Topological Methods in Nonlinear Analysis Volume 55, No. 2, 2020, 565–581 DOI: 10.12775/TMNA.2019.104

© 2020 Juliusz Schauder Centre for Nonlinear Studies Nicolaus Copernicus University in Toruń

## MULTIPLE PERIODIC SOLUTIONS FOR ONE-SIDED SUBLINEAR SYSTEMS: A REFINEMENT OF THE POINCARÉ–BIRKHOFF APPROACH

Tobia Dondè — Fabio Zanolin

ABSTRACT. In this paper we prove the existence of multiple periodic (harmonic and subharmonic) solutions for a class of planar Hamiltonian systems which includes the case of the second order scalar ODE x'' + a(t)g(x) = 0with g satisfying a one-sided condition of sublinear type. We consider the classical approach based on the Poincaré–Birkhoff fixed point theorem as well as some refinements on the side of the theory of topological horseshoes. A Duffing-type equation and an exponential nonlinearity case are studied as applications.

## 1. Introduction and statement of the main results

This work deals with the existence and multiplicity of periodic solutions to sign-indefinite nonlinear first order planar systems of the form

(1.1)  $x' = h(y), \quad y' = -a_{\lambda,\mu}(t)g(x).$ 

<sup>2020</sup> Mathematics Subject Classification. 34C25, 34C28, 54H20.

Key words and phrases. Poincaré–Birkhoff theorem; bend-twist maps; topological horseshoes; periodic solutions; complex oscillations.

This work was performed under the auspices of Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM).