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SEMICLASSICAL STATES FOR SINGULARLY PERTURBED SCHRÖDINGER–POISSON SYSTEMS WITH A GENERAL BERESTYCKI–LIONS OR CRITICAL NONLINEARITY

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Abstract. This paper is concerned with two classes of singularly perturbed Schrödinger–Poisson systems of the form

$$\begin{cases} -\varepsilon^2\Delta u + u + \phi u = f(u), & x \in \mathbb{R}^3, \\ -\Delta\phi = u^2, & x \in \mathbb{R}^3, \end{cases}$$

and

$$\begin{cases} -\varepsilon^2\Delta u + V(x)u + \phi u = g(x,u) + K(x)u^5, & x \in \mathbb{R}^3, \\ -\Delta\phi = u^2, & x \in \mathbb{R}^3, \end{cases}$$

where $\varepsilon > 0$ is a small parameter. We prove that: (1) the first system admits a concentrating bounded state for small ε, where $f \in \mathcal{C}(\mathbb{R},\mathbb{R})$ satisfies Berestycki–Lions assumptions which are almost necessary; (2) there exists a constant $\varepsilon_0 > 0$ determined by V, K and g such that for any $\varepsilon \in (0, \varepsilon_0]$ the second system has a nontrivial solution, where $V, K \in \mathcal{C}(\mathbb{R}^3,\mathbb{R})$, $V(x) \geq 0$, $K(x) > 0$, $g \in \mathcal{C}(\mathbb{R}^3 \times \mathbb{R},\mathbb{R})$ is an indefinite function. Our results improve and complement the previous ones in the literature.

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Key words and phrases. Semiclassical states; Schrödinger–Poisson system; Berestycki–Lions type assumptions; critical Sobolev exponent; variational method.

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