RUCH FILOZOFICZNY
LXXVI 2020 2

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DOI: http://dx.doi.org/10.12775/RF.2020.023

Kotarbiński's Reism Versus the Possibility of the Multiplicity Theory*

How is mathematics possible? That is one of Kant's[1] famous questions that takes on special meaning in the light of the philosophical concept called reism. Mathematics is considered to be the field which constitutes the biggest problem for reism, owing to the fact that the names of its subjects, such as "number", "geometric figure" or "set" are, according to reism, apparent names or – to put it simply – the objects of mathematics simply do not exist. The development of contemporary mathematics has led to the realization that the main field of mathematics or even the basis of mathematics is the so-called theory of multiplicity (set theory). In these circumstances, the question about the possibility of mathematics

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* This text is an elaboration on certain ideas signalled in my other earlier essay on Kotarbiński's philosophy of mathematics entitled: *Reizm a zagadnienie prawdziwości twierdzeń matematyki* (in: *Racjonalność w myśleniu i działaniu. Filozofia Tadeusza Kotarbińskiego*, Bydgoszcz 2017, 35–50).

[1] We do not mean to stress the outstanding importance of Kant for the contemporary philosophy of mathematics – as this is very debatable – but only to emphasize his role in initiating and organizing philosophical reflection on the main spheres of Western science, including mathematics. Kant's question about the possibility of mathematics, even despite the disputable (even doubtful) achievements of the author of the *Critique of Pure Reason* in this matter, may be regarded as a kind of symbol of various important issues present in the philosophy of mathematics, although, for obvious reasons, the meaning of this question and Kant's answer to it are of a special kind, and certainly differ from the one in this text devoted to reism.

must eventually take the form of the question: How is the theory of multiplicity possible in the light of reism?

Kotarbiński experienced this "problem with mathematics" and was clearly aware of its meaning. This paper will not refer to and analyse the basic theses of his reism (it is assumed that readers possess basic knowledge on the subject), nor will his general thesis on the non-existence of mathematical objects be reviewed. If such matters are somehow addressed here, it is done only marginally, as far as the given problematic context requires. In principle, this study will also not directly deal with the issue of the possibility of interpreting the multiplicity theory in reistic categories. Such an interpretation is in principle impossible. The point here is of a different nature: to present possibilities of the multiplicity theory based on certain ideas contained in the writings of Kotarbiński, whose attitude to reism, though quite unclear, does not give an impression of something radically contradictory to reism. The strategy by Kotarbiński has been based on an approach which was quite original, though at the same time slightly puzzling from the point of view of reism, that involved a kind of distancing oneself from the issue of the existence of mathematical "entities". It was not the only idea (some other will be indicated here as well), but it seems the most promising. What exactly does it consist in and what is its value (especially from the point of view of multiplicity theory)? These are the questions we want to address here.

If reism were only a particular ontological or metaphysical position,[2] i.e. forming the claims about the existence (and non-existence) of certain objects, including mathematical ones, there would be nothing extraordinary about it. Numerous equally extreme views have appeared

2 With regard to Kotarbiński's reism, the term "ontology" is used, not "metaphysics". It is possible, however, to have some reservations about it, since reism "is a view on the world, in particular on its existential assets" (Jan Woleński, *Filozoficzna szkoła lwowsko-warszawska* (Warszawa: PWN, 1985), 208), which means that such a theory should rather be called metaphysical than ontological. Reism is not only a network of specific notions about some (possible) reality and its structure, but above all a collection of statements about the existence or non-existence of specific objects in our world. Of course, to a large extent, the choice of such a term is due to certain philosophical beliefs and for these reasons, just like many fundamental issues in philosophy, can be the subject of dispute. This dispute could possibly be alleviated by assuming that, ultimately, every metaphysics contains an ontology of some kind. The use of the term "ontology" in relation to reism might result from some general aversion to metaphysics at the time when this view was being formed, as well as from the popularity of the notion of ontology itself at that time (e.g. Leśniewski's ontology or Heidegger's fundamental ontology et al.). It seems, however, that the sense of reism (and at the same time its relation to Leśniewski's ontology, which Kotarbiński utilised) is best conveyed by the following words of Woleński: "Leśniewski's ontology was an attempt at a 'logistic' – m e t a p h y s i c a l – approach to the general theory of things, and reism [emphasis M. Ch.]", ibidem, 219).

