic grammatical tenses through specialized operators. Tense logic employs a set of operators (*P, H, G, F*) that serve as sentence-forming functors, allowing us to express different possibilities within a given context.

To illustrate Prior's tense operators, let us take the sentence "there is a sea-battle". This sentence can be expressed in different ways to convey various temporal perspectives. By utilizing Prior's tense operators, we can rephrase it in the following five ways, capturing different temporal modifications:

(Pp) It has been the case that there is a sea-battle.

(*Hp*) It has always been the case that there is a sea-battle.

(*p*) There is a sea-battle.

(*Gp*) It will always be the case that there is a sea-battle.

(*Fp*) It will be the case that there is a sea-battle.

From a philosophical standpoint, Prior recognized that the Sea-Battle Tomorrow problem gives rise to a semantic question regarding the nature of the future operator *F*. If we interpret this operator within possible world semantics, an important question follows regarding the properties of such a structure. Specifically, whether such a structure is a linear structure or is it a branching structure.

The conceptualization of moments as a linear order would imply the existence of a single possible future course of moments with respect to any given moment, aligning with the notion of determinism. However, considering moments as forming a tree-like partial order would capture the idea of an open future and indeterminism. Saul Kripke's proposal to represent branching time as a logical system, which was recognized and further developed by Prior, provided a means to articulate this concept. Kripke's explicit idea on branching time is significant as it marks the first-ever presentation of such a logical system, as acknowledged by Prior[12].

[11] Łukasiewicz Jan, "On the Principle of the Excluded Middle", *History and Philosophy of Logic* 8(1987): 67–69.

[12] Øhrstrøm Peter, Hasle Per F. V., *Temporal Logic From Ancient Ideas to Artificial Intelligence* (Dordrecht: Kluwer Academic Publishers, 1995), 189–191.

This observation is not without significance. The reconstructions of the Master Argument can be presented in various formal frameworks, each of which can be interpreted within different mathematical structures. Of particular interest is the possibility of reconstructing the Master Argument in a formal framework that can be interpreted within a branching time structure. Such results would be intriguing because branching structures are typically associated with indeterminism.

Due to the historical puzzles, there were many attempts to provide a reconstruction and a sufficient formal framework of the Master Argument. Such attempts were made in modern times not only by Prior[13], but also, among others, by Rescher[14] or Michael[15]. In our formal reconstruction, we consider propositions as the bearers of logical values, since they express truth or falsehood. True propositions must state or indicate something. Various kinds of things can make a proposition true or false. Among many proposals for the so-called truth-makers, or verifiers, we focus our attention on the concept of state of affairs.

From a temporal point of view, the world can be perceived as a history of consecutive states of affairs. Given state of affairs is taking place if situated somewhere in the history of the world. The history of the world could be understood as the set of temporal sections of the world, called the states of the world. In case of determinism, each proposition has one and constant logical value throughout the whole history. Thus, each proposition has the same logical value in any given cross-section of the history of the world.

In the deterministic view, time is represented by the line, that is, the time is complete in every moment and with the determined future. On the other hand, in the indeterministic view, the situation is quite different. There may exist propositions that have different logical values in at least two distinct cross-sections of the history of the world. The logical representation of such a situation could be based, for example, on the adoption of many-valued logic or a logic that allows one to formulate a time-determined expression that has a logical value dependent on the context. In the indeterministic view, the time is represented by a branching structure that has different alternative futures from the perspective of 'now'.

In order to address the issue of the logical value of future sentences, when using formal tools, it is essential to establish what we mean by a time structure and what properties can be used to describe it. Given that all of the mentioned formal reconstructions utilize a time structure in some form, it is essential to have a proper

[13] Prior Arthur, *Past, Present and Future* (Oxford: Clarendon Press, 1967).

[14] Rescher Nicholas, "On the Logic of Chronological Propositions", *Mind* 77 (1966): 75–96.

[15] Michael Frederick Seymour, "What is the Master Argument of Diodorus Cronus?", *American Philosophical Quarterly* 13 (1976): 229–235.

understanding of this concept for our purposes. To achieve this, we will provide a few precise mathematical definitions pertaining to properties of relations, such as linear or partial ordering.

First and foremost, let us examine four types of ordering relations: partial order, linear order, strict order, and strict linear order. Each of these ordering relations can be characterized by specific properties, including reflexivity, irreflexivity, symmetry, asymmetry, antisymmetry, transitivity, and totality.

Definition 2.1 (Partial Order)**.** $\mathbf{R} \subseteq \mathbf{X}^2$ is a relation of partial order iff for any $x, y \in \mathbf{X}$ it meets the following conditions:

1. $\mathbf{R}(x,x)$;
2. $\mathbf{R}(x,y)\, and\, \mathbf{R}(y,x) \Rightarrow x = y$;
3. $\mathbf{R}(x,y)\, and\, \mathbf{R}(y,z) \Rightarrow \mathbf{R}(x,z)$.

Definition 2.2 (Linear Order)**.** $\mathbf{R} \subseteq \mathbf{X}^2$ is a relation of linear order iff it is the partial order relation and also for any $x, y \in \mathbf{X}$ it meets the condition of totality: $\mathbf{R}(x,y)\, or\, \mathbf{R}(y,x)$.

