FUNCTION AND COLORFUL EXTENSIONS
OF THE KKM THEOREM

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Abstract. We deal with an aggregate version of the KKM theorem. Given n KKM families of special type on an $(n-1)$-dimensional simplex, we show that it is possible to choose a single element from every KKM family to get a KKM family on that simplex. We also introduce and study function KKM families as surrogates of KKM families on simplexes. We show a function version of the KKM theorem. The Coincidence Theorem is our main result, Brouwer's fixed is a special cases of that theorem.

1. Introduction

In 1929, Bronisław Knaster, Kazimierz Kuratowski, and Stefan Mazurkiewicz published a seminal paper [11], where they proved, among others, the following theorem (see also Theorem 2.1): *Each closed KKM family has non-empty intersection.* This result, commonly referred to as *KKM theorem* (or *KKM lemma*), has found a stunning number of applications in various fields of mathematics and beyond (see, e.g., Granas [8], El-Mechaiekh et al. [2], Park [15]). It has even become the subject of a separate theory, called *KKM theory*, initiated and successively developed by Sehie Park [17].

The purpose of this note is to study two aggregate (= colorful and strong colorful) versions of the KKM theorem and to point to some applications.