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HOMOCLINIC SOLUTIONS FOR A CLASS OF AUTONOMOUS SECOND ORDER HAMILTONIAN SYSTEMS WITH A SUPERQUADRATIC POTENTIAL

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Abstract. We will prove the existence of a nontrivial homoclinic solution for an autonomous second order Hamiltonian system $\ddot{q}+\nabla V(q)=0$, where $q\in\mathbb{R}^n$, a potential $V\colon\mathbb{R}^n\to\mathbb{R}$ is of the form $V(q)=-K(q)+W(q)$, K and W are C^1-maps, K satisfies the pinching condition, W grows at a superquadratic rate, as $|q|\to\infty$ and $W(q)=o(|q|^2)$, as $|q|\to 0$. A homoclinic solution will be obtained as a weak limit in the Sobolev space $W^{1,2}(\mathbb{R},\mathbb{R}^n)$ of a sequence of almost critical points of the corresponding action functional. Before passing to a weak limit with a sequence of almost critical points each element of this sequence has to be appropriately shifted.

1. Introduction

This paper concerns the existence of homoclinic solutions for a certain class of autonomous second order Hamiltonian systems. Let us consider

$$\ddot{q}+\nabla V(q)=0, \tag{1.1}$$

where $q\in\mathbb{R}^n$ and a potential $V\colon\mathbb{R}^n\to\mathbb{R}$ satisfies the following conditions:

(H$_1$) $V(q)=-K(q)+W(q)$, where $K,W\colon\mathbb{R}^n\to\mathbb{R}$ are C^1-maps,

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(H_2) there are constants $b_1, b_2 > 0$ such that for all $q \in \mathbb{R}^n$,

$$b_1|q|^2 \le K(q) \le b_2|q|^2,$$

(H_3) $(q, \nabla K(q)) \le 2K(q)$ for all $q \in \mathbb{R}^n$,

(H_4) $2K(q) - (q, \nabla K(q)) = o(|q|^2)$, as $|q| \to 0$,

(H_5) ∇K is Lipschitzian in a neighbourhood of $0 \in \mathbb{R}^n$,

(H_6) $\nabla W(q) = o(|q|)$, as $|q| \to 0$,

(H_7) there is a constant $\mu > 2$ such that for every $q \in \mathbb{R}^n \setminus \{0\}$,

$$0 < \mu W(q) \le (q, \nabla W(q)).$$

Here and subsequently, $(\,\cdot\,,\,\cdot\,)\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denotes the standard inner product in $\mathbb{R}^n$ and $|\cdot|\colon \mathbb{R}^n \to [0, \infty)$ is the induced norm.

Note that if $K\colon \mathbb{R}^n \to \mathbb{R}$ is a C^2-map satisfying (H_2), then (H_4) takes place. Let us also remark that (H_6) and (H_7) imply

$$W(q) = o(|q|^2), \quad \text{as } |q| \to 0. \tag{1.2}$$

Moreover, from (H_7) it follows that for $q \ne 0$ a map given by

$$(0, \infty) \ni \zeta \mapsto W(\zeta^{-1}q)\zeta^{\mu}$$

is nonincreasing. Hence the following inequalities hold

$$W(q) \le W\left(\frac{q}{|q|}\right)|q|^{\mu} \quad \text{if } 0 < |q| \le 1, \tag{1.3}$$

$$W(q) \ge W\left(\frac{q}{|q|}\right)|q|^{\mu} \quad \text{if } |q| \ge 1. \tag{1.4}$$

By (H_2) and (1.4) we get that a potential V grows at a superquadratic rate, as $|q| \to \infty$, i.e.

$$\frac{V(q)}{|q|^2} \to \infty, \quad \text{as } |q| \to \infty.$$

Many mathematicians have written about Hamiltonian systems with a superquadratic potential, for example: V. Coti Zelati, I. Ekeland and E. Séré in [4], H. Hofer and K. Wysocki in [7], V. Coti Zelati and P. Rabinowitz in [5], P. Rabinowitz and K. Tanaka in [14], W. Omana and M. Willem in [11]. Our assumptions on the potential V are natural, since one can immediately produce a lot of examples.

It is easily seen that $q \equiv 0$ is a solution of (1.1). In this work we are interested in the existence of nontrivial homoclinic solutions of (1.1) that emanate from 0 and terminate at 0, i.e. $\lim_{t\to\pm\infty} q(t) = q(\pm\infty) = 0$.