Definition 2.3 (Strict Order)**.** $\mathbf{R} \subseteq \mathbf{X}^2$ is a relation of strict order iff for any $x, y \in \mathbf{X}$ it meets the following conditions:

1. $\sim \mathbf{R}(x,x)$
2. $\mathbf{R}(x,y)\, and\, \mathbf{R}(y,z) \Rightarrow \mathbf{R}(x,z)$.

Definition 2.4 (Strict Linear Order)**.** $\mathbf{R} \subseteq \mathbf{X}^2$ is a relation of strict linear order iff it is a strict order relation and also for any $x, y \in \mathbf{X}$ it meets the following condition of totality: $\mathbf{R}(x,y)\, or\, \mathbf{R}(y,x)\, or\, x = y$.

We will use the notation ≤ to represent the relation of a partial order, and the symbol < to represent the relation of a strict order. This choice of notation aligns with the intuitive understanding of these relations, where ≤ captures the notion of *being earlier or present to*, and < represents the relation of *being strictly earlier than*.

On the basis of the previously described concepts, we can establish the notion of a time structure as a relational system.

Definition 2.5 (Time structure)**.** Time structure is a relational system $\langle \mathbf{T}, \leq \rangle$, where $\mathbf{T} \neq \varnothing$ and $\leq \subseteq \mathbf{T}^2$ is a relation, that meets the following conditions: $\forall_{t_1,t_2,t_3 \in \mathbf{T}}(t_1 \leq t_3 \text{ and } t_2 \leq t_3 \Rightarrow t_1 \leq t_2 \text{ or } t_2 \leq t_1)$ and $\forall_{t_1,t_2 \in \mathbf{T}} \exists_{t_3 \in \mathbf{T}}(t_3 \leq t_1 \text{ and } t_3 \leq t_2)$.

The elements of set $\mathbf{T}$ will be referred to as moments of time or simply moments. With this interpretation, we can provide explanations for the two conditions mentioned in the definition of a time structure. The first condition excludes cases where there are multiple pasts, ensuring a unique past in the time structure. The second condition excludes cases where the time structure lacks uniformity, ensuring a consistent and coherent ordering of moments.

The concept of a time structure allows us to define a branch as a maximal, nonempty, linear substructure within it.

Definition 2.6 (Branch of time structure)**.** The branch of time structure $\langle \mathbf{T}, \leq \rangle$ is a maximal nonempty subset $\mathbf{T}_1 \subseteq \mathbf{T}$ that meets the condition: $\forall_{t_1, t_2 \in \mathbf{T}} (t_1 \leq t_2 \text{ or } t_2 \leq t_1)$.

From the definition of a time structure, several interesting facts can be identified. Firstly, the cardinality of the set of moments is not specified, allowing for time structures with different cardinalities. This implies that the number of moments in a time structure can vary. Secondly, since the relation $\leq$ is interpreted as a relation of *being earlier or present to*, its converse ($\leq^{-1}$) naturally represents the relation of *being later than*.

What is most important for our aim is that in the context of the Sea-Battle Tomorrow, interesting properties of non–linear time structures can be analyzed. Such structures are called branching structures, since they have more than one branch of time. Within these structures, there can be found three different points, of which the first one precedes the other two that are not mutually comparable.

Definition 2.7 (Branching into future)**.** Let $\langle \mathbf{T}, \leq \rangle$ be a time structure. It is said that the structure is branching into future iff it meets the following condition: $\exists_{t_1, t_2, t_3 \in \mathbf{T}} (t_1 \leq t_2 \text{ and } t_1 \leq t_3 \text{ and } \sim t_2 \leq t_3 \text{ and } \sim t_3 \leq t_2)$.

The problem of a truth value of sentences about the future was analyzed and reconstructed using many different logical frameworks, such as many-valued logic, modal logic, tense logic, and positional logic. While we have already briefly focused on the tense logic, in the next section we will dwell on the positional one and then, equipped with all basic notions and its definitions, we will be able to investigate two formal attempts to reconstruct the Master Argument.

3. Positional logic

Positional logic is a family of logical systems characterized by the incorporation of a realization operator (denoted as $\mathbb{R}$) that allows for the evaluation of the logical value of an expression within a specific context. The expression constructed using the realization operator takes the form $\mathbb{R}_t(A)$, where it represents the statement that "A is realized within the context t".

The inception of positional logic dates back to 1947 when Jerzy Łoś, a Polish logician, introduced the pioneering system in his works[16]. This marked a significant milestone not only in logical theory but also in the realm of temporal logic, as the interpretation of context within positional logic was inherently temporal[17]. Subsequently, the study of positional logics was further advanced by notable scholars such as Arthur Prior, who carried on Łoś's legacy, Nicholas Rescher, who formulated systems with broader interpretations known as Topological Logic, and Tomasz Jarmużek, who promoted research on positional logic within Toruń Logic Group.

Positional logic could have many interpretations for the context variables. In fact, context symbols can be interpreted in a very abstract way. In the first works devoted to positional logics, they were interpreted in a temporal and epistemic way. In the later works, context symbols denoted such entities as spatial coordinates and possible worlds. Since the newest developments, those symbols could denote any possible type of context. For instance, $\mathbb{R}_t(\text{Socrates is sleeping})$, where t corresponds to a room in Socrates' house. In this example, the expression could be read as: *sentence 'Socrates is sleeping' comes true in location t*. Although, in our interpretation, positional constants and variables will be interpreted temporally.

