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FURTHER REFLECTIONS ON SENTENCES SAYING OF THEMSELVES STRANGE THINGS*

Abstract. Milne [2005] argued that a sentence saying of itself that it does not have a truthmaker is true but does not have a truthmaker. López de Sa and Zardini [2006] worried that, by parity of reasoning, one should conclude that a sentence saying of itself that it is not both true and short is true but not short. Recently, Milne [2013] and Gołosz [2015] have replied to López de Sa and Zardini's worry, arguing in different ways that the worry is ill-founded. In this paper, I'll address these replies and argue that they fail to dispel López de Sa and Zardini's worry, bringing out in the process some broader points concerning the use of self-referential sentences in arguments in philosophy of logic.

Keywords: diagonalisation; factivity; incompleteness; necessitation; reductio ad absurdum; semantic paradoxes; truthmaking

* These reflections would not have been made had not Dan López de Sa first directed my attention to the kind of argument they are about. The fact that he couldn't co-author this last paper in the series does not implicate that he'd disagree with its substance (nor does the immediately preceding statement implicate that he wouldn't, nor does the immediately preceding statement implicate that he would...). An earlier version of the material in the paper has been presented in 2016 at the LanCog Metaphysics, Epistemology, Logic and Language Seminar (University of Lisbon). I'd like to thank that audience as well as three anonymous referees for very stimulating comments. During the writing of the paper, I've benefitted from the FCT Research Fellowship IF/01202/2013 on *Tolerance and Instability: The Substructure of Cognitions, Transitions and Collections* as well as from partial funds from the project FFI2015-70707-P of the Spanish Ministry of Economy, Industry and Competitiveness on *Localism and Globalism in Logic and Semantics*.

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1. Recap and plan

In [Milne, 2005], Peter Milne argued that the sentence:

M: M does not have a truthmaker

is true but does not have a truthmaker, thus aiming to give a logical refutation of *truthmaker maximalism* (the view that every truth has a truthmaker).[1] The argument (henceforth 'the M_1-argument') went thus:

> Suppose that M has a truthmaker. Then it is true. So what it says is the case is the case. Hence M has no truthmaker. On the supposition that M has a truthmaker, it has no truthmaker. By *reductio ad absurdum*, M has no truthmaker. But this is just what M says. Hence M is a truth without a truthmaker. [Milne, 2005, p. 222]

Ingenious [and influential, see e.g. MacBride, 2013] as it is, the argument did not persuade López de Sa and Zardini [2006]. For, considering the apparently short sentence:

S: S is not [true and short][2]

they couldn't see how the M_1-argument was *any more effective* than the following argument for the conclusion that S is (true but) not short:

> Suppose that S is true and short. Then it is true. So what it says is the case is the case. Hence S is not [true and short]. On the supposition that S is true and short, it is not [true and short]. By *reductio ad absurdum*, S is not [true and short]. But this is just what S says. Hence S is true. Hence, since it is not [true and short], it is not short.[3]
> [Adapted from López de Sa and Zardini, 2006, pp. 154–155]

[1] Part of my interest in this issue is a general philosophical tendency towards *reification*, a tendency to which truthmaker maximalism is congenial; as a particular consequence of that tendency, in [Zardini, 2015b, pp. 507–508, fn 25], I've taken inspiration from my earlier work on truth [see e.g. Zardini, 2011] and vagueness [see e.g. Zardini, 2016a] to provide truthmakers for *universal* sentences, thereby addressing one of the most pressing problems for truthmaker maximalism.

[2] Throughout, I use square brackets to disambiguate constituent structure.

[3] Truthmaker maximalism can be expressed as the view that every truth has the *additional property* of having a truthmaker. Just so, *short maximalism* can be expressed as the view that every truth has the additional property of being short. Short maximalism (as many other F-maximalisms) is obviously false, but cannot be shown to be such by a sentence like S. And, just so, *declarative maximalism* can be expressed as the view that every truth has the additional property of being declarative. Declarative maximalism (as many other F-maximalisms) is obviously true, and cannot be shown not to be such by a sentence like the relevant analogue of S.

López de Sa and Zardini [2006, p. 156] then proceeded to indicate that the problem with M and S has to do with the *interaction of a notion like that of truth with self-reference*, and that one is free to choose an adequate theory of such interaction which would defuse M as a genuine counterexample to truthmaker maximalism.[4,5]

Recently, Milne [2013] and Gołosz [2015] have replied to López de Sa and Zardini's worry, arguing in different ways that it is ill-founded. After discussing in the next section the common reiterated in-passing suggestion that the M_1-argument is similar to certain unproblematically sound arguments, in the remaining two sections of this paper I'll address the gist of these two replies in turn and argue that they fail to dispel López de Sa and Zardini's worry. In the process, I'll bring out some broader points concerning the use of self-referential sentences in arguments in philosophy of logic.

