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BOOK REVIEWS

A formal theory of action

Janusz Czelakowski, **Freedom and Enforcement in Action. A Study in Formal Action Theory**, vol. 42 of the Trends in Logic book series, Springer, Dordrecht, 2015, 261 pages, Print ISBN 978-94-017-9854-9, Online ISBN 978-94-017-9855-6. DOI: 10.1007/978-94-017-9855-6

The book under review introduces an innovative approach to a formal theory of action. Many ideas introduced by the author were presented in some of his earlier papers published in the 90s. Moreover, fragments of the book have been presented at many conferences (one noted by the author is *Logic and its Applications in Philosophy and the Foundations of Mathematics*, a conference organized annually since 1996 in Poland). The book, therefore, presents concepts that are results of many years of work and might be considered as a continuation and an important part of a praxeological tradition introduced by Tadeusz Kotarbiński.

The author discusses mathematical, logical and philosophical aspects of a theory of action. The main issues analysed in the book concern models of actions, the language of action systems, a problem of non-probabilistic and probabilistic performability of actions, relations with theory of algorithms and programming, a deontic logic based on introduced models of the action theory[1], a problem of non-monotonic reasoning and models of knowledge within a framework of action theory. One

[1] Speaking of deontic logic, this year conference of Trends in Logic XVII (Lublin, 12–15.09.2017) is wholly devoted to problems and applications of that branch of philosophical logic. The topic of the conference is: *Traditional and new perspectives on deontic logic and agency modelling.*

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of the biggest virtues of the book is that the analysis is illustrated by intuitive examples which explain how the formal theory works and to what kind of problems it might be applied. All those factors make the book an important and interesting contribution in the study of models of action and their applications.

The book consists of two parts, each of them containing three chapters. The first part of the book introduces relevant definitions and some facts about action theory. It determines some methods of analysis and some special cases of elementary action systems with their formal properties. The second part of the book contains a description of actions in deontic contexts and examines non-monotonic reasoning in epistemic discourse.

In the first chapter the reader finds a definition of an elementary action system, which is a basic notion of the theory. An elementary action system (in short: `EAS`) is a triple consisting a set of states W, a binary relation $R \subseteq W \times W$, which is called relation of direct transition and a set of atomic actions $\mathcal{A}$. An atomic action A is considered as a binary relation between states, so $A \subseteq W \times W$. The basic idea is simple: an action means a change from one state to another according to some rules, restrictions, etc. which determine a possible transition between states. So the transition might be accomplished by some atomic action in an `EAS` if the action changes one state to another, if the action holds (see p. 9). A possible performance of an atomic action might be realized in an `EAS` if a proper transition holds. In the first chapter different families of `EAS`es are classified as separative, normal or equivalential (see p. 13). A compound action is determined as a set of finite sequences of atomic actions of $\mathcal{A}$ from a given `EAS` and might be considered as a language over a set $\mathcal{A}$ (see p. 50). Some set-theoretic operations on compound actions are defined and an algebraic structure on the family of compound actions of an `EAS` is introduced.

The author emphasizes on p. 13 that one of the goals of an `EAS` is to "assure implementation of a given task, that is, to provide a bunch of strings of atomic actions which would lead from Φ-states to Ψ-states", where $\Phi, \Psi \subseteq W$. In the book we find a lot of quite intuitive examples which show how an `EAS` reaches that goal (see p. 13–21). The author considers for example a well-known logical puzzle about missionaries and cannibals who want to cross a river or the simplified situation of the washing in an automatic washing machine.

In the first chapter two approaches to the performability of actions are discussed. According to the first, non-probabilistic one, in reference to a chess game:

> [...] performing an action resembles a chess player's deliberate move across the chessboard — it is a conscious choice of one of many possibilities of the direct continuation of the game. If such possibilities do not exist, e.g., when the nearest squares where the white knight could move are taken by other pieces, performing the actions — white chessmen's move — is impossible in this case. The eventuality of unintended and accidental disturbances of the undertaken actions can be entirely neglected. (p. 28)

According to the second, probabilistic one performability might be explained thus:

> [...] in terms of the *chances* of bringing about the intended effect of the action in the given state and situation. [...] the performability of an action is the resultant of the agent's abilities and the influence of external forces of course of events. It is a kind of fame between the agent an nature. (p. 28)

In order to express the probabilistic approach to performability the notion of a probabilistic action system and the notion of bi-distribution are defined. A probabilistic system is derived from an `EAS` by enriching it "with families of probabilities providing a quantitative measure of the performability of atomic actions" (p. 29). The author also considers some problems with the formalism, which might be briefly described as problems to do with distinction between probabilistic and non-probabilistic systems. Another issue is the performability of a compound action. It is determined as a function of the performability of atomic actions, which are elements of sequences consisting of a given compound action. More precisely, a compound action **A** is performable if some sequence from **A** is performable.

