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STUDY OF A CLASS OF GENERALIZED SCHRÖDINGER EQUATIONS

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ABSTRACT. A class of generalized Schrödinger problems in a bounded domain is studied. A complete overview of the set of solutions is provided, depending on the values assumed by parameters involved in the problem. In order to obtain the results, we combine monotony, bifurcation and variational methods.

1. Introduction

In this work we investigate general conditions for which the stationary generalized Schrödinger problem

$$(\mathbf{P}_{\lambda,q}) \qquad \begin{cases} -\operatorname{div}(\vartheta(u)\nabla u) + \frac{1}{2}\,\vartheta'(u)|\nabla u|^2 = \lambda|u|^{q-1}u & \text{in }\Omega, \\ u = 0 & \text{on }\partial\Omega, \end{cases}$$

has nontrivial solutions, where $\Omega \subset \mathbb{R}^N$, $N \geq 3$, is a bounded smooth domain, q > 0, λ is a real parameter and $\vartheta \colon \mathbb{R} \to [1, \infty)$ is an even C^1 -function satisfying some suitable hypotheses which will be stated later on.

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