[4] To be clear, the point is not that it is impossible to develop a *non-trivial* weird formal theory of truth according to which M is indeed true and indeed does not have a truthmaker. Of course, *that* is possible (just as it is possible to develop a *non-trivial* weird formal theory of truth according to which S is indeed true and indeed not short). The point is rather that, while, admittedly, formal theories *of truth* do not usually include a theory *of truthmaking*, there are plenty of adequate such theories (indeed, all I know of) which, when further naturally so developed as to include a theory of truthmaking, do not deliver the intended conclusion of the M_1-argument. In addition to the cases of hierarchical and non-classical theories discussed in Section 2 and Footnote 26 respectively, it might be worth quickly noting that even the kind of theory that could be thought to come closest to that conclusion — i.e., the "*semantic-descent-without-semantic-ascent*-kind" of theory defended e.g. by Maudlin [2004] — does not actually deliver it. For, when further naturally so developed as to include a theory of truthmaking, that theory does entail, in the way envisaged by the M_1-argument, that M does not have a truthmaker (i.e., entails the sentence M), but it is crucial for the theory that, in general, φ does *not* entail '‘φ’ is true', so that, in this particular case, *although the theory entails that M does not have a truthmaker, it does not entail that M is true*. Thanks to an anonymous referee for prompting me to be more explicit about this point.

[5] López de Sa and Zardini [2007, 2011] then went further and applied the same style of objection to a *wider range of recent influential arguments in philosophy of logic*. Although in this paper I'll mainly focus on the argument concerning truthmaker maximalism, the discussion thus has some broader relevance (for example, Milne [2007] aims at a more ambitious target and uses essentially the same argument as the M_1-argument to give, *pace* Leibniz and Gödel, a logical refutation of the claim that there is an omniscient being, aka God).

2. Maximalism and incompleteness

All of [Milne, 2005, p. 222], [Milne, 2013, pp. 473, 475], and [Gołosz, 2015, pp. 106, 108] suggest in passing that the M_1-argument is similar to *Gödelian incompleteness arguments*. The suggestion might come across as a bit odd. Say that a predicate 'is F' is *factive* iff it unrestrictedly satisfies the principle that, if 'P' is F, then P, and let 'PA' be short for 'Peano Arithmetic'. Then Gödel's [1931] celebrated argument for the first incompleteness theorem does not at all rely on the factivity of 'is provable in PA' (i.e., on PA's *soundness*), but only on PA's ω-*consistency*,[6] whereas the M_1-argument does rely on nothing less than the factivity of 'has a truthmaker'.[7] Charitably read, the suggestion is

[6] In fact, Rosser's [1936] strengthening of Gödel's [1931] results only relies on PA's *consistency*; indeed, also the relevant leg of Gödel's [1931] own argument — the one proving that the *Gödel sentence* G is not provable in PA — only relies on PA's consistency: if G (and so 'G is not provable in PA') is provable in PA, by the properties of 'is provable in PA' in PA 'G is provable in PA' is provable in PA, and so PA is inconsistent.

[7] Wait — that is strictly speaking true, but can't we just modify the M_1-argument so that it is indeed similar to the Gödel argument rehearsed in Footnote 6? No, we can't. For the modification would have to go something like: "If M (i.e., 'M does not have a truthmaker') has a truthmaker, by the properties of 'has a truthmaker' with respect to truthmakers 'M has a truthmaker' has a truthmaker, and so truthmakers are such that a sentence has a truthmaker and its negation has a truthmaker". And, in a couple of significant respects, that is also substantially dissimilar from the Gödel argument rehearsed in Footnote 6. Less importantly, it is not very clear *how bad* it is for truthmakers if a sentence has a truthmaker and its negation has a truthmaker (keep in mind that, as we're not assuming the factivity of 'has a truthmaker', that result does not entail a contradiction): since it is very controversial that having a truthmaker is closed *under conjunction* (as it is very controversial that, for every conjunction whose conjuncts have a truthmaker, there exists an object suitable for being the truthmaker for the conjunction), it is very controversial that the result that a sentence has a truthmaker and its negation has a truthmaker entails that a contradiction has a truthmaker; even granting that, since it is very controversial that having a truthmaker is closed *under single-premise entailment* (as it is very controversial that every truthmaker is a truthmaker for every logical truth), it is very controversial that the result that a contradiction has a truthmaker in turn entails that everything has a truthmaker. This is in stark contrast with the fact that it is very clear that it is *extremely bad* for PA if a sentence is provable in PA and its negation is provable in PA, since it is very clear that PA would then be *trivial*. More importantly, in the presence of self-reference, *it cannot be assumed without further ado* that 'has a truthmaker' has the property with respect to truthmakers which the modification presupposes it to have (just as, in the presence of self-reference, it cannot be assumed without further ado that one can infer that ''φ' is true' is true from φ's being true).

rather appealing to the much less far-reaching but much more simple argument that relies *on PA's soundness*: with G as in Footnote 6, if G (and so 'G is not provable in PA') is provable in PA, by PA's soundness G is not provable in PA, and so, by *reductio ad absurdum*, G is not provable in PA.

