1. Introduction

Our aim in this paper is humble. We want to offer a formal account of the reasoning structure at certain stages of a particular class of problem-solving processes. As a paradigmatic example of these processes we consider reasoning involved in a criminal investigation. We shall present our ideas on a literary rather than real material – detective stories by Arthur Conan Doyle, of which Mr Sherlock Holmes is the main protagonist. We use literary examples rather than real data for a reason: we want to demonstrate that our ideas work on a commonly accepted testbed for erotetic and investigative reasoning. Our claim is that important parts

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1 See e.g. Emmanuel Genot, “Strategies of inquiry. The ‘Sherlock Holmes sense of deduction’ revisited”. *Synthese*, 195(5) (2018): 2065–2088 for similar analyses in terms of Hintikka’s Interrogative Model of Inquiry or David Carson, “The abduc-
of this reasoning may be adequately modelled in terms of Inferential Erotetic Logic, in particular Erotetic Search Scenarios. We start with a short introduction of this formal apparatus in section 2. Then, in section 3, we present the models of two examples taken from the stories: “The Adventure of the Golden Pince-Nez” and “The Adventure of the Six Napoleons”. In section 4 we answer the question: What kind of reasoning, in terms of a classification of reasoning processes, is accounted for by our models.

2. Inferential Erotetic Logic

2.1. Basics

The focus of Inferential Erotetic Logic (IEL) is on inferences which premises or conclusions are questions (the name of the logic stems from Greek erotema – “question”). Wiśniewski offers a detailed presentation of formal aspects of IEL. IEL has found substantial applications in modelling actual cognitive activities. It has proven to be useful in accounting for general problem-solving and reasoning processes, modeling participants’ behavior in dialogue, and cooperative behavior in particular.

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4 Wiśniewski, The Posing of Questions; idem, Questions, Inferences, and Scenarios.


Also, metaepistemic reasoning processes may be successfully accounted for in terms of IEL. Research on applications of IEL gave rise to Erotetic Reasoning Corpus, a dataset for research on natural question processing.

IEL starts with a simple observation that there are cases in which our epistemic needs concerning obtaining some information not already in our possession give rise to questions. This ‘giving rise’ may be erratic and random, but, again, there are cases in which there is some intuitively reasonable justification to it. Let us consider the following example. Suppose that my initial problem consists in identifying a tarts thief (it may be expressed by a question Q1: “Who stole the tarts?”). Suppose also that I managed to establish the following evidence E1: “It is one of the courtiers of the Queen of Hearts attending the afternoon tea-party who stole the tarts”. In order to solve the Q1 problem, and in view of E1 it is intuitively justified to ask the following question Q2: “Which of the Queen of Hearts’ courtiers attended the afternoon tea-party?”. If moreover I know that E2: “Queen of Hearts invites for a tea-party only these courtiers who made her laugh the previous day”, then Q2 and E2 imply (in a non-technical sense of the word, as yet) the question Q3: “Which courtiers made the Queen of Hearts laugh the previous day?”. Now, let us identify what makes these two inferences, from Q1 and E1 to Q2 and from Q2 and E2 to Q3, intuitively justified.

The first factor is, that if the initial question (question-premise) may be truthfully answered (we shall call such questions sound), and if the evidence gathered is true, then the resulting question (question-conclusion) also may be answered truthfully (that is, it is sound as well). Consider the first inference: If the tarts were indeed stolen (which means that there exists a true answer to Q1) and if the range of possible culprits is adequately identified by E1 (which also presumes that some of the courtiers attended the tea party in question), then there must exist a true answer to Q2. The concept of an answer is construed here as the one of a direct answer, which is “directly and precisely responsive to the ques-

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tion, giving neither more nor less information than what is called for”;\textsuperscript{11} under this account, a direct answer need not to be true, it just needs to deliver the required type and amount of information; “It’s 5” is as direct an answer to the question “What is the result of multiplication of 2 by 2?” as “It’s 4”. The second factor is, that answering the question-conclusion must be useful in answering the question-premise in the sense that each answer to the question-conclusion, in view of the available evidence, narrows down the set of possibilities offered by the question-premise. In the case of our first exemplary inference this amounts to the fact that having identified the attendees of the tea party we narrowed down the set of possible tart thieves (provided that E1 is accurate). Notice that both factors are also present in the case of the second inference.

