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BOUNDED RESONANT PROBLEMS DRIVEN BY FRACTIONAL LAPLACIAN

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Abstract. In this paper we study the existence of nontrivial solutions for the fractional Laplacian resonance problem with a bounded nonlinearity via Morse theory and a penalization technique.

1. Introduction

In this paper we study the existence of nontrivial solutions for the following fractional Laplacian problem

$$\begin{cases} (-\Delta)^s u = f(x,u) & x \in \Omega, \\ u = 0, & x \in \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1.1}$$

where $s \in (0,1)$ is fixed, Ω is an open bounded subset of $\mathbb{R}^N$ with Lipschitz boundary, $N > 2s$, $(-\Delta)^s$ is the fractional Laplace operator, which (up to normalization factors) is defined as

$$-(-\Delta)^s u(x) := \int_{\mathbb{R}^N} \frac{u(x+y)+u(x-y)-2u(x)}{|y|^{N+2s}}\, dy, \quad x \in \mathbb{R}^N, \tag{1.2}$$

and the function $f \in C^1(\overline{\Omega} \times \mathbb{R}, \mathbb{R})$ satisfies the subcritical growth condition

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