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HOW MANY SIMPLICES ARE NEEDED TO TRIANGULATE A GRASSMANNIAN?

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Abstract. In this paper we use recently developed methods to compute a lower bound for the number of simplices that are needed to triangulate the Grassmann manifold $G_k(\mathbb{R}^n)$. We first estimate the number of vertices that are needed for such a triangulation by giving a general lower bound and some more precise bounds for $k = 2, 3, 4$. By applying the Lower Bound Theorem (LBT) of Barnette for triangulated manifolds, we then obtain estimates for the number of simplices in all dimensions. For higher-dimensional simplices these estimates can be considerably improved by using the recent progress on the Generalized Lower Bound Theorem for triangulated manifolds, which states that the h''-numbers of triangulated manifolds are unimodal, together with the computation of the Poincaré polynomial. For example, we are able to prove that the number of top-dimensional simplices in a triangulation of $G_k(\mathbb{R}^n)$ grows exponentially with n. Our method can be used to estimate the minimal size of triangulations for other spaces, like Lie groups, flag manifolds, Stiefel manifolds etc.

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