in philosophy, and this is perhaps a distinguishing feature of this field altogether. But the ontological theses of reism transferred onto the epistemological ground or, for example, to the philosophy of science, and mathematics in particular, become the source of fundamental difficulties that a reist would certainly want to avoid, especially if in his "scientific worldview", so to speak, he wanted for some reason to place mathematics as a field in which real scientific problems are considered and whose claims are supposed to have the quality of being true. Faced with these difficulties, a reist must eventually either "suspend" or simply reject his reism altogether, or find an ingenious "circumvention" to preserve both, i.e. his reistic beliefs and his ability to practice mathematics. This is especially important when a reist or someone who has at least some inclination to do so finds in mathematics an interesting field of research. It will be not without significance here that the situation is all the more serious, given that the very important field of mathematics, which has been developing considerably since the beginning of the twentieth century, also assumes certain mathematical categories (e.g. the concept of a set, relations, etc.), and therefore it is also exposed to attacks from reism.[3]

It is quite commonly believed that the multiplicity theory is the part of mathematics with which reism has the greatest problems.[4] It was admitted by Kotarbiński himself. Considering the fundamental significance of the multiplicity theory in contemporary mathematics, this very issue should be recognized as the weakest side of reism, even disqualifying this position from the point of view of the philosophy of mathematics (if, of course, we would like to defend the sensibility of mathematics and we would not be satisfied with the radicalism of the reistic approach). The whole thing is to be already disintegrated at the basic level concerning the concept and existence of a set – the basic category of the multiplicity theory. If, according to reism, sets were not to exist (in other words, if the name "set" were to be an apparent name), a reconstruction of the

[3] Perhaps the greatest importance for the development of metamathematics in Poland was attached to Tarski's achievements. On the other hand, his inclination towards reism is well known; this – owing to the problems mentioned above – must provoke certain questions to which Tarski offered better or worse answers. It is also necessary to remember about some links with reism of a different logician – Leśniewski (especially his theory of names) and, on the other hand, about his contribution to the development of the basics of metamathematics (a distinction between language and meta-language and between the logical system and the commentary to the system). Both Tarski and Kotarbiński, each in a defined area, owed a lot to Leśniewski, which fact they referred to on various occasions.

[4] "The most serious allegations involve the reistic interpretation of plurality theory. [...] in reistic language, the term 'set' is an apparent name; apparent names are also the names of any concepts that are defined in plurality theory using the concept of a set, e.g. 'relation', 'function', 'number' or 'order'" (Woleński, *Filozoficzna szkoła lwowsko-warszawska*, 222).

multiplicity theory would not be possible on the basis of reism. What is more, the fact that, in the light of the contemporary theory of multiplicity, no distinction is made between sets and their elements which are not sets means that from the point of view of this theory only sets exist. And this thesis is certainly no longer acceptable to reism. As Woleński concludes: "An elementary formalization of the multiplicity theory is possible, but at the price of assuming that only sets exist, and this price cannot be paid by a reist under any circumstances".[5] Thus the situation seems hopeless. However, if – for some reasons – reism allowed for the existence of sets, it would have to encounter serious problems with such concepts as "set of sets", "empty sets" or "infinite sets". Kotarbiński also wrote about some of these problems, admitting that in the light of reism they are probably insurmountable difficulties.

