Topological Methods in Nonlinear Analysis Volume 53, No. 1, 2019, 291–307 DOI: 10.12775/TMNA.2019.004

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POSITIVE GROUND STATES FOR A SUBCRITICAL AND CRITICAL COUPLED SYSTEM INVOLVING KIRCHHOFF-SCHRÖDINGER EQUATIONS

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ABSTRACT. In this paper we prove the existence of positive ground state solution for a class of linearly coupled systems involving Kirchhoff-Schrödinger equations. We study the subcritical and critical case. Our approach is variational and based on minimization technique over the Nehari manifold. We also obtain a nonexistence result using a Pohozaev identity type.

1. Introduction

In this article we study the following class of nonlocal linearly coupled systems

$$(S_{\mu}) \begin{cases} \left(a_1 + \alpha'(\|u\|_{E_1}^2)\right)(-\Delta u + V_1(x)u\right) = \mu|u|^{p-2}u + \lambda(x)v & \text{for } x \in \mathbb{R}^3, \\ \left(a_2 + \beta'(\|v\|_{E_2}^2)\right)(-\Delta v + V_2(x)v) = |v|^{q-2}v + \lambda(x)u & \text{for } x \in \mathbb{R}^3, \end{cases}$$

where $a_1, a_2 > 0$, $\alpha, \beta \in C^2(\mathbb{R}_+, \mathbb{R}_+)$ and for each i = 1, 2 we consider the following weighted Sobolev space

$$E_i := \bigg\{ w \in H^1(\mathbb{R}^3) : \int_{\mathbb{R}^3} V_i(x) w^2 \, dx < \infty \bigg\},$$

²⁰¹⁰ Mathematics Subject Classification. Primary: 35J50; Secondary: 35B33, 35Q55. Key words and phrases. Nonlinear Kirchhoff-Schrödinger equations; coupled systems; lack of compactness; ground states.

Research supported in part by INCTmat/MCT/Brazil, CNPq and CAPES/Brazil.