The existence of homoclinic orbits for first and second order Hamiltonian systems has been studied by many authors and the literature on this subject is vast (see [1], [2], [6], [8], [9], [12], [15]), but many questions are still open (see the survey [13] by P. Rabinowitz). Finding homoclinic solutions in Hamiltonian systems can be quite difficult. In the last 20 years, a great progress was made by applying variational methods (see the survey [3] by T. Bartsch and A. Szulkin). For instance, the authors of [4] studied a class of first order Hamiltonian systems using a dual variational transformation and the Mountain Pass Theorem to prove the existence of two distinct homoclinic solutions. P. Rabinowitz in [12] examined a family of second order Hamiltonian systems applying the Mountain Pass Theorem to get a sequence of subharmonic solutions and suitable estimates to pass to a nontrivial limit which occurred to be a nontrivial homoclinic solution (see also [2], [8], [9]).

The theorem which we shall prove is as follows.

THEOREM 1.1. *If the assumptions* (H_1)–(H_7) *are satisfied then the Hamiltonian system* (1.1) *possesses a nontrivial homoclinic solution* $q_0 \in W^{1,2}(\mathbb{R}, \mathbb{R}^n)$ *such that* $\dot{q}_0(\pm\infty) = 0$.

This result is proved in Section 2 by studying the corresponding to (1.1) action functional $I: W^{1,2}(\mathbb{R}, \mathbb{R}^n) \to \mathbb{R}$. Similarly to [14], by a general minimax principle (see Theorem 2.3) we obtain a sequence of almost critical points. However, its weak limit has not to be nontrivial. In order to get a nontrivial homoclinic orbit before passing to a weak limit with a sequence of almost critical points each element of this sequence has to be appropriately shifted.

2. Proof of Theorem 1.1

The proof of Theorem 1.1 will be divided into a sequence of lemmas. Let E be the Sobolev space $W^{1,2}(\mathbb{R}, \mathbb{R}^n)$ with the standard norm

$$\|q\|_E := \left(\int_{-\infty}^{\infty} (|q(t)|^2 + |\dot{q}(t)|^2)\, dt \right)^{1/2}.$$

We first recall two elementary inequalities concerning functions in E.

FACT 2.1. *If* $q: \mathbb{R} \to \mathbb{R}^n$ *is a continuous mapping such that* $\dot{q} \in L^2_{\mathrm{loc}}(\mathbb{R}, \mathbb{R}^n)$, *then for every* $t \in \mathbb{R}$,

$$|q(t)| \leq \sqrt{2} \left(\int_{t-1/2}^{t+1/2} (|q(s)|^2 + |\dot{q}(s)|^2)\, ds \right)^{1/2}. \tag{2.1}$$

The proof of Fact 2.1 can be found in [8]. (See Fact 2.8, p. 385.)

FACT 2.2. *For each* $q \in E$,

$$\|q\|_{L^\infty(\mathbb{R}, \mathbb{R}^n)} \leq \sqrt{2} \|q\|_E. \tag{2.2}$$

Fact 2.2 is a direct consequence of the inequality (2.1).

Let $I\colon E\to\mathbb{R}$ be given by

$$I(q):=\int_{-\infty}^{\infty}\left[\frac{1}{2}|\dot q(t)|^2-V(q(t))\right]dt.$$

By (H_5)–(H_6) it is obvious that $I\in C^1(E,\mathbb{R})$. Moreover,

$$I'(q)w=\int_{-\infty}^{\infty}[(\dot q(t),\dot w(t))-(\nabla V(q(t)),w(t))]\,dt$$

for all $q,w\in E$ and any critical point of I on E is a classical solution of (1.1) with $q(\pm\infty)=0$, as is easy to verify. In order to prove Theorem 1.1, we apply a general minimax principle. Let us remind it.

Theorem 2.3 (see Theorem 4.3 in [10]). *Let K be a compact metric space, $K_0\subset K$ a closed subset, X a Banach space and $\chi\in C(K_0,X)$. Let $\mathcal{M}$ be a complete metric space given by*

$$\mathcal{M}:=\{g\in C(K,X):g(s)=\chi(s)\ \text{if}\ s\in K_0\}$$

with the usual distance. Let $\varphi\in C^1(X,\mathbb{R})$ and let us define

$$c=\inf_{g\in\mathcal{M}}\max_{s\in K}\varphi(g(s)),\quad c_1=\max_{\chi(K_0)}\varphi.$$