Remark. The language of positional logic is constructed from the following sets of symbols:

- logical constants $LC = \{\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \mathbb{R}, \forall, \exists\}$,
- time constants $TC = \{n, a, b, c, ...\}$,
- sentential variables $SV = \{p, q, r, s, ...\}$,
- time variables $TV = \{t_1, t_2, t_3, ...\}$,
- auxiliary symbols $AS = \{(,), [,]\}$.

We will define the set of indices $I = TC \cup TV$.

Definition 3.1. The expressions of positional logic are formed in the following way. The set of formulas $\mathbf{FOR_R}$ is the smallest set X such that:

[16] Łoś Jerzy, "Podstawy Analizy Metodologicznej Kanonów Milla" (Foundations of Methodological Analysis of Mill's Canons), *Annales Universitatis Mariae Curie–Skłodowska* 2.5. F(1947): 269–301; Łoś Jerzy, "Logiki Wielowartościowe a Formalizacja Funkcji Intensjonalnych" (Multivalued Logics and Formalization of Intensional Functions), *Kwartalnik Filozoficzny* 17 (1/2) (1948): 59–78.

[17] Jarmużek Tomasz, Tkaczyk Marcin, "Jerzy Łoś Positional Calculus and the Origin of Temporal Logic", *Logic and Logical Philosophy* 28 (2019): 259–276.

- $\mathbb{R}_t(A) \in X$ if $A \in \mathbf{FOR_{CPL}}$ and $t \in I$, where **CPL** denotes the Classical Propositional Logic,
- $\neg A$, $A \wedge B$, $A \vee B$, $A \rightarrow B$, $A \leftrightarrow B \in X$ if $A, B \in X$,
- $\forall_t A, \exists_t A \in X$ if $A \in X$ and $t \in TV$.

In the context of introducing axiom schemes for positional logic, we will focus on the fundamental component of Łoś's axioms, which have also been employed in the systems developed by Prior, Rescher, and Jarmużek.

Axiom 1. $\mathbb{R}_t(\neg A) \leftrightarrow \neg\mathbb{R}_t(A)$

This axiom states that the negation of expression $\neg A$ within context t is equivalent to the absence of fulfillment of expression A in the same context. In essence, this axiom captures the principle of bivalence.

Axiom 2. $\mathbb{R}_t(A \rightarrow B) \rightarrow (\mathbb{R}_t(A) \rightarrow \mathbb{R}_t(B))$

This axiom expresses the relation between the realization operator and the implication.

Axiom 3. $\forall_t \mathbb{R}_t(A) \rightarrow A$

The final axiom states that if a given sentence is realized at every context, then it can be accepted without any reference to any specific context. This axiom captures the idea that when a sentence holds true under all conditions, its truth value is independent of any particular circumstances.

Both systems developed by Prior and Rescher rely on these three axiom schemes as foundational principles. Moreover, they employ the following rule, commonly used in the systems of positional logic.

Remark. Let $A \in \mathbf{FOR_{CPL}}$: $\dfrac{\vdash A}{\vdash \mathbb{R}_t(A)}$

This rule states that for any formula A expressed in the language of classical propositional logic, which is also a theorem in our system, the formula $\mathbb{R}_t(A)$ is also a theorem.

4. Rescher's reconstruction

In 1966, Nicholas Rescher presented a reconstruction of the Master Argument using positional logic[18]. The objective of his reconstruction was to meet several criteria: incorporating the foundational concepts of Diodorus' original argument, being

[18] Rescher Nicholas, "On the Logic of Chronological Propositions", *Mind* 77 (1966): 75–96; Rescher Nicholas, Urquhart Alasdair, *Temporal Logic* (Wien–New York: Springer Verlag, 1971).

logically persuasive according to contemporary theoretical standards, and accomplishing the intended objectives set forth by Diodorus, that is, obtaining Diodorus' conclusion.

Rescher based his reconstruction on first-order logic (**FOL**), supplemented by the introduction of the realization operator and the first axiom presented in the previous section. In addition to the realization operator, he introduced two relational symbols through the following definitions:

Definition 4.1 ($\neq$)**.** $\forall_{t_1,t_2\in\mathbf{T}}(t_1 \neq t_2 \leftrightarrow \neg t_1 = t_2)$

Definition 4.2 ($<$)**.** $\forall_{t_1,t_2\in\mathbf{T}}(t_1 < t_2 \leftrightarrow t_1 \neq t_2 \wedge t_1 \leq t_2)$

In addition to the aforementioned axioms and definitions, Rescher postulated a specific structure of time. According to his perspective, time must adhere to the conditions of a partially linear relational structure without a final element. This means that the time in Rescher's reconstruction has no end. These properties can be expressed formally as follows:

Definition 4.3 (Rescher's time structure)**.** The time structure $\langle\mathbf{T},\leq\rangle$ assumed by Rescher has the following four conditions of partial linear time structure without the last element:

1. $\forall_{t\in\mathbf{T}}(t \leq t)$ [**T1**]
2. $\forall_{t_1,t_2,t_3\in\mathbf{T}}(t_1 \leq t_2 \text{ and } t_2 \leq t_3 \Rightarrow t_1 \leq t_3)$ [**T2**]
3. $\forall_{t_1,t_2\in\mathbf{T}}(t_1 \leq t_2 \text{ or } t_1 = t_2 \text{ or } t_2 \leq t_1)$ [**T3**]
4. $\forall_{t_1\in\mathbf{T}}\exists_{t_2\in\mathbf{T}}(t_1 < t_2)$, where $t_1 < t_2 \Leftrightarrow t_1 \leq t_2 \text{ and } \sim(t_1 = t_2)$ [**T4**]