On the other hand, however, Kotarbiński did not completely give up the concept of a set and repeatedly showed its applications, including in relation to mathematics. Of course, it was not only as an impartial commentator, e.g. in his lectures on the history of mathematics or logic, that he referred to specific mathematical theories and, for this purpose, he had to use the notion of a set without raising any fundamental doubts about it. He also referred to the concept of a set when it was an attempt to interpret mathematics in accordance with the principles of reism. In one of the passages in which he tried to interpret the concept of a number in a reistic manner, he says, for example, that the symbol "4" could be treated as the name of a set of walls of a room.[6] It is evident that a set is understood here not in a distributive sense, but in a mereological (collective) way, and that reism does permit the existence of such sets.[7] The distinction between sets in a collective (mereological) and distributive sense is considered fundamental from a logical point of view, which of course has consequences for the multiplicity theory, since the latter operates with the distributive concept of a set. A set in the collective sense is understood as an aggregate, a whole composed of parts, and the parts that are part of it can be called "elements" in the sense of being components of this whole. Thus, such a concept of a set is related to the relation of "being part of a given whole". A set is understood differently in the distributive sense. Here this relationship is "the belonging of elements to a set".[8] In the distributive sense, a set is therefore a collection of multiple elements of a particular type, belonging to a set, not as parts of a whole

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[5] Ibidem, 223.

[6] Tadeusz Kotarbiński, *Ontologia, teoria poznania i metodologia nauk* (Wrocław et al.: Ossolineum, 1993), 103.

[7] The collective or mereological concept of a set is the basis of Leśniewski's mereology, which Kotarbiński used and whose categories he assimilated to his reism.

[8] *Mała encyklopedia logiki* (Wrocław et al.: Ossolineum, 1988), 224.

composed of it, but as elements belonging to a class of objects. The terms closest to the concept of a "set" are "class", "collection", "multiplicity", "species" or "type".

A collection in a collective sense includes, for example, all things and organisms, as long as we treat them as a properly organized whole, which seems particularly attractive from the point of view of reism. Since the world of nature, as well as the world of material (and partly social, as will later be discussed) human creations, is made up of such objects, it is full of this type of "sets" in itself, itself as a whole also constituting such a set. In fact, every real thing, that is, which exists for a reist in the proper sense, is, in his opinion, a set (in a collective sense), because it is a "whole" consisting of parts. Paradoxically, one can even see that the very concept of a set from the point of view of reism could be (and such a conditional way of speaking is necessary here, which will be shown further on) that concept which, of all the concepts relating to mathematical objects, is the only one which has a real reference to reality. For the reist, neither numbers nor geometric figures exist; but sets (in a collective sense) certainly do. It is therefore not true that there is no place for sets in reism, if one does not specify what type of set is involved. Unfortunately, in reism, there is no room for sets within the meaning defined by the theory of multiplicity.

Thus, for a reist in the full sense of the word, there exist, not only sets as single, concrete bodies (for this reason, reism is also called somatism and concretism), but also sets as "complex bodies", such as e.g. planetary systems, constellations, bee swarms, or institutions (as long as the latter are not abstracts, but whole bodies composed of people and things).[9] All of this obviously falls under the collective understanding of a set. It is quite commonly accepted that such a notion of a set is used in social sciences. Kotarbiński's reism provides every basis for this. Criticizing hypostasis in the humanities and social sciences, Kotarbiński writes, e.g., "In the strict, fundamental sense, there are only groups of people who are in such a way institutionalized, and in such a way collaboratively functioning, because of such and not other people's dispositions and other constituent things, among others because of such and not other beliefs and aspirations".[10] Thus, society exists, in so far as it is a whole made up of concrete people; on the same principles, as for a reist there exists a nation or a social class.

