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THE STRUCTURE OF POSITIVE SOLUTIONS FOR A SCHRÖDINGER SYSTEM

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Abstract. Using bifurcation analysis we investigate the structure of the set of positive solutions for the coupled nonlinear Schrödinger system

$$\begin{cases} -\Delta u_1 + u_1 = u_1^3 + \beta u_1 u_2^2 & \text{in } \mathbb{R}^N, \\ -\Delta u_2 + \lambda u_2 = \mu u_2^3 + \beta u_2 u_1^2 & \text{in } \mathbb{R}^N, \\ u_1(x), u_2(x) \to 0 & \text{as } |x| \to +\infty, \end{cases}$$

where $N = 1, 2, 3$, μ is a positive constant, λ and β are positive real parameters. We prove the existence of two two-dimensional continua $\mathcal{S}_1$ and $\mathcal{S}_2$ emanating from the two sets of semi-positive solutions which cover some regions in term of $(\beta, \lambda) \in \mathbb{R}^2_+$. To do this, we establish a multi-parameter unilateral global bifurcation theorem.

1. Introduction

In this paper, we study the structure of positive solutions for the following time-independent nonlinear Schrödinger system

$$\begin{cases} -\Delta u_1 + u_1 = u_1^3 + \beta u_1 u_2^2 & \text{in } \mathbb{R}^N, \\ -\Delta u_2 + \lambda u_2 = \mu u_2^3 + \beta u_2 u_1^2 & \text{in } \mathbb{R}^N, \\ u_1(x) \to 0, \quad u_2(x) \to 0 & \text{as } |x| \to \infty, \end{cases} \tag{1.1}$$

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