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THE LONG-TIME BEHAVIOR OF WEIGHTED *p*-LAPLACIAN EQUATIONS

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ABSTRACT. In this work we study weighted *p*-Laplacian equations in a bounded domain with a variable and generally non-smooth diffusion coefficient having at most a finite number of zeroes. The main attention is focused on the case that the diffusion coefficient a(x) in such equations satisfies the inequality $\liminf_{x\to z} |x-z|^{-p}a(x) > 0$ for every $z \in \overline{\Omega}$. We show the existence of weak solutions and global attractors in $L^2(\Omega)$, $L^q(\Omega)$ $(q \ge 2)$ and $D_0^{1,p}(\Omega)$, respectively.

1. Introduction

Let Ω be a bounded smooth domain in \mathbb{R}^n $(n \geq 2)$. We consider weighted *p*-Laplacian equations

(1.1)
$$\begin{cases} \frac{\partial u}{\partial t} - \operatorname{div}(a(x)|\nabla u|^{p-2}\nabla u) + f(u) = g & \text{in } \Omega \times \mathbb{R}^+, \\ u = 0 & \text{on } \partial\Omega \times \mathbb{R}^+, \\ u(x,0) = u_0 & \text{in } \Omega, \end{cases}$$

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