Coming back to the theory of multiplicity, it is necessary to recall the fundamental problem of reism with this theory, which is that while the reist allows for the existence of sets in a collective sense and can benefit from the achievements of mereology, this is of no importance for the

[9] Kotarbiński, *Ontologia, teoria poznania i metodologia nauk*, 169.
[10] Ibidem, 168.

theory of multiplicity, since it operates with the concept of set in the distributive sense.[11] From the point of view of reism, the term "set" in the distributive sense is an apparent name. In fact, the road to a reistic interpretation of the theory of multiplicity seems to be closed. Nevertheless, Kotarbiński "attempted to offer a reistic interpretation of the multiplicity theory, or at least its part based on the distributive concept of a set".[12] However, these attempts will be of limited nature and the final outcome will have to be considered pessimistic. The proposal of the reist is that the expressions containing the distributive concept of a set (the name "set" here has an onomatoidal sense) should be understood as expressions that are incomplete substitutes for the proper ones. In accordance with one of the main directives of reism, such expressions should be translatable into proper expressions. Thus, this concept of set could be conditionally preserved. Accordingly, the sentence "x is an element of the set of M's" is equivalent to the sentence: "x is an M" or "x is one of the M's".[13] For example, the phrase "Socrates is an element belonging to the set of philosophers" means simply: "Socrates is a philosopher" or "Socrates is one of the philosophers". In this way we avoid hypostasis and accepting the existence of some real set of philosophers. However, one should not think that the above technique of eliminating the term "set" is intended to demonstrate that the very concept of set is meaningless and cannot be used. On the contrary, it is precisely the fact that expressions containing the concept of set can be transformed by means of a reistic method into proper expressions that proves that the concept of set itself makes sense, provided, of course, that its substitutive character is kept in mind. This means that, although for the reist, sets do not exist in the distributive sense, phrases containing them are meaningful and statements built on them are true.

The fact that Kotarbiński's reism does not completely negate the validity of using the concept of a set in the distributive sense, of course in its abridged-substitute sense, does not yet determine that a reistic interpretation of the theory of multiplicity in general is possible. The problems of reism with the multiplicity theory are about something else and Kotarbiński knew this well.

The "onomatoidal" interpretation of the term "set" presented above applies only at the basic level of the set theory – set algebra, which treats about individuals being elements of an established full set (e.g. Socrates

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[11] "Kotarbiński [...] admitted, citing experts' opinions, that mereology is not an appropriate substitute for the theory of multiplicity [...]" (Woleński, *Filozoficzna szkoła lwowsko-warszawska*, 222).

[12] Ibidem, 222.

[13] Tadeusz Kotarbiński, *Elementy teorii poznania, logiki formalnej i metodologii nauk* (Warszawa: Ossolineum, 1986), 25.

in the set of philosophers). For a reist, it is difficult to move to a "higher level" of the set theory, e.g. to the concept of a class of classes, although Kotarbiński did not claim that this path is completely closed, but only that it is in the project phase. It is believed that only in the initial phase of his reism did Kotarbiński attach greater hopes to the method of elimination of the concept of a set in a distributive sense by replacing statements containing this concept with equivalent statements that did not contain it. At a later stage, this optimism faded out along with his realization of the difficulties connected with the impossibility of a reistic interpretation of, say, such notions that are as fundamental for mathematics as the definition of a natural number, a rational number or a complex number, which require operating with a higher level of the notion of a set.[14]

Probably the described problems of reism with the multiplicity theory are not so important if one considers an assortment of Kotarbiński's other views on mathematics, and subsequently on a correctly constructed scientific theory. Kotarbiński repeatedly stressed that his reism is as far from Platonism as possible, but can be understood as a direct continuation of nominalism. However, as we know, nominalism is closely connected with empiricism. Under these circumstances, it may come as a big surprise that Kotarbiński approves of the view that mathematics is an a priori science.[15] Apriorism, however, does not intuitively connect with either reism or nominalism. Being aware of these difficulties, Kotarbiński quite cautiously adds that some kind of an agreement between nominalism and apriorism can occur, provided that we are interested in mathematics, and not in the ontological, but in the methodological point of view and its characterization as such.[16] The issue discussed here is included by Kotarbiński in the "philosophy of mathematics", which deals, inter alia, with basic ontological and epistemological issues concerning mathematics. While presenting various positions in this field, Kotarbiński sympathizes with the view that the best way to characterize mathematics is not from the point of view of its subject matter, but

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[14] Klemens Szaniawski, "Filozofia konkretu", *Studia filozoficzne* 3 (1976): 69.