This being noted, I take it that the idea behind Milne's and Gołosz' suggestion is *not* that, given some sort of "*truthmaking hierarchy*" analogous to the arithmetical hierarchy, although M does not have a "truthmaker", it has some sort of "supertruthmaker" (analogously to how, although G is not provable in PA, it is provable in a plausible stronger theory that proves PA's soundness) — that would be hardly worrying for *the thought behind truthmaker maximalism*, analogously to how the facts just mentioned in brackets about G are hardly worrying for *the thought that every arithmetical truth is provable in some sort of way or other*. Rather, the idea must be that the M_1-argument shows that M does not have a truthmaker *of any sort*, where, in the context of Gödelian considerations, that last claim is naturally taken to be equivalent[8] with the claim that no *acceptable* (i.e., sound and possibly satisfying further,

This is in stark contrast with the fact that *it is demonstrable* that 'is provable in PA' has the property in PA which the Gödel argument rehearsed in Footnote 6 presupposes it to have. (One might think that things could be improved in both these respects by first associating truthmakers with a *grand comprehensive theory* TM such that, for every sentence φ, [φ has a truthmaker iff TM entails φ] for then shifting from *what is the case with respect to truthmakers* to *what TM entails*, where one might also think that TM is governed by something like classical logic and that, for every sentence φ, if TM entails φ, TM also entails '‘φ’ has a truthmaker'. However, even setting aside that the latter two thoughts just mentioned are much less conclusive than their analogues about PA are, since — for several well-known reasons — it's unlikely that we can associate truths with any minimally well-behaved grand comprehensive theory T such that, for every sentence φ, [φ is true iff T entails φ] it's similarly unlikely that we can associate truthmakers with some such minimally well-behaved TM in the first place. This is in stark contrast with the fact that we can obviously associate what is provable in PA with a grand comprehensive theory Θ such that, for every sentence φ, [φ is provable in PA iff Θ entails φ].) Having noted all this, we might still accept for some reason or other the additional principles required for the modification to go through. The resulting argument would then undergo a dialectic analogous to the one developed in the next three paragraphs.

[8] Throughout, I use '*implication*' and its relatives to express the *operation* also expressed by the *conditional* and '*biimplication*' and its relatives to express two-way implication, whereas I use '*entailment*' and its relatives to express the converse of the *relation* of *logical consequence* and '*equivalence*' and its relatives to express two-way entailment.

minimal constraints on well-behaved theories) truthmaker theory,[9] however strong, entails that M has a truthmaker.[10]

With Θ any theory, let then 'φ is truthmakeable in Θ' be short for 'Θ entails that φ has a truthmaker', and let Milne-Gołosz Truthmaker Theory (MGTT) be an acceptable truthmaker theory strong enough so that, if M is not truthmakeable in MGTT, it is not truthmakeable in any acceptable truthmaker theory (which, as per the last paragraph, is equivalent with M's not having a truthmaker of any sort). Then, given that, by MGTT's acceptability, the converse direction of that implication also holds, 'has a truthmaker' as it occurs in the M_1-argument can be taken to mean the same as 'is truthmakeable in MGTT' (and, in particular, M can be taken to mean the same as 'M is not truthmakeable in MGTT'). Now, the characterisation of MGTT just given implies that *every principle used in the M_1-argument is also contained in MGTT*, for, if some such principle π were not contained in MGTT, given π's relevance for the issue whether M has a truthmaker, there would be no guarantee that it is not the case that MGTT+π is an acceptable stronger theory that, contrary to MGTT, does entail that M has a truthmaker (after all, if there were such π, it would be *MGTT+π but not MGTT* that entails M, and so certainly it would be more likely that, [if M is not truthmakeable *in MGTT+π*, M is not truthmakeable in any acceptable truthmaker theory] rather than that, [if M is not truthmakeable *in MGTT*, M is

[9] To keep the truthmaking hierarchy as close as possible to the arithmetical hierarchy, we can suppose that every theory Θ in the hierarchy is such that, at least for every sentence φ of a certain kind, if Θ entails φ, Θ also entails ' 'φ' has a truthmaker', that the base theory in the hierarchy includes a suitably strong empirical theory and that, given a theory Θ in the hierarchy, the immediately stronger theory is got by adding to Θ every instance of 'If Θ entails that φ has a truthmaker, then φ' (the restriction on 'φ' in the first supposition — however flawed it may ultimately turn out to be, see Section 3 — is charitably meant to make it not look immediately hopeless to try to establish something to the effect that no acceptable truthmaker theory entails that a certain sentence has a truthmaker by establishing that very sentence within an acceptable truthmaker theory).