If $c>c_1$ then for each $\varepsilon>0$ and for each $h\in\mathcal{M}$ such that

$$\max_{s\in K}\varphi(h(s))\le c+\varepsilon$$

there exists $v\in X$ such that

$$c-\varepsilon\le\varphi(v)\le\max_{s\in K}\varphi(h(s)),\quad \operatorname{dist}(v,h(K))\le\varepsilon^{1/2},\quad \|\varphi'(v)\|_{X^*}\le\varepsilon^{1/2}.$$

Set $\overline{b}_1:=\min\{1,2b_1\}$, $\overline{b}_2:=\max\{1,2b_2\}$, where b_1, b_2 are the constants of the pinching condition (H_2). By definition, $\overline{b}_1\le 1\le\overline{b}_2$. From ($H_2$) we have

$$I(q)\ge\frac{1}{2}\overline{b}_1\|q\|_E^2-\int_{-\infty}^{\infty}W(q(t))\,dt \tag{2.3}$$

for every $q\in E$. By (1.2), (2.2) and (2.3), we conclude that there are constants $\alpha,\varrho>0$ such that

$$I(q)\ge\alpha,\quad \text{if } \|q\|_E=\varrho. \tag{2.4}$$

Take $\nu\in C_0^\infty(\mathbb{R},\mathbb{R}^n)$ such that $|\nu(t)|=1$ for $|t|\le1$ and $\nu(t)=0$ for $|t|\ge2$. Set

$$m:=\inf\{W(q):|q|=1\}.$$

From (1.4), for every $\xi\in\mathbb{R}$ such that $|\xi|\ge1$, we have

$$\int_{-\infty}^{\infty}W(\xi\nu(t))\,dt\ge\int_{-1}^{1}W(\xi\nu(t))\,dt\ge\int_{-1}^{1}W\left(\frac{\xi\nu(t)}{|\xi\nu(t)|}\right)|\xi\nu(t)|^\mu\,dt\ge 2m|\xi|^\mu.$$

Combining this with (H_2) we obtain

$$I(\xi\nu) \leq \frac{1}{2}\bar{b}_2\xi^2\|\nu\|_E^2 - 2m|\xi|^\mu.$$

Since $m > 0$ and $\mu > 2$, for $|\xi|$ sufficiently large, $I(\xi\nu) < 0$. Consequently, there exists $Q \in E$ such that

$$\|Q\|_E > \varrho \quad \text{and} \quad I(Q) < 0 = I(0). \tag{2.5}$$

From now on, let

$$\mathcal{M} := \{g \in C([0,1], E) : g(0) = 0 \text{ and } g(1) = Q\} \tag{2.6}$$

and

$$c := \inf_{g\in\mathcal{M}} \max_{s\in[0,1]} I(g(s)). \tag{2.7}$$

By (2.4)–(2.7), we get $c \geq \alpha > 0$.

Applying Theorem 2.3 we conclude that the following lemma holds.

LEMMA 2.4. *There exists a sequence* $\{q_k\}_{k\in\mathbb{N}}$ *in* E *such that*

$$I(q_k) \to c \quad \textit{and} \quad I'(q_k) \to 0, \quad \textit{as } k \to \infty. \tag{2.8}$$

LEMMA 2.5. *The sequence* $\{q_k\}_{k\in\mathbb{N}}$ *given by* (2.8) *is bounded in* E.

PROOF. By (2.8), for large k,

$$\|I'(q_k)\|_{E^*} < 2 \quad \text{and} \quad |I(q_k) - c| < 1. \tag{2.9}$$

Applying (H_3) and (H_7) we obtain

$$I(q_k) - \frac{1}{2}I'(q_k)q_k \geq \left(\frac{\mu}{2} - 1\right)\int_{-\infty}^{\infty} W(q_k(t))\,dt \tag{2.10}$$

for $k \in \mathbb{N}$. Combining (2.10) with (2.9) we receive

$$c + 1 + \|q_k\|_E \geq \left(\frac{\mu}{2} - 1\right)\int_{-\infty}^{\infty} W(q_k(t))\,dt$$

for large k, and hence

$$\int_{-\infty}^{\infty} W(q_k(t))\,dt \leq \frac{2}{\mu - 2}(c + 1 + \|q_k\|_E). \tag{2.11}$$