[15] Kotarbiński's declarations on his access to apriorism may not be very strong, but they are nevertheless noticeable enough to be considered an important element of his philosophy of mathematics and logic. For instance, when discussing Bolzano's views on the basics of logic and mathematics, he gives this partial assessment of them: "It is a certainly justified access to the purely reflective treatment of the issues and deductions of logic and mathematics (in other terminology: to the a priori nature of these disciplines or to the analytical nature of their statements) […]" (Tadeusz Kotarbiński, "Słowo wstępne", in: Bernard Bolzano, *Paradoksy nieskończoności*, transl. Łucja Pakalska (Warszawa: PWN, 1966), XX). What is more important here, of course, is not how Kotarbiński assesses Bolzano's views, but his own position which he expresses on this occasion.

[16] Kotarbiński, *Elementy teorii poznania, logiki formalnej i metodologii nauk*, 298.

rather from the point of view of the method. And it is in this context that the claim appears that mathematics is an a priori science. "It is assumed that mathematical, and only mathematical, statements proclaim truths which, precisely because of their content, are a priori, i.e. either obvious, or fully justifiable because of obvious truths, and without recourse to perceptive judgements".[17] And because, as our author continues to write about a priori judgements, "their accuracy comes from the meanings of the terms used in them"[18] – we can say that contrary to Kant's opinion, as Kotarbiński points out, mathematics is not a field of synthetic a priori courts, but of analytical a priori judgements. It must be said, however, that Kotarbiński accepts this position with a certain reserve and that the matter is not entirely unequivocal.[19]

Either way, according to Kotarbiński, mathematics is not only not based on empiricism, but in general, as it is going to turn out, is indifferent to empiricism, at least in the sense that the validity of its claims is independent of the issue of existence or non-existence of its objects. This is directly related to what he writes about the existence of objects of a certain in the context of the conditions of its correctness.

Kotarbiński's general conviction in this matter is that the very existence of the "correct theory of the object *P*" does not at all mean that there must exist an object *P*. Kotarbiński also believes that the claims of such a theory in these conditions (i.e. when e.g. the object *P* does not exist) can be considered to be true. In order for a theory (including mathematical one) to be considered as correctly constructed and its theorems to be true, it should meet the following conditions: (1) the theory should be a collection of conditional statements; (2) the strict form of these claims should be as follows: for each *X*, if *X* is *P*, then *X* is *Q*, or similar, and always such that (3) *X* should only be in the predicatives, never in the subject.[20] As for mathematics, this means, according to an example given by Kotarbiński, that the theory of cone intersections can be treated as absolutely correct and its theories as absolutely true even if no such object as hyperbola existed in the world. Of course, the same applies to other mathematical subjects, although the examples given include only geometrical figures and numerical ratios.

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[17] Ibidem, 298.

[18] Ibidem, 299.

[19] "Under the influence of numerous suggestive illustrations, we are rather in favour of the existence of a priori knowledge, obviously not in the sense that an object is a priori knowledge, but in the sense that we know this and that we know it a priori, i.e. not by experience. Let us admit, however, that the boundaries of such knowledge are not clear to us and that the very possibility of such knowledge in fact remains mysterious to us" (ibidem, 299).

[20] Kotarbiński, *Ontologia, teoria poznania i metodologia nauk*, 103.

