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GEODESICS ON $\mathrm{SO}(N)$ AND A CLASS OF SPHERICALLY SYMMETRIC MAPS AS SOLUTIONS TO A NONLINEAR GENERALISED HARMONIC MAP PROBLEM

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Abstract. We address questions on existence, multiplicity as well as qualitative features including rotational symmetry for certain classes of geometrically motivated maps serving as solutions to the nonlinear system

$$\begin{cases} -\mathrm{div}[F'(|x|,|\nabla u|^2)\nabla u] = F'(|x|,|\nabla u|^2)|\nabla u|^2 u & \text{in } \mathbb{X}^n, \\ |u| = 1 & \text{in } \mathbb{X}^n, \\ u = \varphi & \text{on } \partial\mathbb{X}^n. \end{cases}$$

Here $\varphi \in \mathscr{C}^\infty(\partial\mathbb{X}^n, \mathbb{S}^{n-1})$ is a suitable boundary map, F' is the derivative of F with respect to the second argument, $u \in W^{1,p}(\mathbb{X}^n, \mathbb{S}^{n-1})$ for a fixed $1 < p < \infty$ and $\mathbb{X}^n = \{x \in \mathbb{R}^n : a < |x| < b\}$ is a generalised annulus. Of particular interest are spherical twists and whirls, where following [26], a spherical twist refers to a rotationally symmetric map of the form $u\colon x \mapsto \mathrm{Q}(|x|)x|x|^{-1}$ with Q some suitable path in $\mathscr{C}([a,b], \mathrm{SO}(n))$ and a whirl has a similar but more complex structure with only 2-plane symmetries. We establish the existence of an infinite family of such solutions and illustrate an interesting discrepancy between odd and even dimensions.

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