Rescher adopted the classical Aristotelian definition of modalities, that is expressed in the following formulas:

Definition 4.4 (Aristotelian modalities)**.**

1. $\forall_{t\in\mathbf{T}}(\Diamond_t A \leftrightarrow \neg\Box_t \neg A)$ [**M1**]
2. $\forall_{t\in\mathbf{T}}(\Box_t A \leftrightarrow \neg\Diamond_t \neg A)$ [**M2**]

Moreover, there is an additional theorem concerning mentioned modalities. This theorem, with the use of constant denoting 'now' moment, states that every sentence before now is true:

Definition 4.5. $\exists_{t\in\mathbf{T}}(n \leq t \wedge \mathbb{R}_t(A)) \to \Diamond_n A$ [**A1**]

Having that said, we can reconstruct Diodorus' premises in the language of Rescher's formal system.

Definition 4.6 (Diodorus' First Premise)**.** If sentence A is realized at time t_1, then at any later time it is necessary.

$$\forall_{t_1,t_2\in\mathbf{T}}((\mathbb{R}_{t_1}(A)\wedge t_1<t_2)\rightarrow\Box_{t_2}A) \qquad [\mathbf{D1}]$$

Definition 4.7 (Diodorus' Second Premise)**.** If sentence A is possible at time t_1, then it is possible at a later time.

$$\forall_{t_1,t_2\in\mathbf{T}}((\Diamond_{t_1}A\wedge t_1<t_2)\rightarrow\Diamond_{t_2}A) \qquad [\mathbf{D2}]$$

Definition 4.8 (Diodorus' Third Premise)**.** There is such a sentence p_0 that is possible now, but neither now nor at any later time it is realized.

$$\forall_{t_1\in\mathbf{T}}(\Diamond_n p_0\wedge(n\leq t_1\rightarrow\neg\mathbb{R}_{t_1}(p_0))) \qquad [\mathbf{D3}]$$

The reconstruction of the whole Master Argument proposed by Rescher has the following form:

1. $(\mathbb{R}_{t_1}(\neg A)\wedge t_1<t_2)\rightarrow\Box_{t_2}\neg A$ **D1**
2. $\neg\Box_{t_2}\neg A\rightarrow\neg(\mathbb{R}_{t_1}(\neg A)\wedge t_1<t_2)$ 1, **CPL**
3. $\Diamond_{t_2}A\leftrightarrow\neg\Box_{t_2}\neg A$ **M1**
4. $\Diamond_{t_2}A\rightarrow\neg\Box_{t_2}\neg A$ 3, **CPL**
5. $\Diamond_{t_2}A\rightarrow\neg(\mathbb{R}_{t_1}(\neg A)\wedge t_1<t_2)$ 2, 4, **CPL**
6. $\neg(\mathbb{R}_{t_1}(\neg A)\wedge t_1<t_2)\rightarrow(t_1<t_2\rightarrow\neg\mathbb{R}_{t_1}(\neg A))$ **CPL**
7. $\Diamond_{t_2}A\rightarrow(t_1<t_2\rightarrow\neg\mathbb{R}_{t_1}(\neg A))$ 5, 6, **CPL**
8. $(\Diamond_{t_2}A\wedge t_1<t_2)\rightarrow\neg\mathbb{R}_{t_1}(\neg A)$ 7, **CPL**
9. $\neg\mathbb{R}_{t_1}(\neg A)\rightarrow\mathbb{R}_{t_1}(A)$ **Axiom 1**
10. $(\Diamond_{t_2}A\wedge t_1<t_2)\rightarrow\mathbb{R}_{t_1}(A)$ 8, 9, **CPL**
11. $\Diamond_n p_0$ **D3, CPL**
12. $\exists_{t_1\in\mathbf{T}}(n<t_1)$ **T4, FOL**
13. $n<a$ 12, **12, FOL**
14. $((\Diamond_n p_0\wedge n<t_1)\rightarrow\Diamond_{t_1}p_0)$ **D2, FOL**
15. $((\Diamond_n p_0\wedge n<a)\rightarrow\Diamond_a p_0)$ **14, FOL**

16. $\Diamond_n p_0 \wedge n < a$ — 11, 13, **CPL**
17. $\Diamond_a p_0$ — 15, 16**CPL**
18. $(\Diamond_a p_0 \wedge t_1 < a) \rightarrow \mathbb{R}_{t_1}(p_0)$ — 10, **FOL**
19. $(\Diamond_a p_0 \wedge n < a) \rightarrow \mathbb{R}_n(p_0)$ — 18, **FOL**
20. $\Diamond_a p_0 \wedge n < a$ — 17, 13, **CPL**
21. $\mathbb{R}_n(p_0)$ — 19, 20, **CPL**
22. $n \leq n$ — **T1, FOL**
23. $n \leq t_1 \rightarrow \neg\mathbb{R}_{t_1}(p_0)$ — **D3, CPL**
24. $n \leq n \rightarrow \neg\mathbb{R}_n(p_0)$ — 23, **FOL**
25. $\neg\mathbb{R}_n(p_0)$ — 24, 22, **CPL**, *Contradiction* : 21, 25

The presented reconstruction leads to several conclusions. Firstly, it addresses the relationships between modalities, specifically the definitions of possibility and necessity.