Especially in the mouth of a reist, the thesis that the correctness of certain theories (including mathematical ones) and the truthfulness of their claims do not depend on the existence of the objects they talk about is quite surprising. Nevertheless, the conditional form of the statements of mathematical theory and its *a priori* nature free us from the problems of the "ontology of mathematics". Simply the problem of the existence of mathematical subjects and the problem of the truthfulness of their theorems are two completely different issues. In this case, the ontology of reism does not interfere with the sphere of the truthfulness of claims, because here the methodological rules of building of mathematical theory and rules of correct reasoning have a decisive voice. Reism as an ontological position is extremely anti-Platonic and nominalistic, but in its methodological version (at least in the methodology of deductive sciences) its tone is clearly milder. This is a major and clear step in a slightly different direction, allowing one to "suspend oneself" or at least distance oneself from reistic ontology in general. Would it therefore be legitimate to assume that the proposed methodology of mathematics relieves us completely of our ontological obligations? The writings by Kotarbiński do not indicate that there is absolute clarity in this matter. It is also unclear what consequences can be drawn from this in relation to the basic field of mathematics, which is the multiplicity theory.

Let us assume, however, that the general findings presented a moment ago on the issue of the existence of the subjects of mathematics and the conditions of truthfulness of the claims of this science are correct, and let us also try to free the theory of multiplicity from the need to take into account the basic ontological assumptions of reism. Can it be consistently maintained that a correctly constructed set theory does not require the postulate of the existence of its subjects, and yet its theorems meet the truth condition? Can one proceed the way Kotarbiński proposes in relation to geometry and arithmetic?

To put it in the most general terms, the whole matter can be presented as relatively uncomplicated, maybe even trivial, which unfortunately also makes us think about the value of this type of solution. For a reist, the basis for doubting the possibility of the multiplicity theory was the conviction that there is no such thing as sets in the distributive sense. However, if, in accordance with the view presented above, one can "suspend" issues of an ontological nature at this point in time, then nothing stands in the way of building a theory using the distributive notion of a set and, as a result, acknowledging the truthfulness of its claims, while remaining in contradiction with reism. The existence, or rather non-existence, of such an object as a set, as well as the impossibility of interpreting it in a realistic way, adds nothing to the matter and is completely neutral. If this general conclusion were correct, we would come very close to an opposite position, to that of the "Platonists", of which at least some of the proponents of reism were of course aware.[21]

This interpretation avoids the need to seek reistic substitutes for sets in the distributive sense. Even the notion of an infinite set, which was presumably the starting point of the career of the multiplicity theory in the nineteenth century, does not have to pose a problem in Kotarbiński's view. With full approval and appreciation, he can then write about Bolzano's brilliant intuitions on the concept of the infinite set: "[...] Bolzano formulates and makes a general statement that each infinite set (and only an infinite set) remains in a one-to-one correspondence to its own specific part. […] This step only separated this general characteristic of infinite sets from the adoption of the discussed property as a defining characteristic of infinite sets as such".[22] As is evident, the reist has no problem here with using positively not only the concept of a set, but even an infinite set, even a set possessing a paradoxical feature from the point of view of common sense. All three elements of the infinite set theory mentioned here – i.e., the concept of the infinite set as such (in the distributive sense), the infinite set characteristic and the "paradoxical", defining characteristic of such a set – would cause the reist, who approaches these issues in his usual way, basic difficulties resulting from the ontology of reism. If, on the other hand, we take into account the modified (if we may say so) version of "reism" as presented above, aside from strong ontological conditions, such difficulties lose their raison d'être. With reference to the passage quoted above, which presents Bolzano's definition of an infinite set, the following formula (definition of the infinite set) can then be accepted as entirely correct: "If X is a set, and if X remains in the ratio of a one-to-one correspondence with its own specific part, then X is an infinite set". This formula meets the basic requirements for a correctly built theory: (1) its statements are of a conditional nature, (2) the general names ("set") are treated in a predicative manner and (3) it does not give rise to any ontological obligations as to the existence of the object referred to in it.

In a similarly positive way as with Bolzano's ideas, Kotarbiński referred to other important achievements of contemporary logic and mathematics, in particular those related to the concept of class within the relations theory.[23] Among other things, it presents the issue of equi-

[21] Janina Kotarbińska, "Kłopoty z istnieniem. Rozważania z zakresu semantyki", in: eadem, *Z zagadnień torii nauki i teorii języka* (Warszawa: PWN, 1990), 348.

