EXISTENCE AND STABILITY OF STANDING WAVES FOR THE CHOQUARD EQUATION WITH PARTIAL CONFINEMENT

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Abstract. In this paper we study the existence and orbital stability of the Choquard equation with partial confinement. This type equation originates from Fröhlich and Pekar's model of the polaron, where free electrons in an ionic lattice interact with phonons associated with deformations of the lattice or with the polarisation that it creates on the medium (interaction of an electron with its own hole). On the one hand, we prove the existence of global minimizer of the associate energy functional subject to the L^2-constraint. On the other hand, we discuss the orbital stability and asymptotic behavior of the global minimizer.

1. Introduction and main results

In this paper we are concerned with the existence, stability, qualitative and symmetry properties of standing waves associated with the following Cauchy problem

$$(1.1) \quad \begin{cases} -i\partial_t u - \Delta u + (x_1^2 + \ldots + x_{N-1}^2)u = (J_\alpha * |u|^p)\, |u|^{p-2}u, \\ u(0, x) = \phi(x), \quad (t, x_1, \ldots, x_N) \in \mathbb{R} \times \mathbb{R}^N, \end{cases}$$