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BEALL-ING $\mathbb{O}$

Abstract. In "A neglected reply to Prior's dilemma" Beall [2012] presents a Weak Kleene framework where Prior's dilemma for Hume's no-*ought*-from-*is* thesis fails. It fails in the framework because *addition*, the inference rule that one of its horns relies on, is invalid. In this paper, we show that a more general result is necessary for the viability of Beall's proposal — a result, which implies that Hume's thesis holds in the proposed framework. We prove this result and thus show that Beall's proposal is indeed viable.

Keywords: Beall; Hume's Thesis; Prior's Dilemma; Weak Kleene

1. Introduction

Beall [2012] presents a Weak Kleene framework as a response to Prior's dilemma on behalf of the so-called flat-footed account of *ought*-sentences. The flat-footed account implies that an *ought*-sentence is any sentence in which *ought* is used as a main connective. Beall's framework, which we label as 'WK3+$\mathbb{O}$', aims to show that one of the horns of Prior's dilemma fails. It fails because the inference rule it relies on, viz., *addition* — the rule that tells us that from an arbitrary sentence, A, one can validly derive the disjunction, $A \vee B$ — is invalid in WK3+$\mathbb{O}$. Beall argues that this, then, undercuts Prior's alleged derivation of an *ought*-sentence, $A \vee \mathbb{O}B$ from an *ought*-free sentence, A; thus, fails to be a counter-example to Hume's thesis.

The failure of *addition*, however, is only one condition for the viability of Beall's proposal. What is also necessary is to show that there can be no valid argument from an *ought*-free sentence to an *ought*-sentence; i.e., to show that Hume's no-*ought*-from-*is* thesis holds in WK3+$\mathbb{O}$. Beall [2012], however, does not discuss this more general and central condition.

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After rehearsing Prior's dilemma and Beall's proposed WK3+$\mathbb{O}$ framework, we prove that this general condition is met, and so the proposal remains viable.

2. Scene-setting: Prior on Hume

Hume's no-*ought*-from-*is* thesis tells us that no *ought*-sentence (i.e., $\mathbb{O}$-sentence) can be validly derived from a non-*ought* sentence (i.e., $\mathbb{O}$-free sentence). Some theorists interpret this thesis as a kind of entailment barrier that makes such inferences logically out of bounds.[1] Thus, if A is an $\mathbb{O}$-free sentence and $\mathbb{O}B$ is an $\mathbb{O}$-sentence, then $A \nvDash \mathbb{O}B$.[2]

Prior [1960] presents a dilemma where either of its horns counts as a counterexample to Hume's thesis.[3] His case runs as follows:

- Let A be an $\mathbb{O}$-free sentence and $\mathbb{O}B$ be an $\mathbb{O}$-sentence.
- Dilemma: Either $A \vee \mathbb{O}B$ is an $\mathbb{O}$-free sentence or it is an $\mathbb{O}$-sentence.
- Case 1: If $A \vee \mathbb{O}B$ is an $\mathbb{O}$-sentence, then one could derive an $\mathbb{O}$-sentence from an $\mathbb{O}$-free sentence by the following inference: $A \therefore A \vee \mathbb{O}B$.
- Case 2: If $A \vee \mathbb{O}B$ is an $\mathbb{O}$-free sentence, on the other hand, then one could still derive an $\mathbb{O}$-sentence from an $\mathbb{O}$-free sentence by the following inference: $A \vee \mathbb{O}B, \neg A \therefore \mathbb{O}B$
- Conclusion: Either way, Hume's thesis fails.

Following the basic rules of inference, the $\mathbb{O}$-conclusion in Case 1 was derived from the $\mathbb{O}$-free premise using *addition*: $A \therefore A \vee B$. The $\mathbb{O}$-conclusion in Case 2, on the other hand, was derived from the $\mathbb{O}$-free premises using *disjunctive syllogism*: $\neg A, A \vee B \therefore B$.

Prior's main point is that, in either case, one can derive an $\mathbb{O}$-conclusion from an $\mathbb{O}$-free premise; hence, either could be a counterexample to Hume's thesis.[4]

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[1] Russell and Restall [2010] presents this 'entailment barrier' interpretation of Hume's thesis.

[2] Formulating Hume's thesis this way assumes that the categories of being $\mathbb{O}$-free and being $\mathbb{O}$-involving are mutually exclusive and exhaustive.