Corollary 4.1 (Definition of modalities)**.**

1. $\Diamond_n A \leftrightarrow \exists_{t_1 \in \mathbf{T}}(n \leq t_1 \wedge \mathbb{R}_{t_1}(A))$
2. $\Box_n A \leftrightarrow \forall_{t_1 \in \mathbf{T}}(n \leq t_1 \rightarrow \mathbb{R}_{t_1}(A))$

Referring to these definitions, we can state that a sentence is considered possible (necessary) if and only if it is realized in the present or any future moment (present and any future moment). Secondly, examining the prescribed properties for the time structure, we observe that the proposed view of time is characterized as a partially linear ordered structure without the last element, although it does not necessitate specific assumptions regarding the first element, discreteness, density, or continuity of time. The final and most intriguing conclusion is that the axioms do not preclude the possibility of a branching time structure.

However, if we consider branching, partially ordered structure, each parallel branch of time would share identical truth values for corresponding sentences, leading to what could be termed a "trivially branching" time structure. In this particular ontology of time, determinism would persist, despite the presence of a branching time structure, due to the implicit assumption of determinism within Rescher's postulates governing the time structure.

5. The Logic of a Coordinated Time Structure: Jarmużek's reconstruction

In his book, Jarmużek put forward a novel approach to the task of reconstructing the Master Argument[19]. He introduced a formal framework that builds upon the positional logic employed by Rescher but in an expanded form. This system can be regarded as a logic of coordinated time structure, offering a unique perspective on the problem at hand.

Jarmużek's framework is constructed upon three foundational assumptions. First and foremost, it assumes the viability of performing arithmetic operations on temporal moments. Secondly, it posits the existence of a specific temporal point, commonly identified as the "present", within each time structure. Lastly, the framework assumes that the interpretation of the realization operator is adapted to the branching time structure.

The third assumption that sets Jarmużek's framework apart from classical positional logic is worth further elaboration. In this particular system, expressions involving the realization operator are interpreted as follows: the formula $\mathbb{R}_t(A)$ is deemed satisfied at the time point t if and only if there exists a branch of time to which t belongs, and within that branch, there exists a time point b where the expression A evaluates to a specific truth value.

Remark. The language of the Logic of a Coordinated Time Structure is constructed from the following symbols:

- logical constants $LC = \{\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \mathbb{R}, \forall, \exists\}$,
- point constants $PC = \{n, a, b, c, \ldots\}$,
- distance constants $DC = \{f, g, h, \ldots\}$,
- sentential variables $SV = \{p, q, r, s, \ldots\}$,
- point variables $PV = \{t_1, t_2, t_3, \ldots\}$,
- distance variables $DV = \{d_1, d_2, d_3, \ldots\}$,
- auxiliary symbols $AS = \{(,), [,]\}$,
- operational symbols $OS = \{-, +\}$.

We can construct a set of indices I as a smallest set of expressions X such that:

1. $PC \cup PV \subseteq X$,

[19] Jarmużek Tomasz, *On the Sea Battle Tomorrow That May Not Happen. A Logical and Philosophical Analysis of the Master Argument* (Berlin–Warsaw: Peter Lang Publishing, 2018).

2. $\{x : x = k + j \text{ or } x = k - j\} \subseteq X$, where $k \in X$ and $j \in DC \cup DV$.

The process of constructing expressions in the logic of coordinated time follows a similar approach to that of standard positional logic, with one notable difference concerning the scope of quantifiers. In Rescher's positional logic, quantifiers were restricted to range over points of time exclusively. However, in Jarmużek's extended version, quantifiers have a broader scope, encompassing both points of time and distances within the temporal framework.

Remark. Expressions of the logic are formed from the dictionary in the following way. The set formulas $\mathbf{FOR}_{\mathbf{R}_n^+}$ is the smallest set X such that:

- $\mathbb{R}_i(A) \in X$ if $A \in \mathbf{FOR_{CPL}}$ and $i \in I$,
- $\neg A$, $A \wedge B$, $A \vee B$, $A \rightarrow B$, $A \leftrightarrow B \in X$ if $A, B \in X$,
- $\forall_j A, \exists_j A \in X$ for any $j \in PV \cup DV$ and $A \in X$.

The core part of the axiom system in the logic of coordinated time shares a similar form to that of standard positional logic. The first axiom scheme captures the same idea as the rule presented in the previous framework. This axiom can be formulated as follows:

Axiom 1. $\mathbb{R}_i(A)$, where $A \in$ **CPL**

According to this axiom, any theorem of classical logic is realized at every point of the time structure. It is worth noting that this axiom, in conjunction with the concept of realizability, imposes a certain requirement on the time structure. Specifically, it stipulates that from every point in the time structure, there must be an accessible point in time.

The second axiom scheme is analogous to the first axiom of the previous formal framework.

Axiom 2. $\mathbb{R}_i(\neg A) \leftrightarrow \neg\mathbb{R}_i(A)$

The third axiom scheme deals with the distribution of the realization operator over a conjunction. This formula is analogous to the second axiom of the previous system. It enables, along with the previous axiom scheme, to prove the distribution of the operator $\mathbb{R}$ across all the functors of classical logic.