IEL formalizes these intuitions. The first factor is that of transmission of truth/soundness into soundness: if the initial question is sound (i.e., there exists a true direct answer to this question) and all the declarative premises, if there are any, are true, then the question which is the conclusion must be sound as well. The second one is cognitive usefulness: each direct answer to a question which is the conclusion is useful in answering the initial question by narrowing down the set of possibilities offered by the initial question (more precisely: for each direct answer \(B\) to the question which is the conclusion there exists a non-empty proper subset \(Y\) of the set of direct answers to the initial question such that \(Y\) must contain a true direct answer to the initial question if both: \(B\) is true and the declarative premises, if there are any, are true). Taken together these factors form the definition of erotetic implication (e-implication for short) which is a ternary relation holding between a question (the initial one), a set of declarative premises (‘evidence’) and a question (the conclusion); Wiśniewski\textsuperscript{12} offers more precise definition of e-implication. We shall be considering questions which allow for finite number of direct answers only, as this is the type of questions we most commonly use; however, the IEL ideas generalize to the infinite case as well.

E-implication is just one example of a semantic relation which models arising of questions, along with some flavors of it: a weak one\textsuperscript{13} and a falsificationist one\textsuperscript{14}. Another example is evocation of questions,\textsuperscript{15} which models inferences leading from just declarative premises to ques-


\textsuperscript{12} Wiśniewski, \textit{Questions, Inferences, and Scenarios}, 67.

\textsuperscript{13} Urbański et al., “Erotetic Problem Solving”.


\textsuperscript{15} Wiśniewski, \textit{The Posing of Questions}; idem, “An axiomatic account of question evocation: The propositional case”, \textit{Axioms} 5.
tions; it is a case of evocation that information that somebody stole the tarts leads to our initial question Q1: “Who stole the tarts?”.

E-implication allows for modelling many aspects of natural question processing, in which an initial question is internally processed by an agent, and where the outcome is either a new question concerning the subject matter or a strategy of reducing the initial question to auxiliary questions. In both cases, e-implication allows for the description and assessment of the inferences which lead from questions to questions.\(^\text{16}\)

IEL-style logic of questions may be build on top on any declarative logic which meets a rather modest semantic criteria; in particular, it needs to allow for distinguishing true formulas from untrue ones (for details see two chapters on Minimal Erotetic Semantics in Wiśniewski (2013)).\(^\text{17}\) We shall use here just Classical Propositional Calculus. We shall employ Polish prefix notation with N standing for negation, K for conjunction, A for simple disjunction, D for exclusive disjunction, C for implication and E for equivalence.

One more remark is in order here. IEL represents questions according to the set-of-answers methodology.\(^\text{18}\) The idea stems from Hamblin’s postulate: “Knowing what counts as an answer is equivalent to knowing the question”. A question is an expression of the form \(?\{A_1, \ldots, A_n\}\), where \(A_1, \ldots, A_n\) are direct answers to the question. Thus, for example, a formula \(?\{A, NA\}\) represents a simple yes-no question: “Is it the case that \(A\) or is it the case that non-\(A\)?”; this kind of questions we shall abbreviate by \(?A\).

### 2.2. Erotetic Search Scenarios

When solving problems by questioning more often than not we ask sequences of questions, in which each consecutive one is somehow motivated by the previous questions as well as by the answers obtained to them, and possibly some declarative premises. Moreover, sometimes we form plans for questioning: anticipating possible answers we may design a kind of scenario, which nodes are questions, and different paths are determined by different answers to them.

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\(^{16}\) Łupkowski et al., “Epistemic Erotetic Search Scenarios”, 611.


Let us continue our initial example. Suppose that in addition to the previous evidence I managed to establish the following E3: “Three courtiers made the Queen of Hearts laugh the other day: the Knave of Hearts, the Duchess, and one of the rose painters”. Also, gathering information about these suspects, I collected the following E4: “The rose-painter stole the tarts if and only if there are crumbs near the rose-bushes”, E5: “If the Duchess didn’t steal the tarts she is not looking very happy”, E6: “If the Knave of Hearts never spoke during the tea-party it is him who stole the tarts”. Now I’m in a position to lay out a plan for further investigation, which will be determined by the sequences of questions and possible answers to them. It may be represented by the tree-like structure (Fig. 1).

