Topological Methods in Nonlinear Analysis Volume 56, No. 2, 2020, 589–606 DOI: 10.12775/TMNA.2020.035

© 2020 Juliusz Schauder Centre for Nonlinear Studies Nicolaus Copernicus University in Toruń

## COMPUTATIONS OF THE LEAST NUMBER OF PERIODIC POINTS OF SMOOTH BOUNDARY-PRESERVING SELF-MAPS OF SIMPLY-CONNECTED MANIFOLDS

Grzegorz Graff — Jerzy Jezierski — Adrian Myszkowski

ABSTRACT. Let r be an odd natural number, M a compact simplyconnected smooth manifold, dim  $M \geq 4$ , such that its boundary  $\partial M$  is also simply-connected. We consider f, a  $C^1$  self-maps of M, preserving  $\partial M$ . In [G. Graff and J. Jezierski, Geom. Dedicata 187 (2017), 241–258] the smooth Nielsen type periodic number  $D_r(f; M, \partial M)$  was defined and proved to be equal to the minimal number of r-periodic points for all maps preserving  $\partial M$  and  $C^1$ -homotopic to f. In this paper we demonstrate a purely combinatorial method of calculation of the invariant and illustrate it in various cases.

## 1. Introduction

Let  $f: M \to M$  be a self-map of a simply-connected manifold and r be a given non-negative integer. The natural generalization of the classical Nielsen number is Nielsen periodic number of order r, which estimates the number of r-periodic points in the homotopy class of f (cf. [10]). The smooth branch of this theory, in which one considers smooth (i.e.  $C^1$ ) map and smooth homotopies, differs very strongly from the continuous one. Roughly speaking, the smoothness

<sup>2020</sup> Mathematics Subject Classification. Primary: 37C25, 55M20; Secondary: 37C05.

Key words and phrases. Periodic points; smooth maps; Nielsen number; fixed point index; Lefschetz number.

Research supported by the National Science Centre, Poland within the grant Sheng 1 UMO-2018/30/Q/ST1/00228.