

**NONTRIVIAL CRITICAL GROUPS
IN p -LAPLACIAN PROBLEMS
VIA THE YANG INDEX**

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ABSTRACT. We construct and variationally characterize by a min-max procedure involving the Yang index a new sequence of eigenvalues of the p -Laplacian, and use the structure provided by this sequence to show that the associated variational functional always has a nontrivial critical group. As an application we obtain nontrivial solutions for a class of p -superlinear problems.

1. Introduction

Let Φ be a C^1 functional defined on a Banach space W . In Morse theory the local behavior of Φ near an isolated critical point u_0 at the level c is described by the critical groups

$$(1.1) \quad C_q(\Phi, u_0) = H_q(\Phi^c \cap U, \Phi^c \cap U \setminus \{u_0\})$$

where $\Phi^c = \{u \in W : \Phi(u) \leq c\}$, U is a neighbourhood of u_0 containing no other critical point, and H denotes singular homology. Critical groups distinguish between different types of critical points and are extremely useful for obtaining multiple solutions of variational problems (see e.g. [3]).

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In this paper we study the critical groups of

$$(1.2) \quad I_\lambda(u) = \int_\Omega |\nabla u|^p - \lambda |u|^p, \quad u \in W = W_0^{1,p}(\Omega)$$

at the origin, where Ω is a bounded domain in \mathbb{R}^N , $N \geq 1$, $1 < p < \infty$, and λ is a real parameter. Nonzero critical points of I_λ are the eigenfunctions of the nonlinear eigenvalue problem

$$(1.3) \quad \begin{cases} -\Delta_p u = \lambda |u|^{p-2} u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ is the p -Laplacian, so when $\lambda \notin \sigma(-\Delta_p)$, 0 is the only critical point of I_λ and hence $C_q(I_\lambda, 0)$ are defined.

In the semilinear case $p = 2$, I_λ is a C^2 functional defined on the Hilbert space $H_0^1(\Omega)$ and 0 is a nondegenerate critical point of Morse index l if $\lambda \in (\mu_l, \mu_{l+1})$, where $\{\mu_l\}_{l \in \mathbb{N}}$ are the eigenvalues of $-\Delta$ repeated according to their multiplicity, so

$$(1.4) \quad C_q(I_\lambda, 0) = \delta_{ql} \mathcal{G}$$

where \mathcal{G} is the coefficient group (see e.g. [3]). In contrast, the critical groups seem difficult to compute in the quasilinear case $p \neq 2$ for a variety of reasons. To begin with, we are no longer working in a Hilbert space, so the standard tools such as the splitting lemma and the shifting theorem do not apply. Moreover, except for the fact that there is an unbounded sequence of min-max eigenvalues $\{\mu_l\}_{l \in \mathbb{N}}$, very little is known about the spectrum, and there are no eigenspaces to work with. The only results that the author is aware of in this case are those of Dancer and the author himself (see [8]) showing that

$$C_q(I_\lambda, 0) = \begin{cases} \delta_{q0} \mathcal{G} & \text{if } \lambda \in (-\infty, \mu_1), \\ \delta_{q1} \mathcal{G} & \text{if } \lambda \in (\mu_1, \mu_2), \\ 0 & \text{if } \lambda \in (\mu_2, \infty) \setminus \sigma(-\Delta_p) \text{ and } q = 0 \text{ or } 1. \end{cases}$$

In particular, it is not known whether there is a nontrivial critical group when $\lambda \in (\mu_2, \infty) \setminus \sigma(-\Delta_p)$. We will construct an unbounded sequence of variational eigenvalues $\{\lambda_l\}_{l \in \mathbb{N}}$ such that

PROPOSITION 1.1. *If $\lambda \in (\lambda_l, \lambda_{l+1}) \setminus \sigma(-\Delta_p)$, then*

$$(1.6) \quad C_l(I_\lambda, 0) \neq 0.$$

REMARK 1.2. (1.4) and (1.6) also hold for $l = 0$ if we set $\mu_0 = \lambda_0 = -\infty$.

