Topological Methods in Nonlinear Analysis Volume 53, No. 1, 2019, 271–289 DOI: 10.12775/TMNA.2019.003

© 2019 Juliusz Schauder Centre for Nonlinear Studies Nicolaus Copernicus University

## $\begin{array}{c} {\bf NEW~RESULTS} \\ {\bf OF~MIXED~MONOTONE~OPERATOR~EQUATIONS} \end{array}$

TIAN WANG — ZHAOCAI HAO

ABSTRACT. In this article, we study the existence and uniqueness of fixed points for some mixed monotone operators and monotone operators with perturbation. These mixed monotone operators and monotone operators are e-concave-convex operators and e-concave operators respectively. Without using compactness or continuity, we obtain the existence and uniqueness of fixed points by monotone iterative techniques and properties of cones. Our main results extended and improved some existing results. Also, we applied the results to some differential equations.

## 1. Introduction and preliminaries

Throughout the paper, E is a real Banach space with norm  $\|\cdot\|$ . P is a cone in E if it satisfies:

- (1) if  $x \in P$ ,  $\lambda \ge 0$  then  $\lambda x \in P$ ;
- (2) if  $x \in P$ ,  $-x \in P$  then  $x = \theta$ ,

where  $\theta$  is zero in E,  $P^+ = P - \{\theta\}$ .

<sup>2010</sup> Mathematics Subject Classification. 34B16, 34B18.

 $<sup>\</sup>it Key\ words\ and\ phrases.$  Fixed point; e-concave-convex operator; e-concave operator; mixed monotone.

Supported financially by the National Natural Science Foundation of China (11571296, 11371221), the Fund of the Natural Science of Shandong Province (ZR2014AM034), and Colleges and universities of Shandong province science and technology plan projects (J13LI01), University outstanding scientific research innovation team of Shandong province (Modeling, optimization and control of complex systems) and Qufu Normal University Fund(XJ201126).