Topological **M**ethods in **N**onlinear **A**nalysis
Volume 54, No. 1, 2019, 131–139
DOI: 10.12775/TMNA.2019.028

KRASNOSEL'SKIĬ–SCHAEFER TYPE METHOD IN THE EXISTENCE PROBLEMS

Calogero Vetro — Dariusz Wardowski

Abstract. We consider a general integral equation satisfying algebraic conditions in a Banach space. Using Krasnosel'skiĭ–Schaefer type method and technical assumptions, we prove an existence theorem producing a periodic solution of some nonlinear integral equation.

1. Introduction and preliminaries

Let $(\mathcal{B}, \|\cdot\|)$ be the Banach space of continuous Γ-periodic functions $\varphi\colon \mathbb{R}\to\mathbb{R}$ with $\Gamma > 0$ and the supremum norm. In this paper, we study the following integral equation

$$\varphi(t) = f(t, \varphi(t)) - \int_{t-\alpha}^{t} D(t, s) g(s, \varphi(s))\, ds, \tag{1.1}$$

where $\alpha > 0$, $f, g, D\colon \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ are continuous functions satisfying the following assumptions:

- (A1) $f(t + \Gamma, x) = f(t, x)$, $D(t + \Gamma, s + \Gamma) = D(t, s)$, $g(t + \Gamma, x) = g(t, x)$ for all $s, t, x \in \mathbb{R}$,
- (A2) $D(t, t-\alpha) = 0$, $D_{st}(t, s) \le 0$, $D_s(t, s) \ge 0$ for all $t \in \mathbb{R}$ and $s \in (t-\alpha, t)$,
- (A3) the function D_{st} is continuous,

2010 *Mathematics Subject Classification.* Primary: 47H10, 47N20.

Key words and phrases. F-contraction; compact operator; nonlinear integral equation; Krasnosel'skiĭ–Schaefer fixed point theorem.

The second author was financially supported by the National Science Center, Poland. Grant with a registration number 2017/01/X/ST1/00390.