Recall that a connected component of $\{x \in \Omega : u(x) \neq 0\}$ is called a *nodal domain* of u . We will also obtain the following estimate on the number of nodal domains of an eigenfunction.

PROPOSITION 1.3. *If $\lambda \in \sigma(-\Delta_p)$ has an associated eigenfunction with l nodal domains, then $\lambda \geq \lambda_l$. In particular, any eigenfunction of λ_l has at most l nodal domains if $\lambda_l < \lambda_{l+1}$.*

After some preliminaries on the Yang index in the next section, we will prove these propositions in Section 3. The usual Lusternik–Schnirelmann characterization of μ_l involves a min-max over a class of sets of genus $\geq l$, but we will define λ_l using the subclass of sets of Yang index $\geq l - 1$, which have the advantage of having nontrivial reduced homology groups in dimension $l - 1$. This also implies that $\lambda_l \geq \mu_l$.

In the ODE case $n = 1$, μ_l is simple and has an eigenfunction with l nodal domains (see e.g. [7]), so we also have $\lambda_l \leq \mu_l$ by Proposition 1.3. When $p = 2$, $(\lambda_{l-1}, \lambda_l) \subset (\mu_{l-1}, \mu_l)$ by (1.4) and (1.6), so $\lambda_l = \mu_l$ again. Similarly, $\lambda_1 = \mu_1$ and $\lambda_2 = \mu_2$ for all p by (1.5) and (1.6). So we have

PROPOSITION 1.4. $\lambda_l = \mu_l$ in the cases: (a) $n = 1$, (b) $p = 2$, (c) $l = 1, 2$.

In the last section we consider as an application the p -superlinear problem

$$(1.7) \quad \begin{cases} -\Delta_p u = f(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where f is a Carathéodory function on $\Omega \times \mathbb{R}$ satisfying

- (f₁) $|f(x, t)| \leq C(|t|^{q-1} + 1)$ for some $q < Np/(N - p)$ if $p < N$ and ∞ if $p \geq N$,
- (f₂) $0 < \mu F(x, t) \leq tf(x, t)$ for $|t|$ large, where $F(x, t) = \int_0^t f(x, s) ds$, for some $\mu > p$,
- (f₃) the limit $\lambda = \lim_{t \rightarrow 0} f(x, t)/|t|^{p-2} t$ exists uniformly in x .

We will prove

THEOREM 1.5. *If $\lambda \notin \sigma(-\Delta_p)$, then (1.7) has a nontrivial solution.*

REMARK 1.6. This also follows from the mountain-pass lemma when $\lambda < \lambda_1$, and the case $\lambda_1 < \lambda < \lambda_2$ was proved by Liu (see [11]).

REMARK 1.7. Fadell and Rabinowitz in [10] have used the Yang index to obtain results on the number of solutions of variational bifurcation problems in Hilbert spaces. See Coffman [4]–[6] for other uses and a different definition of the index.

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2. Yang index

In this section we briefly recall the definition and some properties of the Yang index. Yang (see [12]) considered compact Hausdorff spaces with fixed-point-free

continuous involutions and used the Čech homology theory, but for our purposes here it suffices to work with closed symmetric subsets of Banach spaces that do not contain the origin and singular homology groups.

