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GLOBAL CONTINUATION IN EUCLIDEAN SPACES OF THE PERTURBED UNIT EIGENVECTORS CORRESPONDING TO A SIMPLE EIGENVALUE

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Abstract. In the Euclidean space $\mathbb{R}^k$, we consider the perturbed eigenvalue problem $Lx + \varepsilon N(x) = \lambda x$, $\|x\| = 1$, where ε, λ are real parameters, L is a linear endomorphism of $\mathbb{R}^k$, and $N\colon S^{k-1} \to \mathbb{R}^k$ is a continuous map defined on the unit sphere of $\mathbb{R}^k$. We prove a global continuation result for the *solutions* $(x, \varepsilon, \lambda)$ of this problem. Namely, under the assumption that $x_* \in S^{k-1}$ is one of the two unit eigenvectors of L corresponding to a simple eigenvalue $\lambda_* \in \mathbb{R}$, we show that, in the set of all the solutions, the connected component containing $(x_*, 0, \lambda_*)$ is either unbounded or meets a solution $(x^*, 0, \lambda^*)$ having $x^* \neq x_*$. Our result is inspired by a paper of R. Chiappinelli concerning the local persistence property of eigenvalues and eigenvectors of a perturbed self-adjoint operator in a real Hilbert space.

1. Introduction

Let $T\colon H \to H$ be a self-adjoint bounded operator in a real Hilbert space H, and $N\colon S \to H$ be a Lipschitz continuous map defined on the unit sphere of H.

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