Axiom 3. $\mathbb{R}_i(A) \wedge \mathbb{R}_i(B) \rightarrow \mathbb{R}_i(A \wedge B)$

The next two axiom schemes of the logic express the relationship between the realization operator and operational symbols. The fourth axiom scheme captures the idea that going back in the branching time structure does not alter the realization of the expression on the initial branch. The fifth axiom scheme states that if an expression is realized on a subsequent branch from the initial one, then it is also realized on the initial branch. These axioms are formulated as follows:

Axiom 4. $\mathbb{R}_k(A) \rightarrow \mathbb{R}_{k-j}(A)$

Axiom 5. $\mathbb{R}_{k+j}(A) \rightarrow \mathbb{R}_k(A)$

These two axioms characterize coordinated time structures. They establish that what is realized in time k is also realized in any time earlier than k. However, there is no requirement for it to be realized in moments later than k.

The following axiom captures the idea that if two point symbols refer to the same object, according to real number arithmetic, then the expression A is realized in both of those points.

Axiom 6. $\mathbb{R}_i(A) \rightarrow \mathbb{R}_j(A)$ as long as $i = j$ is true in real number arithmetic

The last axiom states that if an expression holds true for a given context and also holds true for any context that comes earlier or later by any amount of time, then that expression holds true in every moment of time.

Axiom 7. $\exists_{t_1} \forall_{d_1} (A(t_1 - d_1) \wedge A(t_1) \wedge A(t_1 + d_1)) \rightarrow \forall_{t_2} A(t_2)$

Although a thorough discussion of the interpretations of phrases in the language of the logic of coordinated time would involve introducing various additional concepts related to formula comprehension, we will concentrate on defining the truth value of a sentence of the form $\mathbb{R}_i(A)$:

Definition 5.1. We say that $\mathbb{R}_i(A)$ is true if and only if within the time structure there $\exists_{t \in T}(t \leq d(i)$ or $d(i) \leq t$ and $V(t, A) = 1)$.

In the constructed logic, the first six axioms hold true in any time structures that conform to this logic. However, the last axiom serves to differentiate linear structures from other types of structures.

Jarmużek's approach to the Master Argument bears resemblance to Rescher's reconstruction, but it is rooted in the language of the logic of coordinated time discussed earlier. Like Rescher, Jarmużek builds his reconstruction on first-order logic with the addition of the realization operator. However, Jarmużek's reconstruction diverges from Rescher's not only in terms of the language employed but also in certain implications it entails.

Of particular significance in Jarmużek's reconstruction is the departure from viewing time as a linear structure. Instead, temporal precedence of moments in Jarmużek's approach is represented through the addition or subtraction of distances within the coordinating structure. The relational symbol capturing this notion can be introduced as follows:

Definition 5.2. $\forall_{t_1,t_2\in\mathbf{T}}(t_1 < t_2 \Leftrightarrow \exists_{d\neq 0} f(t_1) + d = f(t_2))$

This definition states that if two distinct moments within a structure are related as $t_1 < t_2$, there exists a non-zero number d, referred to as the distance, such that adding d to the numerical representation of point t_1 yields the numerical representation of the subsequent point t_2.

Besides the logical axioms and conditions of an assumed time structure, Jarmużek takes into account philosophical assumptions that were employed in the reasoning, such as Aristotle's definitions of modalities:

Definition 5.3 (Aristotelian modalities)**.**

1. $\forall_{t\in\mathbf{T}}(\Diamond_t A \leftrightarrow \neg\Box_t \neg A)$ **[M1]**
2. $\forall_{t\in\mathbf{T}}(\Box_t A \leftrightarrow \neg\Diamond_t \neg A)$ **[M2]**

Moreover, we employ a following theorem analogous to the one presented for Rescher's reconstruction.

Definition 5.4. $\mathbb{R}_n(A) \vee \exists_d(\mathbb{R}_{n+d}(A)) \rightarrow \Diamond_n A$ **[A1]**

Having that said, we can reconstruct Diodorus' premises in the language of Jarmużek's formal system.

Definition 5.5 (Diodorus' First Premise)**.** If sentence A is realized before temporal point t, then at time t it is necessary

$\forall_{t\in\mathbf{T}}(\mathbb{R}_{t-d}(A) \rightarrow \Box_t A)$ **[D1]**

Definition 5.6 (Diodorus' Second Premise)**.** If sentence A is possible at time t_1, then it is possible at a later time

$\forall_{t_1,t_2\in\mathbf{T}}((\Diamond_{t_1} A \wedge t_1 < t_2) \rightarrow \Diamond_{t_2} A)$ **[D2]**

Definition 5.7 (Diodorus' Third Premise)**.** There is such a sentence p_0 that is possible now, but neither now nor at any later time it is realized

$\Diamond_n p_0 \wedge \neg\mathbb{R}_n(p_0) \wedge \neg\exists_d(\mathbb{R}_{n+d}(p_0))$ **[D3]**

As we can observe, the three Diodorean premises share a similar form to the formalization in Rescher's language, albeit with some differences. Specifically, the above formalization introduces the use of operational symbols.