Following [12], we first construct a special homology theory defined on the category of all pairs of closed symmetric subsets of Banach spaces that do not contain the origin and all continuous odd maps of such pairs. Let (X, A) , $A \subset X$ be such a pair and $C(X, A)$ its singular chain complex with \mathbb{Z}_2 coefficients, and denote by $T_{\#}$ the chain map of $C(X, A)$ induced by the antipodal map $T(x) = -x$. We say that a q -chain c is symmetric if $T_{\#}(c) = c$, which holds if and only if $c = c' + T_{\#}(c')$ for some q -chain c' . The symmetric q -chains form a subgroup $C_q(X, A; T)$ of $C_q(X, A)$, and the boundary operator ∂_q maps $C_q(X, A; T)$ into $C_{q-1}(X, A; T)$, so these subgroups form a subcomplex $C(X, A; T)$. We denote by

$$(2.1) \quad Z_q(X, A; T) = \{c \in C_q(X, A; T) : \partial_q c = 0\},$$

$$(2.2) \quad B_q(X, A; T) = \{\partial_{q+1} c : c \in C_{q+1}(X, A; T)\},$$

and

$$(2.3) \quad H_q(X, A; T) = Z_q(X, A; T)/B_q(X, A; T)$$

the corresponding cycles, boundaries, and homology groups. A continuous odd map $f: (X, A) \rightarrow (Y, B)$ of pairs as above induces a chain map $f_{\#}: C(X, A; T) \rightarrow C(Y, B; T)$ and hence homomorphisms

$$(2.4) \quad f_*: H_q(X, A; T) \rightarrow H_q(Y, B; T).$$

EXAMPLE 2.1 ([12, Example 1.8]). For the l -sphere,

$$(2.5) \quad H_q(S^l; T) = \begin{cases} \mathbb{Z}_2 & \text{for } 0 \leq q \leq l, \\ 0 & \text{for } q > l. \end{cases}$$

Let X be as above, and define homomorphisms $\nu: Z_q(X; T) \rightarrow \mathbb{Z}_2$ inductively by

$$(2.6) \quad \nu(z) = \begin{cases} \text{In}(c) & \text{for } q = 0, \\ \nu(\partial c) & \text{for } q > 0, \end{cases}$$

if $z = c + T_{\#}(c)$, where the index of a 0-chain $c = \sum_i n_i \sigma_i$ is defined by $\text{In}(c) = \sum_i n_i$. As in [12], ν is well defined and $\nu B_q(X; T) = 0$, so we can define the *index homomorphism* $\nu_*: H_q(X; T) \rightarrow \mathbb{Z}_2$ by $\nu_*([z]) = \nu(z)$.

PROPOSITION 2.2 ([12, Proposition 2.8]). *If F is a closed subset of X such that $F \cup T(F) = X$ and $A = F \cap T(F)$, then there is a homomorphism*

$$\Delta: H_q(X; T) \rightarrow H_{q-1}(A; T)$$

such that $\nu_*(\Delta[z]) = \nu_*([z])$.

Taking $F = X$ we see that if $\nu_* H_l(X; T) = \mathbb{Z}_2$, then $\nu_* H_q(X; T) = \mathbb{Z}_2$ for $0 \leq q \leq l$. We define the *Yang index* of X by

$$(2.7) \quad i_Y(X) = \inf\{l \geq -1 : \nu_* H_{l+1}(X; T) = 0\},$$

taking $\inf \emptyset = \infty$. Clearly, $\nu_* H_0(X; T) = \mathbb{Z}_2$ if $X \neq \emptyset$, so $i_Y(X) = -1$ if and only if $X = \emptyset$.

EXAMPLE 2.3 ([12, Example 3.4]). $i_Y(S^l) = l$.

PROPOSITION 2.4 ([12, Proposition 2.4]). *If $f: X \rightarrow Y$ is as above, then $\nu_*(f_*([z])) = \nu_*([z])$ for $[z] \in H_q(X; T)$, and hence $i_Y(X) \leq i_Y(Y)$. In particular, this inequality holds if $X \subset Y$.*

Recall that the *Krasnosel'skiĭ Genus* of X is defined by

$$(2.8) \quad \gamma(X) = \inf\{l \geq 0 : \text{there exists a continuous odd map } f: X \rightarrow S^{l-1}\}$$

(see e.g. [3]). By Example 2.3 and Proposition 2.4 we have following propositions.