[3] The presentation here follows [Beall, 2012].

[4] There have been many responses to Prior's dilemma in the literature. For a useful discussion see [Pigden, 2010].

3. Beall's weak Kleene proposal

Beall [2012] proposes a framework where Prior's Case 1 fails. It fails because under a 'funny' interpretation of $\mathbb{O}$ and the Boolean operators, *addition* is invalid. Such a funny interpretation is couched in terms of a point-based WK3+$\mathbb{O}$ semantic framework. We could construct the semantics as follows.

3.1. WK3+$\mathbb{O}$ semantics for $\mathbb{O}$

Let W be a nonempty set of points and R be an accessibility relation on W (i.e., subset of $W \times W$). Each atomic sentence, A maps into a trivalent set of valuations, V: $\{1, 0, .5\}$ relative to a given point, w ($\in W$), where 1 is *true*, 0 is *false*, and .5 is *funny*.

Given this, $\mathbb{O}$ will have the following truth conditions:

$$v_x(\mathbb{O}A) = \begin{cases} 1 & \text{if } v_y(A) = 1 \text{ for all y such that Rxy} \\ 0 & \text{if } v_y(A) = 0 \text{ for all y such that Rxy} \\ .5 & \text{otherwise} \end{cases}$$

This gives us an *intensional* reading of $\mathbb{O}$-sentences. Following, Beall [2012], we stay neutral as to how the accessibility relation, R would be restricted.

3.2. WK3+$\mathbb{O}$ semantics for Boolean connectives

Boolean-made compounds are defined in the usual recursive way, and will have the following w-relative truth-conditions:

$\neg$	
1	0
.5	.5
0	1

$\vee$	1	.5	0
1	1	.5	1
.5	.5	.5	.5
0	1	.5	0

$\wedge$	1	.5	0
1	1	.5	0
.5	.5	.5	.5
0	0	.5	0

Notice that the connectives behave classically if all subsentences are treated classically. On the other hand, the entire compound would be funny (or have a .5 value) if at least one subsentence is funny.

3.3. WK3+$\mathbb{O}$ validity

Finally, WK3+$\mathbb{O}$-validity is defined as follows:

$A \vDash B$ iff there is no WK3+$\mathbb{O}$ model on which $v(A) = 1$ but $v(B) \neq 1$.

A WK3+$\mathbb{O}$ model is a structure, $M = \langle W, R, V \rangle$, where W, R and V are defined as above. As might be expected, WK3+$\mathbb{O}$-validity is classical if the subsentences are classical; otherwise, it would admit a bit of funny business.

3.4. Prior's Case 1 in WK3+$\mathbb{O}$

Now given Beall's WK3+$\mathbb{O}$, we could show that *addition* is invalid. A simple counter-model is a valuation where $v(A) = 1$ and $v(B) = .5$.

Furthermore, we could show that Prior's Case 1 is also invalid. Let $W = \{@, w\}$, and $R@@$ and $R@w$. Suppose $v_@(A) = 1$ and $v_w(B) = .5$. Then $v_@(A) = 1$ but $v_@(A \vee \mathbb{O}B) = .5$. This, then, would be its counter-model.

4. A general result

Showing that Prior's Case 1 fails, however, is only one condition for the viability of Beall's proposed framework. What is also necessary is to show that Hume's thesis holds in WK3+$\mathbb{O}$; i.e. that $A \nvDash \mathbb{O}B$ is WK3+$\mathbb{O}$-valid. Although Beall [2012] does not discuss this more general and central condition, we can show that his proposed framework already has the resources to prove this. We show it thus.

4.1. Definitions

We say that a sentence is a *primary* $\mathbb{O}$*-sentence* iff $\mathbb{O}$ is the main connective in the sentence (where *main connective* is defined as usual via the recursive definition of sentences). A sentence is a *secondary* $\mathbb{O}$*-sentence* iff it contains a primary $\mathbb{O}$-sentence as a proper subsentence.[5]

We say that a sentence is a *first-degree* $\mathbb{O}$*-sentence* iff it is a primary $\mathbb{O}$-sentence $\mathbb{O}A$ where A is $\mathbb{O}$-free. Similarly, we say that a sentence is a