Based on the previously provided definitions and axioms, we can now present Jarmużek's reconstruction of the Master Argument. The proof proceeds as follows:

1. $\mathbb{R}_{t-d}(\neg p_0) \rightarrow \Box_t \neg p_0$ **D1**
2. $\neg \Box_t \neg p_0 \rightarrow \neg \mathbb{R}_{t-d}(\neg p_0)$ 1, **CPL**
3. $\Diamond_t p_0 \leftrightarrow \neg \Box_t \neg p_0$ **M1**
4. $\Diamond_t p_0 \rightarrow \neg \mathbb{R}_{t-d}(\neg p_0)$ 3, 2, **CPL**
5. $\Diamond_t p_0 \rightarrow \mathbb{R}_{t-d}(p_0)$ 4, **Axiom 2, CPL**
6. $\Diamond_n p_0$ **D3, CPL**
7. $\neg \mathbb{R}_n(p_0) \wedge \neg \exists_d(\mathbb{R}_{n+d}(p_0))$ **D3, CPL**
8. $\Diamond_n p_0 \rightarrow \Diamond_{n+d} p_0$ **D2, FOL**
9. $\Diamond_{n+d} p_0$ 8, 6, **CPL**
10. $\Diamond_{n+d} p_0 \rightarrow \mathbb{R}_{(n+d)-d}(p_0)$ 5, **FOL**
11. $\mathbb{R}_{(n+d)-d}(p_0)$ 10, 9, **CPL**
12. $\mathbb{R}_n(p_0)$ 11, **Axiom 6**
13. $\neg \mathbb{R}_n(p_0)$ 7, **CPL**, $Contradiction: 12, 13$

Several conclusions can be drawn from the presented reconstruction. Firstly, the definitions of modalities within the framework are noteworthy.

Corollary 5.1 (Definition of modalities)**.**

1. $\Diamond_t A \leftrightarrow \mathbb{R}_t(A) \vee \exists_d(\mathbb{R}_{t+d}(A))$
2. $\Box_t A \leftrightarrow \mathbb{R}_t(A) \wedge \forall_d(\mathbb{R}_{t+d}(A))$

Based on this definition, it can be inferred that a sentence is considered possible (necessary) if and only if it is realized in the present or any future moment (present and any future moment). Secondly, considering the axioms, it is apparent that the presented view of time does not have a defined end or beginning. Furthermore, no assumptions are made regarding the discreteness, density, or continuity of time. The most intriguing conclusion is that the axioms do not rule out the possibility of a branching time structure.

This result distinguishes Jarmużek's reconstruction from other reconstructions using positional logic, as it allows us to interpret Diodorus' argument within a non–trivial branching time structure. This is unexpected because the argument itself is regarded as a defense of determinism. On the other hand, non-trivial branching time structures are typically employed to model an indeterministic ontology of time. Hence, this outcome is particularly surprising and unique.

6. Topicality of the Master Argument

Up to this point, our primary emphasis has been on examining the challenges posed by the logical tools of temporal logic and the logical structures of time inherent in the reconstructions of Diodorus' argument. Through a comparison between Jarmużek's proposition and Rescher's reconstruction, we have elucidated the types of structures that must be presupposed for the inferences of the Master Argument to hold validity. This analysis has enabled us to explore the formal attributes that align with philosophical perspectives on determinism and indeterminism regarding the nature of time.

Given the predominant focus of our paper on the logical analysis of the ancient philosophical problem, this section endeavors to broaden our perspective. Initially, we advocate for the joint use of historical and logical tools in such analyses. Subsequently, we briefly highlight that inquiries into the interplay between time and determinism, a subject of contemplation since Aristotle's era, have consistently remained enduring and perennial problems within philosophical discourse.

6.1. Neither historical scepticism nor sole formalism

From a historical perspective, our knowledge of Diodorus' philosophy is limited. The main sources of information about him come from doxography[20]. However, there are reliable testimonies that indicate Diodorus formulated concepts related to time and modality that are crucial for the logical reasoning we have discussed[21]. Without these testimonies, our previous discussion would lack substance.

It becomes evident that the Aristotelian Sea-Battle problem should also be understood in the context of the polemics surrounding Diodorus' reasoning. In fact, focusing on Aristotle's argument is not only of secondary importance but also helps us to better understand the ancient paradigm of the debate concerning modalities, time, and determinism/indeterminism.

Given the limited historical sources available regarding Diodorus and the extensive application of logical tools by various logicians in reconstructing the Master Argument, it becomes essential to contemplate the nature of the analysis presented herein from a broader methodological standpoint. One might inquire whether its purpose is to offer a faithful historical reconstruction or whether it inadvertently aligns

[20] Döring Klaus, *Die Megariker. Kommentierte Sammlung der Testimonien* (Amsterdam: Verlag B. R. Grüner N. V., 1972), 39-41, 133-134, 214-215.

[21] Jarmużek Tomasz, *On the Sea Battle Tomorrow That May Not Happen. A Logical and Philosophical Analysis of the Master Argument* (Berlin–Warsaw: Peter Lang Publishing, 2018), 131-156.

with the concept of "whig history of science", depicting the Master Argument not in its original conception but as it is perceived today through logical reconstruction. It is essential to acknowledge that the choice between these two options does not cover the entire spectrum of possibilities, but rather highlights two extremes.

While our paper does not delve into the detailed examination of methodological principles for textual or historical explanations, it is relevant for our purposes to differentiate a "historical history" that focuses on the specific historical context and events surrounding the argument and a "logical history" that emphasizes the logical development and analysis of the problem at hand[22].

The "historical history" should primarily employ standard methods of historical research, including textual and contextual analysis, as well as relying on philology. However, when it comes to the Master Argument, we have highlighted the limitations of this approach, as the available historical sources are scarce. In contrast, the "logical history" prioritizes a theoretical framework that aids in generating or supplementing the historical reconstruction. In our paper, we have specifically concentrated on the intricacies of two chosen formal approaches to reconstructing the Master Argument.