[22] Kotarbiński, "Słowo wstępne", XVIII.

[23] From a certain point of view, the concept of class can be understood as a substitute for the concept of set. (See e.g.: Szaniawski, "Filozofia konkretu", 68). As Kotarbiński writes: "'Class'' is so much as: 'a set of all,' more specifically: 'a class of M's' – i.e. 'a set of all M's'" (Kotarbiński, *Elementy teorii poznania, logiki formalnej i metodologii nauk*, 26).

numerous classes (of sets) as well as the concept of cardinality based on the idea of the class of equipotent sets.[24] What is interesting is that these discussions are devoid of critical comments or even doubts typical of Kotarbiński resulting from his general reistic beliefs. They are even omitted in the context of the concept of the class of sets, though at the same time Kotarbiński knew well that the otherwise mereological and reistic interpretation of this term, if at all possible, represents at most an interesting programme rather than a tangible achievement.[25]

It is difficult to state unequivocally whether this relaxed and lacking "reistic engagement" approach by Kotarbiński when discussing selected issues of the set theory was a consequence of his view that a properly constructed theory of an object (including the set theory?) is possible even with complete indifference to the issue of the existence of such an object. Or maybe it was just a kind of "a standard trick of a mathematician with philosophical inclinations towards nominalism",[26] as Woleński accurately calls it, whose remark, although made in relation to Tarski, has in fact a universal character. Namely, seeing a problem with demonstrating the ontological basis of some concepts (e.g. those related to infinity), this nominalist (reist) uses a tried-and-tested formula of logicians and mathematicians, starting his argument with "let's assume that it exists", without bothering at all with any ontological validation of this assumption. Both reasons are very similar, if not identical, at least in their general sense – both "ignore" in one way or another the problem of the existence of mathematical or logical objects. While this "typical trick" is most comprehensible in mathematical and logical practice, from the point of view of the coherence of Kotarbiński's philosophical system, it poses a certain problem since reism has based its main philosophical "message" on the issue of the existence of specific objects. For this reason, the attempts of some people to modify reism in the direction of a possible "reconciliation" of its main idea with the requirements of mathematical theory seem to be completely understandable.

Such is the meaning of the proposals by Janina Kotarbińska, who has seen the main goal of her proposed significant modifications within the methodological doctrine of reism in defence of mathematics, including the multiplicity theory. She postulated that the meaningfulness of statements about abstracts (especially mathematical objects) should be made independent of their translation into reistic language, the language of things.[27] As she demonstrated, this is possible in two ways (i.e. either with a multi-category or single-category name concept), but the result is

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[24] Tadeusz Kotarbiński, *Wykłady z dziejów logiki* (Warszawa: PWN, 1985), 158.

[25] Ibidem, 164.

[26] Jan Woleński, *Epistemologia* (Warszawa: PWN, 2005), 152.

[27] Kotarbińska, 345.

always the same: "A reist may recognize the existence of abstract objects of all sorts, without risking being inconsistent with his own doctrine, regardless of whether the relevant existential sentences can be interpreted as statements about things".[28] Thus, he may speak both about the existence of individuals, as well as about the existence of an individual class or a class of individual classes, etc.[29] However, it is not without significance that, according to this proposal, the existence of these "objects" is referred to in the fundamental sense of the word "to exist", whereas the word "to exist" can still be used in the fundamental sense only in relation to things. The distinction between the essential and fundamental meaning of the term "to exist", as can be seen, plays a fairly important role here and allows this proposal to remain in elementary accord with the general idea of reism. Kotarbińska therefore maintains her opinion on the fundamental validity of this doctrine, not necessarily in its ontological dimension, but rather as a methodological programme aimed at freeing oneself from apparent names. In this respect, criticism of the multiplicity theory is maintained, although the issue of the existence or non-existence of its objects is no longer addressed, but rather focuses on accusations of a different nature. Specifically, the conceptual apparatus of the theory of plurality is unintuitive, which is to be understood as far from the common language, and thus elusive from the point of view of the concrete language. Hence, the fundamental criticism of the theory of multiplicity is maintained, although its foundations are already of a different nature than ontological ones. On the other hand, this whole solution bears the signs of something intrinsically contradictory. First, it was postulated that the meaningfulness of abstract statements should be made independent of their translation into reistic language, and now the accusation is raised that statements from the multiplicity theory cannot be translated into sentences about concretes. It is difficult to assess unambiguously the value of the modification of reism proposed by Kotarbińska.

