

ASYMPTOTIC BEHAVIOR FOR NONAUTONOMOUS FUNCTIONAL DIFFERENTIAL INCLUSIONS WITH MEASURES OF NONCOMPACTNESS

NGUYEN VAN DAC — TRAN DINH KE

ABSTRACT. We study the asymptotic behavior of nonautonomous differential inclusions with delays in Banach spaces by analyzing their pullback attractors. Our aim is to give a recipe expressed by measures of noncompactness to prove the asymptotic compactness of the process generated by our system. This approach is effective for various differential systems regardless of the compactness of the semigroup governed by linear part.

1. Introduction

We consider the following problem:

$$(1.1) \quad u'(t) \in Au(t) + F(t, u(t), u_t) \quad \text{for } t \geq \tau,$$

$$(1.2) \quad u(t) = \varphi^\tau(t - \tau) \quad \text{for } t \in [\tau - h, \tau],$$

where the state function u takes values in a separable Banach space X , A is a closed linear operator which generates a strongly continuous semigroup $\{S(t)\}_{t \geq 0}$ on X , F is a multivalued function defined on $[\tau, \infty) \times X \times C([-h, 0]; X)$, u_t is the history of the state function up to the time t , i.e. $u_t(s) = u(t + s)$ for $s \in [-h, 0]$, and φ^τ is an element of $C([-h, 0]; X)$.

2010 *Mathematics Subject Classification.* Primary: 35B41, 37L30; Secondary: 47H08, 47H10.

Key words and phrases. Measure of noncompactness; MNC estimate; functional differential inclusion; pullback attractor; condensing operator.

The work of the second author was supported by Vietnam Institute for Advanced Study in Mathematics — VIASM.

Differential inclusions of the form (1.1) emerge from a number of problems. In the monograph [20], Filippov presented a useful way to deal with differential equations with discontinuous right-hand sides, in which a regularized procedure leads to differential inclusions. Differential inclusions appear also in the control problems whose control factor is taken in the form of multivalued feedback. The presence of delayed terms in these problems is an inherent feature.

One of the most important questions concerning system (1.1)–(1.2) is to figure out the behavior of its solutions at large time, i.e. when $t - \tau \rightarrow +\infty$. In dealing with asymptotic behavior of differential equations without uniqueness or differential inclusions in autonomous form, there have been introduced and investigated such notions as generalized semiflows due to Ball [5], [6], multivalued semiflows due to Melnik and Valero [26]. A comparison of these two approaches was given in [14]. Thanks to the framework of Melnik and Valero, there have been many works devoted to the investigation of asymptotics for various classes of partial differential equations (PDEs) without uniqueness (see, e.g. [2], [3], [23], [30], [31]). We also refer to the theory of trajectory attractors developed by Chepyzov and Vishik [16] which is a fruitful way to study the long-time behavior of solutions of PDEs for which the uniqueness is unavailable. In order to study asymptotic behavior of nonautonomous differential systems, Melnik and Valero [27] proposed the framework of uniform global attractors for multivalued semiprocesses. Alternatively, the theory of pullback attractors has been developed for both nonautonomous and random dynamical systems in multivalued case by Caraballo et al. [8], [9] and [10].

In all frameworks, an essential step in formulating global attractors is to verify the asymptotic compactness condition for corresponding semiflows/processes. This condition holds if the semigroup governed by principal parts (i.e. $S(t) = e^{tA}$) is compact. However, for PDEs in unbounded domains the latter requirement is unrealistic. In these cases, one can use a nice condition expressed by measures of noncompactness (MNC), namely the ω -limit compact condition. We mention some typical works [24], [25], [36], [37] for single-valued dynamical systems, and [18], [35], [34] for multivalued ones, in which the ω -limit compactness was employed as a crucial condition. In concrete models formed by PDEs without delays, the testing of the ω -limit compact condition is usually replaced by checking the flattening condition, which is possible if one can construct a basis in phase spaces (see, e.g. [18], [25], [36], [37]). Unfortunately, it is impractical to check the latter condition for PDEs with delays since the corresponding phase spaces have complicated structure, i.e. it is impossible to find their basis. So our objective in this paper is to propose an effective way to verify the asymptotic compactness of multivalued nonautonomous dynamical systems (MNDS) generated by