PROPOSITION 2.5. $\gamma(X) \geq i_Y(X) + 1$.

PROPOSITION 2.6. *If $i_Y(X) = l \geq 0$, then the reduced homology group $\tilde{H}_l(X) \neq 0$.*

PROOF. By (2.7),

$$(2.9) \quad \nu_* H_q(X; T) = \begin{cases} \mathbb{Z}_2 & \text{for } 0 \leq q \leq l, \\ 0 & \text{for } q > l. \end{cases}$$

We show that if $[z] \in H_l(X; T)$ is such that $\nu_*([z]) \neq 0$, then $[z] \neq 0$ in $\tilde{H}_l(X)$. Arguing indirectly, assume that $z \in B_l(X)$, say, $z = \partial c$. Since $z \in B_l(X; T)$, $T_\#(z) = z$. Let $c' = c + T_\#(c)$. Then $c' \in Z_{l+1}(X; T)$ since $\partial c' = z + T_\#(z) = 2z = 0 \pmod 2$, and $\nu_*([c']) = \nu(c') = \nu(\partial c) = \nu(z) \neq 0$, contradicting $\nu_* H_{l+1}(X; T) = 0$. \square

3. Variational eigenvalues and critical groups

As is well-known, the eigenvalues of (1.3) are the critical values of

$$(3.1) \quad I(u) = \int_\Omega |\nabla u|^p, \quad u \in S = \{u \in W : \|u\|_p = 1\},$$

which satisfies the Palais–Smale condition (PS) (see e.g. [9]). Denote by \mathcal{A} the class of closed symmetric subsets of S , let

$$(3.2) \quad \mathcal{F}_l = \{A \in \mathcal{A} : i_Y(A) \geq l - 1\},$$

and set

$$(3.3) \quad \lambda_l := \inf_{A \in \mathcal{F}_l} \max_{u \in A} I(u).$$

PROPOSITION 3.1. λ_l is an eigenvalue of $-\Delta_p$ and $\lambda_l \nearrow \infty$.

PROOF. If λ_l is not a critical value of I , then there is an $\varepsilon > 0$ and an odd homeomorphism $\eta: S \rightarrow S$ such that $\eta(I^{\lambda_l + \varepsilon}) \subset I^{\lambda_l - \varepsilon}$ by a lemma of Bonnet ([2]) (the standard first deformation lemma is not sufficient here as the manifold S is not of class $C^{1,1}$ when $p < 2$). Take $A \in \mathcal{F}_l$ with $\max I(A) \leq \lambda_l + \varepsilon$ and set $\tilde{A} = \eta(A)$. Then $\tilde{A} \in \mathcal{A}$ since η is an odd homeomorphism and $i_Y(\tilde{A}) \geq i_Y(A) \geq l - 1$ by Proposition 2.4, so $\tilde{A} \in \mathcal{F}_l$, but $\max I(\tilde{A}) \leq \lambda_l - \varepsilon$, a contradiction.

Since $\mathcal{F}_l \supset \mathcal{F}_{l+1}$, $\lambda_l \leq \lambda_{l+1}$. To see that $\lambda_l \rightarrow \infty$, recall that this holds for the Lusternik–Schnirelmann eigenvalues $\mu_l := \inf_{A \in \mathcal{G}_l} \max_{u \in A} I(u)$ where $\mathcal{G}_l = \{A \in \mathcal{A} : \gamma(A) \geq l\}$. $\mathcal{F}_l \subset \mathcal{G}_l$ by Proposition 2.5, so $\lambda_l \geq \mu_l$. \square