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Q1
  E1
  Q2
  E2
  Q3
  E3 ApAqr
  E4 Ers
  E5 CNqNt
  E6 Cup

Q4 ?s
  s Ns

  Apq

    ?\{u, t\}
    u t

    p q
```

Figure 1. An erotetic search scenario for the tarts problem.
This is an Erotetic Search Scenario (ESS) for Q1 relative to E1, E2. A scenario may be defined either in terms of a labelled tree or in terms of a family of erotetic derivations (which are paths of the tree); again, more precise definition of an ESS is offered by Wiśniewski. Such scenarios provide in fact conditional instructions for solving the initial problem – answering the initial question, in our case Q1 – relative to declarative evidence, E1, E2, and dependent upon the auxiliary questions, Q2 to Q4, in particular upon answers obtained to the queries (these are auxiliary questions answered in the scenario, in our example Q3 and Q4). With the exception of the first one, each question in a scenario is e-implied by a previous one (and, possibly, some declaratives). Each declarative which is neither a piece of initial evidence (relatively to which the scenario is constructed), nor an answer to a query, is implied by preceding declaratives. The last element of each path is an answer to the initial question. One important property of ESS is that if the initial question is sound and if all the declarative evidence is true, then at least one path of a scenario is such that all the questions on it are sound and all the declarative formulas are true – including the last one, which is an answer to the initial question. This is the essence of the Golden Path Theorem.

3. Stories and Models

Our claim is that Erotetic Search Scenarios are good formal accounts of the reasoning structure of certain stages of a particular class of problem-solving processes. In order to support this claim we shall refer to paradigmatic examples of allegedly applying deduction in solving problems – Sherlock Holmes detective stories by Arthur Conan Doyle.

For these readers who are not familiar with Doyle’s detective stories a warning is in place. We are going to spoil them a little bit – obviously – however, not entirely. We shall not present them in detail, just use the elements necessary to build the models; inevitably, these include solutions.

3.1. The Adventure of the Golden Pince-Nez

There is a couple of related problems Holmes is solving in this case, which include: (1) how did the murder of Mr Willoughby Smith, a sec-

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20 Wiśniewski, Questions, Inferences, and Scenarios, 113 and 116.
21 Ibidem, 116.
retary to prof. Coram, happen? (2) who is the culprit? (3) where is the culprit? (4) what was the motive? As for (1), the murder weapon was a sealing-wax knife belonging to prof. Coram. As for (2), Holmes manages to determine the culprit is a female. Then (3) becomes crucial and Holmes arrives at the following question:

(PQ1) $\{p, Kqr\}$: Did the culprit ran from the house by the garden path or has she run to prof. Coram’s room and is still hiding there?

This question is sound relatively to the ‘evidence’, which in this particular case is a hypothesis, that either the first or the second answer to PQ1 is true:

(PE1) $DpKqr$

Notice, that slightly weaker formula, $ApKqr$, is a presupposition of PQ1.

There are two further pieces of information Holmes manages to establish as relevant:

(PE2) $CKqrs$: If the culprit has run to prof. Coram’s room and is hiding there, then prof. Coram gets additional food portions.

(PE3) $EtKqr$: There is a hiding place in prof. Coram’s room with visible traces of use if an only if the culprit run to prof. Coram’s room and is hiding there.

And that’s all. What is left is to ask relevant questions, answers to which will lead to solution of the initial problem PQ1. The whole scenario is presented in Fig. 2.

There is one question, PQ2: $\{p, Kqr, Ns\}$, not explicitly present in the story. Its presence in the scenario is required by one peculiar property of e-implication: it is not transitive. This is quite an intuitive property: that I know that a question Qk gives rise to a question Qm and that Qm gives rise to Qn is not tantamount to me knowing that Qk gives rise to Qn; as a result, this helps to avoid in IEL an interrogative counterpart to the logical omniscience paradox. However, in order to retain e-implication, in some reasoning reconstructions we need to resort to employment of auxiliary questions which are not queries. Using PQ2 as a kind of bridging device we may now identify the following e-implications in the Golden Pince-Nez scenario:

(1) PQ1 on the basis of PE2 e-implies PQ2;
(2) PQ2 on the basis of PE1 e-implies $s$
(3) PQ1 on the basis of PE3 e-implies $t$
“How did you know that, Holmes?”. Inferential Erotetic Logic in Formal Modelling

\[ \{p, Kqr\} \]
\[ DpKqr \]
\[ CKqrs \]
\[ EtKqr \]
\[ \{p, Kqr, Ns\} \]

Figure 2. An erotetic search scenario for the Golden Pince-Nez problem.

What is worth noticing is that from a purely formal point of view the question \(?s\) is redundant. Holmes might have just asked \(?t\) and reach solution on the basis of the answer, as in the alternative scenario (Fig. 3), no auxiliary no-queries required.