By the use of (H_2), (H_3) and (H_7), we get

$$I'(q_k)q_k \leq \bar{b}_2\|q_k\|_E^2 - \mu\int_{-\infty}^{\infty} W(q_k(t))\,dt \tag{2.12}$$

for $k \in \mathbb{N}$. From (2.3) and (2.12) it follows that

$$\frac{1}{\bar{b}_1}I(q_k) - \frac{1}{\mu\bar{b}_2}I'(q_k)q_k \geq \left(\frac{1}{2} - \frac{1}{\mu}\right)\|q_k\|_E^2 - \left(\frac{1}{\bar{b}_1} - \frac{1}{\bar{b}_2}\right)\int_{-\infty}^{\infty} W(q_k(t))\,dt \tag{2.13}$$

for $k \in \mathbb{N}$. By (2.9) and (2.13), for large k,

$$\frac{1}{\overline{b}_1}(c+1) + \|q_k\|_E \geq \left(\frac{1}{2} - \frac{1}{\mu}\right)\|q_k\|_E^2 - \left(\frac{1}{\overline{b}_1} - \frac{1}{\overline{b}_2}\right)\int_{-\infty}^{\infty} W(q_k(t))\,dt. \tag{2.14}$$

Finally, from (2.11) and (2.14), for large k,

$$\left(\frac{1}{2} - \frac{1}{\mu}\right)\|q_k\|_E^2 \leq \frac{1}{\overline{b}_1}(c+1) + \|q_k\|_E + \frac{2}{\mu-2}\left(\frac{1}{\overline{b}_1} - \frac{1}{\overline{b}_2}\right)(c+1+\|q_k\|_E). \tag{2.15}$$

Since $\mu > 2$, (2.15) shows that $\{q_k\}_{k\in\mathbb{N}}$ is bounded in E. $\square$

For each $k \in \mathbb{N}$ there is $\tau_k \in \mathbb{R}$ such that a map $q_{\tau_k}\colon \mathbb{R} \to \mathbb{R}^n$ given by

$$q_{\tau_k}(t) := q_k(t+\tau_k),$$

where $t \in \mathbb{R}$, achieves a maximum at $0 \in \mathbb{R}$, i.e.

$$\max\{|q_{\tau_k}(t)|\colon t \in \mathbb{R}\} = |q_{\tau_k}(0)|. \tag{2.16}$$

Then $q_{\tau_k} \in E$ and it is easy to check that $\|q_{\tau_k}\|_E = \|q_k\|_E$, $I(q_{\tau_k}) = I(q_k)$ and $\|I'(q_{\tau_k})\|_{E^*} = \|I'(q_k)\|_{E^*}$. In consequence, by Lemma 2.4,

$$I(q_{\tau_k}) \to c \quad \text{and} \quad I'(q_{\tau_k}) \to 0, \tag{2.17}$$

as $k \to \infty$, and by Lemma 2.5, the sequence $\{q_{\tau_k}\}_{k\in\mathbb{N}}$ is bounded in E. Since E is a reflexive Banach space, $\{q_{\tau_k}\}_{k\in\mathbb{N}}$ possesses a weakly convergent subsequence in E.

Let q_0 denote a weak limit of a weakly convergent subsequence of $\{q_{\tau_k}\}_{k\in\mathbb{N}}$. Without loss of generality, we will write

$$q_{\tau_k} \rightharpoonup q_0 \quad \text{in } E, \tag{2.18}$$

as $k \to \infty$, which implies $q_{\tau_k} \to q_0$ in $L^\infty_{\mathrm{loc}}(\mathbb{R}, \mathbb{R}^n)$, as $k \to \infty$.

Lemma 2.6. *q_0 given by* (2.18) *is a homoclinic solution of* (1.1).

Proof. Since $q_0 \in E$, we see that $q_0(t) \to 0$, as $t \to \pm\infty$, by Fact 2.1. Therefore, it is sufficient to show that $I'(q_0) = 0$. Fix $w \in C_0^\infty(\mathbb{R}, \mathbb{R}^n)$ and assume that for some $A > 0$, $\mathrm{supp}(w) \subset [-A, A]$. We have

$$I'(q_{\tau_k})w = \int_{-A}^{A} [(\dot{q}_{\tau_k}(t), \dot{w}(t)) - (\nabla V(q_{\tau_k}(t)), w(t))]\,dt$$

for each $k \in \mathbb{N}$. From (2.17) it follows that $I'(q_{\tau_k})w \to 0$, as $k \to \infty$. On the other hand,