[5] A sentence B is a proper subsentence of sentence A iff B is a subsentence of A but is not A itself. (Subsentences are defined per usual via recursive definition of sentences.)

second-degree $\mathbb{O}$*-sentence* iff it is a primary $\mathbb{O}$-sentence $\mathbb{O}A$ where A is a first-degree $\mathbb{O}$-sentence. And so on for all finite n, so that a sentence is n-degree iff it is a primary $\mathbb{O}$-sentence $\mathbb{O}A$ where A is an $(n-1)$-degree $\mathbb{O}$-sentence.[6]

Let $\{w, w'\} \subseteq W$ in any WK3+$\mathbb{O}$ model. Then we say that w' is w*-accessible* iff w is R-related to w' where R is the accessibility relation in the model. (In other words, w' is a w-accessible point iff $\langle w, w' \rangle \in R$ in the given model.)

We say that a point $w \in W$ of any WK3+$\mathbb{O}$ model is *narcissistic* iff the only w-accessible point is w. (In other words: iff Rww and for any $w' \in W$ if Rww' then $w = w'$.)

Let A be any sentence in the language of WK3+$\mathbb{O}$. We say that a point $w \in W$ of any WK3+$\mathbb{O}$ model is *funny with respect to* A iff $v_w(A) = 0.5$. ('wrt' abbreviates 'with respect to'.)

Let $w \in W$ be a point in any WK3+$\mathbb{O}$ model. We say that w is *fully atomically funny* iff w is funny with respect to all atomics.

4.2. Facts

FACT 1 (Existence: fully atomically funny points). *There are* WK3+$\mathbb{O}$ *models with fully atomically funny points.*

PROOF (SKETCH). The semantics do not preclude (and thereby allow) a point at which a valuation assigns .5 to every atomic sentence. $\square$

FACT 2 (Fully funny $\mathbb{O}$-free points). *Let w be a point in any* WK3+$\mathbb{O}$ *model. If w is fully atomically funny then w is fully funny with respect to all* $\mathbb{O}$*-free sentences.*

PROOF (SKETCH). This follows from the 'infectiousness' of 'funniness' (viz., value 0.5) in the semantics of the $\mathbb{O}$-free language (in particular, standard connectives). The proof is by induction on the complexity of $\mathbb{O}$-free sentences. $\square$

From Facts 1 and 2 we obtain existence of fully funny $\mathbb{O}$-free points:

COROLLARY 1. *There are* WK3+$\mathbb{O}$ *models with fully funny* $\mathbb{O}$*-free points.*

[6] One could define $\mathbb{O}$-free A to be a 0-degree $\mathbb{O}$-sentence A for ease.

4.3. Central lemmas

With the foregoing definitions and Fact 1 the following two lemmas are critical:

LEMMA 1 (Narcissism and $\mathbb{O}$). *If w is a narcissistic point in a* WK3+$\mathbb{O}$ *model then $v_w(\mathbb{O}A) = n = v_w(A)$.*

PROOF. Let w be a narcissistic point in such a model. By the semantics for $\mathbb{O}$, we have that $v_w(\mathbb{O}A) = 1$ *iff* $v_{w'}(A) = 1$ for all w-accessible points w'. Given that w is narcissistic, $w = w'$, and hence we have that $v_w(\mathbb{O}A) = 1$ *iff* $v_w(A) = 1$. The case in which $v_w(\mathbb{O}A) = 0$ is exactly the same (*mutatis mutandis*). Finally, by the semantics for $\mathbb{O}$, if $v_w(\mathbb{O}A) \notin \{1, 0\}$ then $v_w(\mathbb{O}A) = .5$ and, hence, from (contraposition on the biconditionals above), $v_w(A) = .5$. Since these three cases exhaust the possible values for n (viz., 1, 0, .5), we have that, for any $n \in \{1, 0, .5\}$, $v_w(\mathbb{O}A) = n = v_w(A)$. □

Let w be a point in any WK3+$\mathbb{O}$ model. We say that w is *fully funny* iff w is funny with respect to *all* sentences (atomic and compound).

LEMMA 2 (Sufficiency for fully funny points). *Let w be a point in any* WK3+$\mathbb{O}$ *model satisfying the following two conditions:*[7]

1. *w is fully atomically funny*
2. *w is narcissistic*

Then w is fully funny (simpliciter).

