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## ON THE TOPOLOGICAL PRESSURE OF THE SATURATED SET WITH NON-UNIFORM STRUCTURE

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ABSTRACT. We derive a conditional variational principle of the saturated set for systems with the non-uniform structure. Our result applies to a broad class of systems including  $\beta$ -shifts, S-gap shifts and their subshift factors.

## 1. Introduction

Most results in multifractal analysis are applied to study the local asymptotic quantities, such as Birkhoff averages, Lyapunov exponents, local entropies, and pointwise dimensions, which reveal information about a single point or trajectory. It is of interest to study the level set for these quantities. A topological dynamical system  $(X, d, \sigma)$  (or  $(X, \sigma)$  for short) consists of a compact metric space (X, d) and a continuous map  $\sigma \colon X \to X$ . For a continuous function  $\psi \colon X \to \mathbb{R}$ , we always consider the following set:

$$X(\psi,\alpha) = \left\{ x \in X : \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \psi(\sigma^i x) = \alpha \right\}.$$

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