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STRICT C^1-TRIANGULATIONS IN O-MINIMAL STRUCTURES

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Abstract. Inspired by the recent articles of T. Ohmoto and M. Shiota [9], [10] on $\mathcal{C}^1$-triangulations of semialgebraic sets, we prove here by using different methods the following theorem: *Let R be a real closed field and let an expansion of R to an o-minimal structure be given. Then for any closed bounded definable subset A of R^n and a finite family $B_1, \dots, B_r$ of definable subsets of A there exists a definable triangulation $h\colon |\mathcal{K}| \to A$ of A compatible with $B_1, \dots, B_r$ such that $\mathcal{K}$ is a simplicial complex in R^n, h is a C^1-embedding of each (open) simplex $\Delta \in \mathcal{K}$ and h extends to a definable C^1-mapping defined on a definable open neighborhood of $|\mathcal{K}|$ in R^n.* This improves Ohmoto–Shiota's theorem in three ways; firstly, h is a $\mathcal{C}^1$-embedding on each simplex; secondly, the simplicial complex $\mathcal{K}$ is in the same space as A and thirdly, our proof is performed for any o-minimal structure. The possibility to have h with the first of these properties was stated by Ohmoto and Shiota as an open problem (see [9]).

1. Introduction

Our present article is inspired by the recent results of T. Ohmoto and M. Shiota [9], [10] on $\mathcal{C}^1$-triangulations of semialgebraic sets. We propose here a different proof giving a stronger theorem.

Assume that R is any real closed field and an expansion of R to some o-minimal structure is given. Throughout the paper we will be talking about definable

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