In case of the notion of an `EAS` such factors as time, locations, order of actions, knowledge of the agents and so on are omitted. But in the second chapter the author discusses how one can enrich an `EAS` to a situational action system in order to take into account some contextual aspects of actions. According to the formalism he introduces, a situation is an ordered pair consisting of a state of an `EAS` and a label which refers to some contextual aspect like time or location. This part of the book also presents some remarks concerning algorithms of action based on an

EAS and determines some relations between the action theory and theory of algorithms.

The last chapter of the first part presents some important facts concerning fixed points and their possible applications in the context of the action theory. This part introduces the notion of an ordered action system which is a result of combing a poset with an EAS and determines relations between ordered and situational action systems.

The first chapter of the second part presents a deontology of actions. The chapter starts with some analysis of norms and presents them as deontological rules of conduct:

> The issue whether a norm is a rule, a prescription, etc. depends on the category (or rather a class) of distinguished action systems — the character of a given class of action systems determines the respective category of associated norms. (p. 148)

And on the next page:

> Norms define the ways and circumstances in which actions are performed, e.g. by specifying the place, time, and order of their performance, or by forbidding their use for the second time. (p. 149)

The author introduces a deontic logic based on his action theory and the logic of atomic norms. In such a system deontic operators enable one to form sentences with action variables instead of propositional ones. The logic is defined syntactically as a set of schemata closed under modus ponens. The first three axiom schemata are adequate to classical logic while the last two characterize deontic operators. The first of them states that what is obligated is also permitted, i.e., $\mathbf{O}\alpha \to \mathbf{P}\alpha$, where α ranges actions, and the second one states that what is forbidden is what and only what is not permitted, i.e., $\mathbf{F}\alpha \leftrightarrow \neg\mathbf{P}\alpha$. A model of the language is a Kripke model of modal logic enriched by a valuation which assigns to each action a binary relation. The logic is proved to be sound and complete. Additionally, an effective method for determining theses is introduced. Furthermore, the author discusses the closure principle which says that if an action is not forbidden, then it is permitted. One of the last issues is a logic of righteous norms and the problem of the consistency of elementary norms is analysed.

The second chapter is focused on the stit operator and stit semantics. The author shows how to reconstruct stit models as a class of situational actions systems. The understanding of 'action' undergoes a change,

> [...] actions are seen [...] as selections of preexistent histories or trajectories of the system in time. Actions available to an agent in a given state (moment) u are simply the cells of a partition of the set of possible histories passing through u. Stit semantics is therefore time oriented and time, being a situational component of actions, plays a privileged role in this approach. (p. 196)

The last chapter presents an approach to epistemic actions which "concern mental states of agents — they [epistemic actions] bring about changes of agents' knowledge or beliefs about environment as well as about beliefs" (p. 209).

One of the biggest problems analysed in this chapter is how to define a model of knowledge compatible with the dynamics of mental states. In that context three tasks are emphasized by the author. Firstly, a temporalization of the systems of epistemic logic expressed by the formalism presented in the book. Secondly, a formalization of complex belief structures with a collective agent. And thirdly, a development of the action theory so that it would not presuppose the omniscience of an agent. The first two issues must be taken into account if a dynamic model of knowledge-states is considered, since they are especially related to such problems as the forming of and changing of beliefs. The last problem matters if we want to have a realistic system of knowledge, where agents might be wrong and make mistakes.

The author starts, however, with some general remarks concerning knowledge and its properties; for instance, the Gettier problem is discussed. Next, a model of knowledge as a action system is defined. One of the most important issues is the frame problem, which leads to the question of formalisation of non-monotonic reasoning within the action theory.

> In reference to action theory the frame problem can be articulated as the problem of identification and description of the conditions (state of affairs, etc.) which are irrelevant or not affected by the actually performed action. (p. 218)

From a formal point of view the frame problem raises a question of how to define "a set of logical formulas describing the results of actions without the need for taking into account factors being «non-effect of those actions»" (p. 219). One of the solutions is to introduce some frame axioms which specify these „non-effects of actions" and assume that a situation is not changed by an action unless there is an evidence

that it was affected by an action. However, as the author emphasizes, as far as the logic is concerned, there appears to be a serious problem. The consequence operation is usually assumed to be monotonic. Yet new information might invalidate old conclusions. Monotonicity means that we will not be able to exclude these conclusions. It is for this reason that non-monotonic reasoning needs to be introduced into the formalism. The frames for non-monotonic reasoning presented by the author are the bases for systems that allow for further analysis of the three aforementioned tasks.

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