\[ \{p, Kqr\} \]
\[ DpKqr \]
\[ EtKqr \]

Figure 3. An alternative erotetic search scenario for the Golden Pince-Nez problem.
Why not, then? The reason is of a pragmatic character. Establishing the answer to ?t required carrying out a time-consuming experiment (smoking a great number of cigarettes and dropping the ash all over the space in front of the suspected hiding place). An answer to ?s could have been obtained simply by chatting to the housekeeper, who is eager to provide information. While the negative answer to ?s would lead to an answer to the initial question, only the positive one would require further investigation. Although the first scenario is more complex than the alternative one there is one advantage to it: It contains a path which requires the least amount of resources to be engaged, and the remaining paths are comparable in that respect to the alternatives. Thus it is reasonable to give it a shot.

The scenario represents the problem’s solving space. Which of its paths becomes actualized depends on obtained answers to the queries. In this particular case answers to both queries of the first scenario, ?s and ?t, turned out to be affirmative ones. As a result, the leftmost path of the scenario became actualized and the culprit was indeed found in the hiding place in prof. Coram’s room.

3.2. The Adventure of the Six Napoleons

What we are about to model is solving the problem of where is the black pearl of the Borgias. Holmes managed to narrow down the set of possibilities to be considered to three, and to ask the following question:

(SQ1) \(?p, q, r\): Is Beppo the Italian in the possession of the pearl, or is the pearl in the plaster Napoleon bust owned by Mr Brown, or in the one owned by Mr Sandeford?

As these are mutually exclusive possibilities, there is an obvious declarative premise:

(SE1) ENpDqr: Beppo is not in the possession of the pearl if and only if the pearl is in the plaster Napoleon bust owned by Mr Brown, or in the one owned by Mr Sandeford.

The next premise is a kind of gamble: Mr Brown’s house is located much closer to the crime scene of the previous burglary than Mr Sandeford’s:

(SE2) EsNp: Beppo will soon arrive at Mr Brown’s house if and only if the pearl is not in his possession.

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23 This story is available at https://en.wikisource.org/wiki/The_Adventure_of_the_Six_Napoleons and the plot is summarized at https://bakerstreet.fandom.com/wiki/The_Adventure_of_the_Six_Napoleons; access 8.05.2019.
If time was no constraint there would be a simple systematic course of action to be undertaken, the one ignoring SE2 and consisting in eliminating consecutive possibilities. We may represent it by the ESS presented in Fig. 4.

\[
\begin{align*}
?\{p, q, r\} \\
\text{ENpDqr} \\
\text{q} &\quad \text{Nq} \\
?r &\quad \text{Nr} \\
\text{NDqr} \\
p
\end{align*}
\]

Figure 4. An erotetic search scenario for the Six Napoleons problem – no time constraints.

However, time was a constraint in this case. Also, Beppo proven to be quite a dangerous man, so catching him as soon as possible became a goal as important as determining the location of the pearl. Thus Holmes made use of SE2 and his reasoning may be modelled by another scenario (Fig. 5).

Notice, that both SE1 and SE2 are in fact just hypothetical ones. However, only SE2 is identified as a gamble: SE1 is well-justified in view of meticulously crafted hypothesis, expressed via SQ1. The development of events allowed to establish the affirmative answer to ?s and the negative one to ?q (the middle path of the scenario became actualized). As a result, to much astonishment of Dr Watson and inspector Lestrade, Holmes was able to identify the location of the pearl and in his typical theatrical manner smash down the Sandeford’s Napoleon bust in front of them just in order to find the gem among the shards.
4. What Kind of Reasoning Is That?

It is a well-known fact, often ascertained by Doyle, that Holmes’s reasoning is a deductive one. It is becoming well-known also, that deduction is not all what is to this reasoning; there is a strong abductive component in it.\textsuperscript{24} In abduction we account for puzzling or otherwise interesting phenomena by deriving hypotheses which make them coherent with our knowledge or beliefs.\textsuperscript{25} In Holmes’s own words these are the cases in which “we have been compelled to reason backward from effects to causes”.\textsuperscript{26} This last claim captures only one of possible faces of abduc-


\textsuperscript{26} In \textit{The Adventure of the Cardboard Box”,} in: Doyle, \textit{The Complete Sherlock Holmes.}
tion, the causal explanatory one. Nevertheless, it accounts well for the type of reasoning involved in solving Holmesian cases.

Abduction is a compound type of reasoning in the sense of Ajdukiewicz. It consists of both generation and evaluation of hypotheses, as in this making sense of puzzling phenomena we are not interested just in showdown of ideas but in finding the best (even if only locally and not globally) account on the phenomenon in question. In particular, in the evaluation phase testing of prospective hypotheses involves deductive reasoning. So, what type of reasoning exactly is modelled by our Erotetic Search Scenarios in the discussed examples?

