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ASYMPTOTIC DYNAMICS OF NON-AUTONOMOUS FRACTIONAL REACTION-DIFFUSION EQUATIONS ON BOUNDED DOMAINS

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Abstract. In this paper, we consider the asymptotic dynamics of non-autonomous fractional reaction-diffusion equations of the form

$$u_t + (-\Delta)^s u + f(u) = g(t)$$

complemented with the Dirichlet boundary condition on a bounded domain. First, we obtain some higher-attraction results for pullback attractors, that is, without any additional t-differentiability assumption on the forcing term g, for any space dimension N and any growth power $p \geq 2$ of f, the known $(L^2(\Omega), L^2(\Omega))$ pullback attractor can indeed attract every $L^2(\Omega)$-bounded set in the $L^{2+\delta}(\Omega)$-norm for every $\delta \in [0, \infty)$ as well as in the $W_0^{s,2}(\Omega)$-norm. Then, we construct a family of Borel probability measures $\{\mu_t\}_{t\in\mathbb{R}}$, whose supports satisfy the higher-attraction results. Finally, we investigate the relationship between such the Borel probability measures and time-dependent statistical solutions for this fractional Laplacian equation.

1. Introduction

It is well known that the fractional operators as infinitesimal generators of Lèvy stable diffusion processes arise naturally in many different problems in the plasma, flames propagation and chemical reactions in liquids, population

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