Several questions, nonetheless, still remain open. How should the two perspectives, the historical and the logical, interact? How can we uphold the integrity of historical data without distorting the true history of the problem, while also ensuring that the logical reconstruction effectively addresses the crucial theoretical issues and their evolution within the debate? It is imperative that both perspectives interact, as neither historical skepticism, stemming from the scarcity of sources, nor the mere application of modern logical tools, considered solely an analytical aid, can serve as the sole approach to understanding the Master Argument.

Moreover, logical reconstructions of the Master Argument, even if they may seem not very appropriate or faithful enough to deal with the ancient sources, give further insight into crucial aspects of Diodorus' reasoning. Such reconstructions turn out to be very helpful in identifying what we have called at the beginning, the perennial philosophical questions.

[22] Marcacci Flavia, "History of Science, Epistemology and Ontology", in: *Science between Truth and Ethical Responsibility. Evandro Agazzi in the Contemporary Scientific and Philosophical Debate*, ed. M. Alai, M. Buzzoni, G. Tarozzi (Cham–Heidelberg–New York–Dordrecht–London: Springer, 2015): 231–241.

6.2. Indeterminism and determinism still discussed

The perennial nature of philosophical questions, when viewed collectively from both the "historical history" and "logical history" perspectives, arises from the profound engagement of human knowledge confronted with fundamental theoretical challenges. Within the context of our examination of the Master Argument, one such fundamental challenge revolves around the concept of branching and non-branching time structures, which correspond to philosophical notions of determinism and indeterminism regarding the nature of time.

The question of indeterminism/determinism – whether the future is open or whether there is just one real possibility for it – is one of the crucial topics of the philosophy of science[23]. Basically, among the many reasons to ask whether a certain scientific theory is deterministic or not may be the fact that, on the one hand, we often employ theories as giving us the correct picture of the world (metaphysical point of view), and on the other, we use theories to predict or retrodict phenomena (epistemological point of view).

As some authors point out, there are three dominant approaches to determinism of theories of physics in current literature, that is, based on: the study of differential equations, mappings between temporal realizations, and on branching models[24]. In the latter case, the basic idea is that alternative future possibilities are analyzed by means of branching histories. Last but not least, some authors argue that branching is a natural representation of a theory's indeterminism since it "represents exactly the kind of structure that is needed to assess a theory's determinism or indeterminism" in contrast to the former approaches[25].

Although this naturalness, coming from the greater simplicity and conceptual primacy of the branching approach to determinism, is emphasized in the philosophy of physics as an argument for its adoption, it seems that it also operates within the domain of logical reconstructions of the Master Argument. Moreover, Jarmużek's reconstruction has shown that it is quite unexpected to articulate the argument for determinism within a logical framework that could be interpreted in such an indeterministic way.

Both from the logical and philosophy of physics point of view, it seems that the exploration of branching structures leads to novel insights and a more

[23] Gyenis Balázs, "Determinism, Physical Possibility, and Laws of Nature", *Found Phys* 50 (2020): 568–581.

[24] Müller Thomas, Placek Tomasz, "Defining Determinism", *British Journal for the Philosophy of Science* 69(1) (2018): 215–252.

[25] Ibidem, 250.

comprehensive understanding of the determinism and indeterminism debate. From the "historical history" point of view it means that, although there is the lack of textual sources regarding the Master Argument, Diodorus in his reasoning "hits the nail on the head", that is, he appropriately identified the perennial – meta-historical – root of the problem.

7. Conclusions

In recent years, there have been several formal reconstructions of the Master Argument. The availability of diverse, intricate, and nonstandard formal frameworks has greatly contributed to the advancement of historical and philosophical investigations in this area. Among the various sophisticated modern systems, one notable example is positional logic, which experienced its peak period in the 1960s and 1970s. During that time, Rescher utilized positional logic as a formal framework to reconstruct the Master Argument. Today, the Toruń Logic Group has revived positional logic, breathing new life into this formal system. With this revival, a fresh solution to the Master Argument has emerged, presenting another modern reconstruction employing the mentioned logic.

We have attempted to present Jarmużek's reconstruction as a fascinating approach with several noteworthy properties. Firstly, positional logic, which has been largely forgotten over the years, is brought to the forefront in this reconstruction. Jarmużek's unique approach goes beyond Rescher's utilization of positional logic by employing an extended version of the system that offers a broader range of capabilities.

Secondly, in contrast to Rescher's reconstruction, we made an effort to highlight the properties of the time structure assumed by the logical frameworks themselves, as well as the axioms and Diodorean premises. Through Jarmużek's reconstruction, we demonstrated that two positional logic-based reconstructions can yield different ontological consequences. Rescher's framework, for instance, was confined to a partially linear structure without the inclusion of the last element. Furthermore, in the case of branching, all branches within such a structure would be trivial, rendering Rescher's reconstruction unable to interpret the Master Argument in a fully branching time structure.

Jarmużek's reconstruction brings a fresh perspective to the topic. In addition to exploring different properties of the time structure, his reconstruction enables the representation of a non-trivial branching structure. As a result, it was quite unexpected to articulate the argument for determinism within an indeterministic temporal framework.

As demonstrated in the previous section, engaging in logical investigations when historical sources are scarce can provide a valuable opportunity to expand the perspective on a given topic. In the case of the Master Argument reconstruction, this approach has not only raised intriguing ontological and logical questions but has also prompted a deeper examination of the nature of time and its connection to determinism. By combining logical reasoning with the exploration of foundational concepts, such investigations can lead to novel insights and a more comprehensive understanding of the subject matter.

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