PROOF OF PROPOSITION 1.1. We can take $U = W$ in (1.1) as 0 is the only critical point of I_λ ;

$$(3.4) \quad C_l(I_\lambda, 0) = H_l(I_\lambda^0, I_\lambda^0 \setminus \{0\})$$

where $I_\lambda^0 = \{u \in W : I_\lambda(u) \leq 0\}$. Since I_λ is positive homogeneous, I_λ^0 is radially contractible to 0 and $I_\lambda^0 \setminus \{0\}$ is homotopic to $I_\lambda^0 \cap S$ via the radial projection onto S , so it follows from the long exact sequence of reduced homology groups for the pair $(I_\lambda^0, I_\lambda^0 \setminus \{0\})$ that

$$(3.5) \quad H_l(I_\lambda^0, I_\lambda^0 \setminus \{0\}) \cong \tilde{H}_{l-1}(I_\lambda^0 \cap S) = \tilde{H}_{l-1}(I^\lambda)$$

where the last equality follows from $I_\lambda|_S = I - \lambda$. Since I is even, $I^\lambda \in \mathcal{A}$, and since $\lambda > \lambda_l$, there is an $A \in \mathcal{F}_l$ such that $A \subset I^\lambda$, so $i_Y(I^\lambda) \geq i_Y(A) \geq l - 1$ by Proposition 2.4. On the other hand, $I^\lambda \notin \mathcal{F}_{l+1}$ since $\lambda < \lambda_{l+1}$, so $i_Y(I^\lambda) < l$. Hence $i_Y(I^\lambda) = l - 1$, and $\tilde{H}_{l-1}(I^\lambda) \neq 0$ by Proposition 2.6. \square

PROOF OF PROPOSITION 1.3. Take an eigenfunction u of λ with nodal domains Ω_i , $i = 1, \dots, l$, and define a continuous odd map $f: S^{l-1} \rightarrow S$ by

$$(3.6) \quad f(\xi_1, \dots, \xi_l)(x) = \begin{cases} |\xi_i|^{2/p-1} \xi_i \frac{u(x)}{\|u\|_{L^p(\Omega_i)}} & \text{if } x \in \Omega_i, \\ 0 & \text{if } x \notin \bigcup_{i=1}^l \Omega_i. \end{cases}$$

The image $A = f(S^{l-1}) \in \mathcal{A}$ and $i_Y(A) \geq i_Y(S^{l-1}) = l - 1$ by Example 2.3 and Proposition 2.4, so $A \in \mathcal{F}_l$, and $I = \lambda$ on A . \square

4. Proof of Theorem 1.5

The condition (f_2) , originally introduced in the semilinear case $p = 2$ by Ambrosetti and Rabinowitz ([1]), implies that

$$(4.1) \quad |f(x, t)| \geq C |t|^{\mu-1}, \quad F(x, t) \geq C |t|^\mu$$

and that the variational functional

$$(4.2) \quad \Phi(u) = \int_{\Omega} |\nabla u|^p - p F(x, u), \quad u \in W$$

associated with (1.7) satisfies (PS) (see e.g. [11]).

First we construct a perturbed functional $\tilde{\Phi}$ that has the same critical points as Φ and equals the asymptotic functional I_λ near zero and Φ near infinity.

LEMMA 4.1. *There are $\rho > 0$ and $\tilde{\Phi} \in C^1(W, \mathbb{R})$ such that*

$$(4.3) \quad \tilde{\Phi}(u) = \begin{cases} I_\lambda(u) & \text{for } \|u\| \leq \rho, \\ \Phi(u) & \text{for } \|u\| \geq 2\rho, \end{cases}$$

and 0 is the only critical point of Φ and $\tilde{\Phi}$ with $\|u\| \leq 2\rho$.

PROOF. Let

$$g(x, t) = f(x, t) - \lambda |t|^{p-1}t, \quad G(x, t) = \int_0^t g(x, s) ds, \quad \Psi(u) = p \int_{\Omega} G(x, u),$$

so that $\Phi = I_\lambda - \Psi$. Since $\lambda \notin \sigma(-\Delta_p)$, I_λ satisfies (PS) and has no critical points on the unit sphere S_1 in W , so $\delta := \inf_{S_1} \|I'_\lambda\| > 0$.

