

**SINGULARLY PERTURBED N -LAPLACIAN PROBLEMS
WITH A NONLINEARITY
IN THE CRITICAL GROWTH RANGE**

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ABSTRACT. We consider the following singularly perturbed problem:

$$-\varepsilon^N \Delta_N u + V(x)|u|^{N-2}u = f(u), \quad u(x) > 0 \quad \text{in } \mathbb{R}^N,$$

where $N \geq 2$ and $\Delta_N u$ is the N -Laplacian operator. In this paper, we construct a solution u_ε which concentrates around any given isolated positive local minimum component of V , as $\varepsilon \rightarrow 0$, in the Trudinger–Moser type of subcritical or critical case. In the subcritical case, we only impose on f the Berestycki and Lions conditions. In the critical case, a global condition on the nonlinearity f is imposed. However, any *monotonicity* of $f(t)/t^{N-1}$ or *Ambrosetti–Rabinowitz* type conditions are not required.

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