The proposals we find in the works by Andrzej Grzegorczyk have a different character. In his logical and mathematical practice he also applies the conceptual apparatus of the plurality theory without major limitations, although at the same time, as a philosopher sympathizing with rheism, he is aware of certain difficulties concerning the generally understood non-intuitive nature of the concept of set according to the multiplicity theory. According to Grzegorczyk, classical mathematics is based on a kind of intuition common to all mathematicians, which is the basis of both the concept of a natural number as well as the related notion of a set, in particular the finite set. On the other hand, the concept

[28] Ibidem, 348.

[29] Ibidem.

of any set of natural numbers, including the infinite set, can already be considered controversial.[30] At the same time, it should be stressed that this controversy in Grzegorczyk's understanding is not connected with typical reistic objections to mathematical objects and that, as a result, the author's deliberations will go beyond the problem plane natural for reism. This is owing to the fact that Grzegorczyk takes into account what is called "their mathematical being"[31] in relation to mathematical objects which immediately takes us to a completely different level of considerations. This is a typical way of understanding "existence" for mathematics, according to which a mathematician claims e.g. that there exist natural numbers, complex numbers, etc., while a set of numbers that would include numbers that are at the same time even and odd does not exist. And in connection with the multiplicity theory, the problem is e.g. whether we can accept the existence of an abstract object called a set, if we do not possess a method of resolution through a finite effective process. As can be seen, the controversy in this case concerns something different from what the reists traditionally struggled with and is related to the (fundamental for mathematics) issue of constructivism. A possible closer connection with reism would consist in the fact that since mathematical subjects exist for us in the form of inscriptions, the mentioned method of resolution would consist in performing a finite series of operations on these inscriptions. This new "ontology" of mathematics may be considered a certain consequence of the general reistic attitude in the philosophy of mathematics, but at the same time it may be regarded as very different because of the assumption of some kind of "mathematical being" of the objects of mathematics, which in principle goes beyond the level of the dispute about the existence characteristic of reism.

Regardless of how we evaluate all these proposals, they point to one thing first and foremost: among the ideas stemming from Kotarbiński's concept, there are also those which, although remaining in an unclear relation to the main theses of reism, but are at the same time not in complete contradiction with it, open up this philosophy to issues to which it officially claimed to have no access.

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[30] Andrzej Grzegorczyk, *Zarys arytmetyki teoretycznej* (Warszawa: PWN, 1983), 305.

[31] Ibidem.

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Summary

Essentially speaking, this essay is not about the problem of interpreting set theory in accordance with the assumptions of reism. This interpretation is unlikely, in fact impossible. It is about something different: about presenting the possibilities of reconciling set theory with reism on another plane, based on certain ideas or ideas present in Kotarbiński's writings. Despite the fact that the attitude of these ideas to reism is unclear it does not seem to be radically contradictory to reism. Kotarbiński's strategy, on which we want to focus, was based on an original intervention which consisted in acquiring the distance to the problem of existing mathematical objects. It was not the only idea – other ideas will also be signalled here – but this one seems to be the most promising. What exactly is it about? What is its value, and is it really so significant a solution for the philosophy of mathematics? – These are the questions we want to answer here.

Keywords: reism, Tadeusz Kotarbiński, set theory, philosophy of mathematics