

ON MULTIPLICITY OF EIGENVALUES AND SYMMETRY OF EIGENFUNCTIONS OF THE p -LAPLACIAN

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ABSTRACT. We investigate multiplicity and symmetry properties of higher eigenvalues and eigenfunctions of the p -Laplacian under homogeneous Dirichlet boundary conditions on certain symmetric domains $\Omega \subset \mathbb{R}^N$. By means of topological arguments, we show how symmetries of Ω help to construct subsets of $W_0^{1,p}(\Omega)$ with suitably high Krasnosel'skiĭ genus. In particular, if Ω is a ball $B \subset \mathbb{R}^N$, we obtain the following chain of inequalities:

$$\lambda_2(p; B) \leq \dots \leq \lambda_{N+1}(p; B) \leq \lambda_{\ominus}(p; B).$$

Here $\lambda_i(p; B)$ are variational eigenvalues of the p -Laplacian on B , and $\lambda_{\ominus}(p; B)$ is the eigenvalue which has an associated eigenfunction whose nodal set is an equatorial section of B . If $\lambda_2(p; B) = \lambda_{\ominus}(p; B)$, as it holds true for $p = 2$, the result implies that the multiplicity of the second eigenvalue is at least N . In the case $N = 2$, we can deduce that any third eigenfunction of the p -Laplacian on a disc is nonradial. The case of other symmetric domains and the limit cases $p = 1$, $p = \infty$ are also considered.

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