

EXISTENCE OF POSITIVE GROUND SOLUTIONS FOR BIHARMONIC EQUATIONS VIA POHOŽAEV–NEHARI MANIFOLD

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ABSTRACT. We investigate the following nonlinear biharmonic equations with pure power nonlinearities:

$$\begin{cases} \Delta^2 u - \Delta u + V(x)u = u^{p-1}u & \text{in } \mathbb{R}^N, \\ u > 0 & \text{for } u \in H^2(\mathbb{R}^N), \end{cases}$$

where $2 < p < 2^* = 2N/(N-4)$. Under some suitable assumptions on $V(x)$, we obtain the existence of ground state solutions. The proof relies on the Pohožaev–Nehari manifold, the monotonic trick and the global compactness lemma, which is possibly different to other papers on this problem. Some recent results are extended.

1. Introduction

This paper is to study the existence of positive ground state solutions of the following biharmonic equation with pure power nonlinearities:

$$(EQ) \quad \begin{cases} \Delta^2 u - \Delta u + V(x)u = u^{p-1}u & \text{in } \mathbb{R}^N, \\ u > 0 & \text{for } u \in H^2(\mathbb{R}^N), \end{cases}$$

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Key words and phrases. Biharmonic equations; ground state solutions; concentration-compactness principle; Pohožaev manifold; Nehari manifold.

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