Topological Methods in Nonlinear Analysis Volume 52, No. 1, 2018, 311–335 DOI: 10.12775/TMNA.2018.027

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## RELATIVE ENTROPY METHOD FOR MEASURE-VALUED SOLUTIONS IN NATURAL SCIENCES

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ABSTRACT. We describe the applications of the relative entropy framework introduced in [10]. In particular the uniqueness of an entropy solution is proven for a scalar conservation law, using the notion of measure-valued entropy solutions. Further we survey recent results concerning measure-valued-strong uniqueness for a number of physical systems — incompressible and compressible Euler equations, compressible Navier–Stokes, polyconvex elastodynamics and general hyperbolic conservation laws, as well as long-time asymptotics of the McKendrick–Von Foerster equation.

## 1. Introduction

The origins of the relative entropy method can be traced back to physics. The underlying principle behind it is the simple idea to measure in a certain way how much two evolutions of a given physical system, whose initial states

 $<sup>2010\</sup> Mathematics\ Subject\ Classification.\ 35Q35.$ 

Key words and phrases. Measure-valued solution; weak-strong uniqueness; scalar conservation laws.

This work was partially supported by the Simons Foundation grant 346300 and the Polish Government MNiSW 2015-2019 matching fund.

T.D. acknowledges the support of the National Science Centre, DEC-2012/05/E/ST1/02218.

The research was partially supported by the Warsaw Center of Mathematics and Computer Science

P.G. and A.Ś.-G. received support from the National Science Centre (Poland), 2015/18/M/ST1/00075.