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## TOPOLOGICAL SHADOWING AND THE GROBMAN–HARTMAN THEOREM

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ABSTRACT. We give geometric proofs for the Grobman–Hartman theorem for diffeomorphisms and ODEs. Proofs use covering relations and cone conditions for maps and isolating segments and cone conditions for ODEs. We establish topological versions of the Grobman–Hartman theorem as the existence of some semiconjugaces.

## 1. Introduction

The goal of this paper is to give a new geometric proof of the Grobman–Hartman theorem [8]–[10] for diffeomorphisms and ODEs in finite dimension. By the 'geometric proof' we understand the proof which works in the phase space of the system under consideration and uses concepts of qualitative geometric nature.

We focus on the global version of the Grobman–Hartman theorem, which in the case maps states that, if  $A \colon \mathbb{R}^n \to \mathbb{R}^n$  is a hyperbolic linear isomorphism and if  $g \colon \mathbb{R}^n \to \mathbb{R}^n$  is given by

$$g(x) = Ax + h(x),$$

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