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SIGN-CHANGING SOLUTIONS FOR COUPLED SCHRÖDINGER EQUATIONS WITH MIXED COUPLING

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Abstract. We consider the following nonlinear elliptic systems:

$$(*)\qquad \begin{cases} -\Delta u_1 + \lambda_1 u_1 = \mu_1 u_1^3 + \beta_{12} u_1 u_2^2 + \beta_{13} u_1 u_3^2 & \text{in } \Omega, \\ -\Delta u_2 + \lambda_2 u_2 = \mu_2 u_2^3 + \beta_{12} u_2 u_1^2 + \beta_{23} u_2 u_3^2 & \text{in } \Omega, \\ -\Delta u_3 + \lambda_3 u_3 = \mu_3 u_3^3 + \beta_{13} u_3 u_1^2 + \beta_{23} u_3 u_2^2 & \text{in } \Omega, \\ \qquad\qquad \vec{u} = (u_1, u_2, u_3) \in H_0^1(\Omega)^3, \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ $(n \leq 3)$ is a bounded domain with smooth boundary, $\lambda_j, \mu_j > 0$ $(j = 1, 2, 3)$, $\beta_{12} > 0$, and $\beta_{13}, \beta_{23} \leq 0$. For this model case of coupled Schrödinger equations with mixed coupling, for sufficiently large $\beta_{12} > 0$, we show that there exists a solution $(u_{1\beta}, u_{2\beta}, u_{3\beta})$ of $(*)$ such that $u_{1\beta} > 0, u_{2\beta} > 0$ and $u_{3\beta}$ changes sign exactly once. We also show that, for any given $k \in \mathbb{N}$, there exist k vector solutions $(u_{1\beta}^\ell, u_{2\beta}^\ell, u_{3\beta}^\ell)$ $(\ell = 1, \ldots, k)$ and these solutions are characterized by the genus with respect to a partial symmetry $\sigma(u_1, u_2, u_3) = (-u_1, -u_2, u_3)$.

1. Introduction

In this paper, we consider nonlinear elliptic systems with mixed coupling. In order to consider the case of mixed coupling, the systems need to have three or

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