$$\int_{-A}^{A} (\dot{q}_{\tau_k}(t), \dot{w}(t))\,dt \to \int_{-A}^{A} (\dot{q}_0(t), \dot{w}(t))\,dt,$$

as $k \to \infty$, by (2.18), and

$$\int_{-A}^{A} (\nabla V(q_{\tau_k}(t)), w(t))\,dt \to \int_{-A}^{A} (\nabla V(q_0(t)), w(t))\,dt,$$

as $k \to \infty$, because $q_{\tau_k} \to q_0$ uniformly on $[-A, A]$. Thus $I'(q_{\tau_k})w \to I'(q_0)w$, as $k \to \infty$, and, in consequence, $I'(q_0)w = 0$. Since $C_0^\infty(\mathbb{R}, \mathbb{R}^n)$ is dense in E, we get $I'(q_0) = 0$. □

LEMMA 2.7. *Let q_0 be given by* (2.18). *Then $\dot{q}_0(t) \to 0$, as $t \to \pm\infty$.*

PROOF. From Fact 2.1, we obtain

$$|\dot{q}_0(t)|^2 \leq 2\int_{t-1/2}^{t+1/2} |\ddot{q}_0(s)|^2\,ds + 2\int_{t-1/2}^{t+1/2} (|q_0(s)|^2 + |\dot{q}_0(s)|^2)\,ds.$$

For this reason, it suffices to notice that

$$\int_r^{r+1} |\ddot{q}_0(s)|^2\,ds \to 0,$$

as $r \to \pm\infty$. Since q_0 satisfies (1.1), we have

$$\int_r^{r+1} |\ddot{q}_0(s)|^2\,ds = \int_r^{r+1} |\nabla V(q_0(s))|^2\,ds.$$

Take $\varepsilon > 0$. By (H_5) and (H_6), there is $\eta > 0$ such that for $|q| < \eta$, $|\nabla V(q)| < \varepsilon$. Moreover, there is $\delta > 0$ such that, if $|s| > \delta$, then $|q_0(s)| < \eta$. Hence, if $|r| > \delta + 1$, then

$$\int_r^{r+1} |\nabla V(q_0(s))|^2\,ds < \varepsilon^2,$$

which completes the proof. □

To finish the proof of Theorem 1.1, we have to show that $q_0 \neq 0$.

On the contrary, suppose that $q_0 \equiv 0$. Consequently, we have $q_{\tau_k}(0) \to 0$, as $k \to \infty$. From (2.16) it follows that $q_{\tau_k} \to 0$ uniformly on $\mathbb{R}$, as $k \to \infty$. By (2.17) and the boundedness of $\{q_{\tau_k}\}_{k\in\mathbb{N}}$ in E, we get $2I(q_{\tau_k}) - I'(q_{\tau_k})q_{\tau_k} \to 2c > 0$, as $k \to \infty$. On the other hand, by (H_4), (H_6) and (1.2),

$$\begin{aligned}
2I(q_{\tau_k}) - I'(q_{\tau_k})q_{\tau_k} &= \int_{-\infty}^{\infty} [(\nabla V(q_{\tau_k}(t)), q_{\tau_k}(t)) - 2V(q_{\tau_k}(t))]\,dt \\
&= \int_{-\infty}^{\infty} [2K(q_{\tau_k}(t)) - (\nabla K(q_{\tau_k}(t)), q_{\tau_k}(t))]\,dt \\
&\quad + \int_{-\infty}^{\infty} [(\nabla W(q_{\tau_k}(t)), q_{\tau_k}(t)) - 2W(q_{\tau_k}(t))]\,dt \to 0,
\end{aligned}$$

as $k \to \infty$. Indeed. Take $\varepsilon > 0$. From (H_4), (H_6) and (1.2), we deduce that there is $\delta > 0$ such that if $|q| < \delta$, then $|2K(q) - (\nabla K(q), q)| \leq \varepsilon|q|^2$, $|\nabla W(q)| \leq \varepsilon|q|$ and $|W(q)| \leq \varepsilon|q|^2$. Since $q_{\tau_k} \to 0$ uniformly on $\mathbb{R}$, there is $k_0 \in \mathbb{N}$ such that for $k > k_0$ and for $t \in \mathbb{R}$, $|q_{\tau_k}(t)| < \delta$. Hence $|2I(q_{\tau_k}) - I'(q_{\tau_k})q_{\tau_k}| \leq 4\varepsilon\|q_{\tau_k}\|_E^2$ for $k > k_0$, which contradicts (2.17).

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