PROOF. By Fact 2, w is funny with respect to all $\mathbb{O}$-free sentences since w is fully atomically funny. But, now, since w is narcissistic, Lemma 1 implies that $v_w(\mathbb{O}A) = v_w(A)$ for all A. Since $\mathbb{O}$-free A are one and all funny at w, so too are all n-degree $\mathbb{O}$-sentences (beginning with 1-degree, then 2-degree, etc., feeding each through the Lemma 1 equation). In turn, since all secondary $\mathbb{O}$-sentences are constructed from n-degree $\mathbb{O}$-sentences, and (as above) all n-degree sentences are funny at w, it follows, from the 'infectiousness' of the funny value in the semantics, that all secondary $\mathbb{O}$-sentences are funny at w too. But, now, since all sentences in the language are either $\mathbb{O}$-free sentences or n-degree $\mathbb{O}$-sentences or secondary $\mathbb{O}$-sentences – and all of those are funny at w per

[7] The narcissism is not strictly required here but we invoke it because of the narcissist lemma regarding funniness. (All that is required is that w itself be w-accessible together with w's being fully funny wrt $\mathbb{O}$-free sentences.)

above – it follows that all sentences are funny at w, and hence that w is a fully funny point. ☐

4.4. The main theorem

The central result required by Beall's original proposed reply to Prior's dilemma is that *no* $\mathbb{O}$-sentence can be derived from an $\mathbb{O}$-free sentence; i.e., that Hume's thesis holds in WK3+$\mathbb{O}$. This can now be proven.

THEOREM 3 (Hume's Thesis). *Where* $\vDash$ *is the* WK3+$\mathbb{O}$ *consequence relation, and where* A *is any* $\mathbb{O}$*-free sentence*: $A \nvDash \mathbb{O}B$.

PROOF. Consider any WK3+$\mathbb{O}$ model such that $v_@(A) = 1$, and where for some $w \in W$, the following three conditions hold:

1. w is fully atomically funny
2. w is narcissistic
3. w is @-accessible

By Lemma 2, conditions (1) and (2) imply that w is a fully funny point. But, then, since w is an @-accessible point whereat $v_w(B) = 0.5$ for all sentences B, the semantics for $\mathbb{O}$ imply that $v_@(\mathbb{O}B) = 0.5$. Hence, there is a model in which A has value 1 but $\mathbb{O}B$ has a value other than 1. ☐

4.5. A comment on the proof

The proof does its job, but it is an overkill. The model requires the funniness of *all* $\mathbb{O}$-sentences (at the base point, which models the actual world). Searching for a more natural model without this defect would be fruitful, but this can be left to further research.

5. An open question about $\mathbb{O}$'s funniness

With the foregoing discussion, we have proven that Beall's proposed WK3+$\mathbb{O}$ framework is indeed viable. It remains an open question, however, as to what exactly it means for $\mathbb{O}$ to be funny.

The standard view is the *meaningless* interpretation due to [Bochvar and Bergmann, 1981]. According to this view, funny-sentences are meaningless and infectious; thus, compounding them would likewise result to further meaninglessness. If $\mathbb{O}$-involving sentences are funny in this way, then perhaps ethical discourses function like how moral noncognitivists think of them: they are simply not truth-involving.

On the other hand, 'funny' might be interpreted in terms of a kind of error theory where funny-sentences are just systematically false. Beall [2012] shows that given this interpretation, an $\mathbb{O}$-involving sentence would have the value, .5 regardless of the value of its atomic, $\mathbb{O}$-free content. If .5 just means systematically false, then, perhaps, this interpretation implies that ethical discourses are likewise systematically false.

Finally, Beall [2016] has recently suggested that 'funny' might mean being off-topic. According to this view, funny-sentences are off-topic just in case they do not preserve the topic currently under discussion. Accordingly, given any topic of discourse, any $\mathbb{O}$-involving sentence made in it is similarly off-topic. As a consequence, for any topic of discourse, making ethical sentences would be off-topic *simpliciter*.[8]

Whatever the right interpretation of the funniness of $\mathbb{O}$ might turn out to be, one thing is for sure. Any interpretation must still abide by Hume's thesis that *no* 'ought' can be derived from an 'is'.

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[8] Francez [2019], however, have presented problems with this suggested off-topic interpretation.

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