

## NIELSEN NUMBER, IMPULSIVE DIFFERENTIAL EQUATIONS AND PROBLEM OF JEAN LERAY

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**ABSTRACT.** We will show that, unlike to usual (i.e. non-impulsive) differential equations, the Nielsen theory results for single-valued as well as multivalued maps on tori can be effectively applied to impulsive differential equations and inclusions. With this respect, two main aims will be focused, namely: (i) multiplicity results for harmonic periodic solutions, (ii) the coexistence of subharmonic periodic solutions with various periods. In both cases, we will try to contribute at least partly to the problem posed already in 1950 by Jean Leray. A dynamic complexity of the related maps, measured in terms of entropy, will be also examined.

### 1. Introduction

As already pointed out several times by Brown (see e.g. [16], [17]), Leray [36] posed at the first International Congress of Mathematicians held after World War II in Cambridge, Mass., in 1950, the problem of adapting the Nielsen theory to the needs of nonlinear analysis. In particular, he had in mind especially its application to differential equations for obtaining multiplicity results.

This problem is still rather delicate and the obtained results in this field are rather rare (see e.g. the survey papers [2], [24] and [19, Chapter III.12.5]).

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