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GLOBAL AND LOCAL STRUCTURES OF OSCILLATORY BIFURCATION CURVES WITH APPLICATION TO INVERSE BIFURCATION PROBLEM

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Abstract. We consider the bifurcation problem

$$-u''(t) = \lambda(u(t) + g(u(t))), \quad u(t) > 0, \quad t \in I := (-1, 1), \quad u(\pm 1) = 0,$$

where $g(u) = g_1(u) := \sin\sqrt{u}$ and $g_2(u) := \sin u^2 (= \sin(u^2))$, and $\lambda > 0$ is a bifurcation parameter. It is known that λ is parameterized by the maximum norm $\alpha = \|u_\lambda\|_\infty$ of the solution u_λ associated with λ and is written as $\lambda = \lambda(g, \alpha)$. When $g(u) = g_1(u)$, this problem has been proposed in Cheng [4] as an example which has arbitrary many solutions near $\lambda = \pi^2/4$. We show that the bifurcation diagram of $\lambda(g_1, \alpha)$ intersects the line $\lambda = \pi^2/4$ infinitely many times by establishing the precise asymptotic formula for $\lambda(g_1, \alpha)$ as $\alpha \to \infty$. We also establish the precise asymptotic formulas for $\lambda(g_i, \alpha)$ $(i = 1, 2)$ as $\alpha \to \infty$ and $\alpha \to 0$. We apply these results to the new concept of inverse bifurcation problems.

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