Alas, neither IEL in general nor ESSs in particular offer a systematic account on creativity, or a logic of discovery. Notice, that our initial questions in both cases expressed in fact competing explanatory hypotheses and scenarios characterized the logical space of testing these hypotheses. The second author developed a schema for Holmesian problem-solving, in which the place of ESSs is clearly indicated (Fig. 6). There are two reasoning modules in this model. Problem-solving starts with deduction on gathered data (due to reflexivity of logical entailment relation memory retrieval may be interpreted as a case of deduction, too). If it fails to deliver a solution to the problem – and it always fails, that’s the point of detective stories – the role of the second module is to produce hypotheses which correctness is not guaranteed by the data, but which are worth considering in view of the data – which means, that they undergo some preliminary testing. The ones deemed most promising are then chosen for a more minute scrutiny. This is quite close to Peircean concept of abduction as providing preliminary insights to be entertained for the time being.

However, in order to be tested a hypothesis needs first to be arrived at. On this our scenarios tell nothing. Still, there is at least one important factor of Holmesian reasoning overlooked as yet, which is precisely captured by ESSs, and e-implication in particular: the goal-directedness.

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30 Peirce, *Collected Works*, 5.188.

Figure 6. A schema of Holmesian problem-solving.

Were Holmesian hypotheses tests just deductive, it would be justified to arrive at any logically entailed conclusion, however irrelevant with respect to the matter being considered it could be. This is obviously not the case. But how to account for this goal-directedness in terms of deductive reasoning? E-implication helps. The first condition of the definition of e-implication, transmission of truth/soundness into soundness, mirrors, in an erotetic setting, the definitory property of deduction. The second condition, cognitive usefulness, adds something new: it warrants that each consecutive e-implied question narrows down the set of possibilities entertained by the previous e-implying ones, directing the whole problem-solving process towards solution. According to this model, only such questions may enter the scenario which are helpful in answering the previous ones – ultimately, only such questions are allowed answers to which enable achieving the goal of answering the initial question. As a sidenote, let us point out that even in the deductive module of the above model there must be some erotic component involved. This is due to the fact that deductions, which are aimed at deriving solution to the given problem based on data gathered are goal-directed, too.

Admittedly, e-implication inherits all the weaknesses of declarative deductive reasoning: in particular, the Golden Path Theorem offers just a conditional warrant of arriving at a true answer to the initial question. If the assumptions are incorrect, even Holmes may err; this happens in the story “The Yellow Face”, at the end of which, after failing to get even close to the correct solution of the mystery, Holmes asks Watson: “if it should ever strike you that I am getting a little over-confident in my powers, or giving less pains to a case than it deserves, kindly whisper
'Norbury’ in my ear, and I shall be infinitely obliged to you”. But this is the doing of logic, erotetic or declarative: it is providing the forms, and the substance the reasoner puts into them is a different matter.

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Bibliography


**Summary**

We offer a formal account of the reasoning structure at certain stages of a particular class of problem-solving processes, by means of Inferential Erotetic Logic. Our ideas are presented on a commonly accepted testbed for erotetic and investigative reasoning, detective stories by Arthur Conan Doyle. We also address the issue of what kind of reasoning is accounted for by our models: our claim is that they capture goal-directed deductions.
Streszczenie

”Jak na to wpadłeś, Holmesie?”. Inferencyjna Logika Pytań w formalnym modelowaniu rozwiązywania problemów w dochodzeniach Sherlocka Holmesa

Przedstawiamy formalne ujęcie struktury rozumowań zaangażowanych w rozwiązywanie szczególnej klasy problemów, wykorzystując w tym celu Inferencyjną Logikę Pytań. Nasze przykłady czerpiemy ze źródła powszechnie wykorzystywanego do testowania modeli rozumowań erotetycznych i dochodzeniowych: opowiadań Arthura C. Doyle’a. Odnosimy się również do zagadnienia, jaki typ rozumowania jest modelowany w proponowanych przez nas rozwiązaniach, twierdząc, że jest to nastawiona na cel dedukcja.

Słowa kluczowe: Inferencyjna Logika Pytań, scenariusze poszukiwań dla pytań, dedukcja nastawiona na cel, rozumowania śledcze, historie detektywistyczne, Sherlock Holmes