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POSITIVE SOLUTIONS OF KIRCHHOFF–HÉNON TYPE ELLIPTIC EQUATIONS WITH CRITICAL SOBOLEV GROWTH

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Abstract. We investigate the following Kirchhoff–Hénon type equation involving the critical Sobolev exponent with Dirichlet boundary condition:

$$-\bigg(a+b\bigg(\int_\Omega |Du|^2\,dx\bigg)^{(p-2)/2}\bigg)\Delta u=\Psi u^{q-1}+|x|^\alpha u^{2^*-1}$$

in Ω included in a unit ball under several conditions. Here, $a,b\geq 0$, $a+b>0$, $2<p<q<2^*$ and $\Psi\in L^\infty(\Omega)\setminus\{0\}$ is a given non-negative function with several conditions. We show that, if either $N=3$ with $4<q<2^*=6$ or $N\geq 4$, there exists a positive solution for small $\alpha\geq 0$. Our methods includes the mountain pass theorem and the Talenti function.

1. Introduction

We investigate the following equation.

$$\begin{cases} -\bigg(a+b\bigg(\displaystyle\int_\Omega |Du|^2\,dx\bigg)^{(p-2)/2}\bigg)\Delta u=\Psi u^{q-1}+|x|^\alpha u^{2^*-1} & \text{in } \Omega,\\ u>0 & \text{in } \Omega,\\ u=0 & \text{on } \partial\Omega. \end{cases}\tag{1.1}$$

We use $2^*=2N/(N-2)$ to denote the critical Sobolev exponent for $N\geq 3$. $\Omega\subset\mathbb{R}^N$ is a piecewise C^1-class bounded domain with $\Omega\subset B(0,1)$. Here,

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