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EQUIVALENCE BETWEEN UNIFORM $L^{2^\star}(\Omega)$ A-PRIORI BOUNDS AND UNIFORM $L^\infty(\Omega)$ A-PRIORI BOUNDS FOR SUBCRITICAL ELLIPTIC EQUATIONS

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ABSTRACT. We provide sufficient conditions for a uniform $L^{2^*}(\Omega)$ bound to imply a uniform $L^{\infty}(\Omega)$ bound for positive classical solutions to a class of subcritical elliptic problems in bounded C^2 domains in \mathbb{R}^N . We also establish an equivalent result for sequences of boundary value problems.

1. Introduction

We consider the existence of $L^{\infty}(\Omega)$ a priori bounds for classical positive solutions to the boundary value problem

(1.1)
$$-\Delta u = f(u), \quad \text{in } \Omega, \qquad u = 0, \quad \text{on } \partial \Omega,$$

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