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ON THE EXISTENCE OF FIXED POINTS FOR TYPICAL NONEXPANSIVE MAPPINGS ON SPACES WITH POSITIVE CURVATURE

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Abstract. We show that the typical nonexpansive mapping on a small enough subset of a CAT(κ)-space is a *contraction in the sense of Rakotch*. By typical we mean that the set of nonexpansive mapppings without this property is a σ-porous set and therefore also of the first Baire category. Moreover, we exhibit metric spaces where strict contractions are not dense in the space of nonexpansive mappings. In some of these cases we show that all continuous self-mappings have a fixed point nevertheless.

1. Introduction

The celebrated fixed point theorem of Brouwer asserts that every continuous self-mapping of a bounded, closed and convex subset of a Euclidean space has a fixed point. Unfortunately, this result does not generalise to bounded, closed and convex subsets of infinite-dimensional Banach spaces, as in this case the unit

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