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POSITIVE SOLUTIONS FOR SCHRÖDINGER–POISSON–SLATER SYSTEM WITH COERCIVE POTENTIAL

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Abstract. We study the following Schrödinger–Poisson–Slater type system:

$$\begin{cases} -\Delta u + V(x)u + \lambda\phi(x)u = |u|^{p-1}u, & x \in \mathbb{R}^3, \\ -\Delta\phi = u^2, \quad \lim\limits_{|x|\to+\infty} \phi(x) = 0, \end{cases} \tag{0.1}$$

where $\lambda > 0$ is a parameter, $p \in (1,2)$, $V \in C(\mathbb{R}^N, \mathbb{R}_0^+)$. If V is a constant, or $p \in (2,5)$ with $V \in L^\infty(\mathbb{R}^3)$, the above system has been considered in many papers. In this paper, we are interested in the case: $p \in (1,2)$ and V being a coercive potential, i.e., $\lim\limits_{|x|\to+\infty} V(x) = \infty$. We prove that there exists $\lambda_0 > 0$ such that system (0.1) has at least two positive solutions u_λ^0 and u_λ^1 for any $\lambda \in (0,\lambda_0)$. Moreover, u_λ^0 is a ground state (i.e., the least energy solution) which must blow up as $\lambda \to 0$. Particularly, when $p \in (11/7,2)$ and $\lambda > 0$ is small enough, we show that the ground state of (0.1) must be non-radially symmetric even if $V(x) = V(|x|)$, such as $V(x) = |x|^2$.

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