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## DYNAMICS OF THE BBM EQUATION WITH A DISTRIBUTION FORCE IN LOW REGULARITY SPACES

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ABSTRACT. The Benjamin–Bona–Mahony equation with a distribution force on torus is studied in low regularity spaces. The global well-posedness and the existence of a global attractor in  $\dot{H}^{s,p}(\mathbb{T})$  are proved.

## 1. Introduction

There are a lot of studies devoted to the global attractor of dynamical systems generated by nonlinear partial differential equations. The dynamical system, due to the damped effect, is usually dissipative in some Banach space X, namely it has a bounded absorbing set in X. To prove the compact property of solution semigroup, one may try to control the nonlinear term by Sobolev embedding and the dissipative bound in X. This is the reason why some growth restrictions need to be posed on the nonlinear terms. Following this line, roughly speaking, the growth restrictions on nonlinear term can be relaxed if the phase space is more regular, see [2] for a discussion on this topic for reaction diffusion equations. In a given phase space, it is very interesting to find the critical exponent of growth order for the nonlinear term, which has been done in [1] and [17]. However,

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 $<sup>\</sup>label{eq:condition} \textit{Key words and phrases.} \ \ \text{Global attractor}; \ \ \text{Benjamin-Bona-Mahony equation}; \ \ \text{low regularity}.$ 

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