

## CONNECTED COMPONENT OF POSITIVE SOLUTIONS FOR SINGULAR SUPERLINEAR SEMI-POSITONE PROBLEMS

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ABSTRACT. Bifurcation theory is used to prove the existence of connected components of positive solutions for some classes of singular superlinear semi-positone problems with nonlinear boundary conditions.

### 1. Introduction

Consider the boundary value problem

$$(1.1) \quad \begin{cases} -\Delta u = \lambda K(|x|)f(u) & \text{in } B, \\ \frac{\partial u}{\partial n} + \tilde{c}(u)u = 0 & \text{on } |x| = r_0, \\ u(x) \rightarrow 0 & \text{as } |x| \rightarrow \infty, \end{cases}$$

where  $B = \{x \in \mathbb{R}^N : |x| > r_0 > 0\}$ ,  $N > 2$ ,  $K : [r_0, \infty) \rightarrow (0, \infty)$ ,  $\tilde{c} : [0, \infty) \rightarrow (0, \infty)$ ,  $f : (0, \infty) \rightarrow \mathbb{R}$  are continuous, and  $\lambda$  is a positive parameter. Here  $n$  denotes the outer unit normal vector on  $\partial B$ .

By letting  $r = |x|$  and  $t = (r/r_0)^{2-N}$ , (1.1) becomes

$$(1.2) \quad \begin{cases} -u''(t) = \lambda h(t)f(u(t)), & t \in (0, 1), \\ u(0) = 0, \quad u'(1) + c(u(1))u(1) = 0, \end{cases}$$

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