By homogeneity, $\inf_{S_\rho} \|I'_\lambda\| = \rho^{p-1} \delta$, while it follows from (f_3) that

$$\sup_{S_\rho} |\Psi| = o(\rho^p) \quad \text{and} \quad \sup_{S_\rho} \|\Psi'\| = o(\rho^{p-1})$$

as $\rho \rightarrow 0$, so

$$(4.4) \quad \inf_{S_\rho} \|\Phi'\| \geq \rho^{p-1}(\delta + o(1)) > 0$$

for all sufficiently small $\rho > 0$. Take a smooth function $\varphi: [0, \infty) \rightarrow [0, 1]$ such that

$$(4.5) \quad \varphi(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq 1, \\ 0 & \text{for } t \geq 2, \end{cases}$$

and set

$$(4.6) \quad \tilde{\Phi}(u) = \Phi(u) + \varphi(\|u\|/\rho) \Psi(u).$$

Since $\|d(\varphi(\|u\|/\rho))\| = O(\rho^{-1})$, (4.3) holds with Φ replaced by $\tilde{\Phi}$ also, and the conclusion follows. □

Next we turn to the behavior of $\tilde{\Phi}$ at infinity.

LEMMA 4.2. *There is an $a_0 < 0$ such that for all $a < a_0$, $\tilde{\Phi}_a$ is homotopic to S_1 and hence contractible.*

PROOF. For $u \in S_1$ and $t \geq 2\rho$,

$$(4.7) \quad \tilde{\Phi}(tu) = t^p - p \int_{\Omega} F(x, tu) \leq t^p - Ct^{\mu} \|u\|_{\mu}^{\mu} \rightarrow -\infty \quad \text{as } t \rightarrow \infty$$

by (4.1), and

$$(4.8) \quad \frac{d}{dt} \tilde{\Phi}(tu) = p \left(t^{p-1} - \int_{\Omega} u f(x, tu) \right) = \frac{p}{t} \left(\tilde{\Phi}(tu) + \int_{\Omega} H(x, tu) \right)$$

where $H(x, t) := pF(x, t) - tf(x, t) \leq -(\mu - p)F(x, t) < 0$ for $|t|$ large by (f₂). Let

$$(4.9) \quad a_0 = \min \left\{ -\sup_{\Omega \times \mathbb{R}} H, \inf_{B_{2\rho}} \tilde{\Phi} \right\}.$$

If $a < a_0$ and $\tilde{\Phi}(tu) \leq a$, then $t \geq 2\rho$ and $d\tilde{\Phi}(tu)/dt < 0$, so there is a unique $t_0 = t_0(u) \geq 2\rho$ such that $\tilde{\Phi}(tu) > a$ for $0 \leq t < t_0$, $\tilde{\Phi}(t_0u) = a$, and $\tilde{\Phi}(tu) < a$ for $t > t_0$, and the map $t_0: S_1 \rightarrow [2\rho, \infty)$ is C^1 by the implicit function theorem. It follows that $\tilde{\Phi}_a = \{tu : u \in S_1, t \geq t_0(u)\}$ has the homotopy type of S_1 . \square

Now we are ready to prove Theorem 1.2. We have $\lambda_l < \lambda < \lambda_{l+1}$ for some l , so

$$(4.10) \quad C_l(\tilde{\Phi}, 0) = C_l(I_{\lambda}, 0) \neq 0$$

by Lemma 4.1 and Proposition 1.1. On the other hand, taking a as in Lemma 4.2,

$$(4.11) \quad C_q(\tilde{\Phi}, \infty) = H_q(W, \tilde{\Phi}_a) = 0 \quad \text{for all } q.$$

Since $C_l(\tilde{\Phi}, 0) \not\cong C_l(\tilde{\Phi}, \infty)$, $\tilde{\Phi}$ must have a second critical point (see e.g. [3]).

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