[10] Left-to-right: if a truthmaker theory entails [that a sentence has a truthmaker] whereas that sentence does not have one of any sort, the theory is not sound and so not acceptable. Right-to-left: if a sentence does have a truthmaker of some sort or other, there must be an acceptable truthmaker theory that entails that that sentence has a truthmaker (at worst, take any acceptable truthmaker theory and add to it the claim that the sentence has a truthmaker — that can hardly turn the theory into a not acceptable one given that that claim holds). Thanks to Ricardo Santos for discussion of this equivalence.

not truthmakeable in any acceptable truthmaker theory]).[11] Therefore, *the background theory in which the M_1-argument is run must be weaker than or identical with MGTT*.

But then, focussing without loss of generality on the latter case, the suggested similarity with the Gödelian incompleteness argument that relies on PA's soundness would exist *only if the background theory in which that argument is run were PA*, and so only if the argument went something like: "Working in PA, if G (and so 'G is not provable in PA') is provable in PA, by PA's soundness G is not provable in PA, and so, by *reductio ad absurdum*, G is not provable in PA". But that is emphatically *not* how the Gödelian incompleteness argument that relies on PA's soundness goes. Indeed, assuming the consistency of PA, the argument just mentioned is invalid in PA at its first step (the one that, appealing to PA's soundness, goes from 'G is not provable in PA' being provable in PA to G's not being provable in PA), given that, assuming the consistency of PA, by *Löb's theorem* ([Löb, 1955], with a little help from an anonymous referee who happened to be Henkin himself [see e.g.

[11] This is in crucial contrast with the uncontroversial way in which one can know that a certain *predeterminate* sentence φ (even a self-referential one) is not provable in any sort of way (or, *mutatis mutandis*, truthmakeable in any sort of way, though I leave to the reader the details of the *mutanda*). For φ will already be "canonically" specified as being something of the form '... is provable in A^α...' (with A^α one of the theories in the relevant arithmetical hierarchy), and so will already belong to a determinate level in the hierarchy, which will in turn determine that there is a theory A^β stronger than A^α and such that one knows that, if φ is not provable in A^β, it is not provable in any theory in the hierarchy. One can then establish the antecedent of that conditional in a theory A^γ stronger than A^β, for there is no requirement that every principle used in the argument for that conclusion is also contained in A^β, as A^β is not characterised by the condition that, if 'φ is not provable in A^β' is not provable in A^β, it is not provable in any theory in the hierarchy (it is rather characterised by the condition that, if φ (i.e., '... is provable in A^α...') is not provable in A^β, it is not provable in any theory in the hierarchy). On the contrary, for M to serve the purposes of the M_1-argument (i.e., for M to be legitimately taken to entail that no acceptable truthmaker theory entails that M has a truthmaker), it must be "non-canonically" specified as being something of the form 'M is not truthmakeable in Θ' where Θ is *at the same time* supposed to be such that, if M is not truthmakeable in Θ, it is not truthmakeable in any acceptable truthmaker theory (so that Θ is supposed to play both the role of A^α and the role of A^β): given that M is identical with 'M is not truthmakeable in Θ', that condition is tantamount to the condition that, if 'M is not truthmakeable in Θ' is not truthmakeable in Θ, it is not truthmakeable in any acceptable truthmaker theory, which, as I'm explaining in the text, implies that Θ includes the background theory in which the M_1-argument is run (so that Θ is also supposed to play the role of A^γ).

Halbach and Visser, 2014, p. 257]), PA does not entail that 'is provable in PA' is factive.[12,13]

3. Is this sentence necessitated by something?

Moving on to the gist of the replies to López de Sa and Zardini's worry, Milne [2013] has replied by offering an argument (henceforth 'the M_2-argument') that does not overtly involve the notion of truth. That's supposed to be still relevant for the debate on truthmaking, since a truthmaker for 'P' is now understood (by Milne [2013] and so by myself in this section) simply to be an object x such that x *necessitates* its being the case that P (in the sense, henceforth assumed, that x exists and it is necessary that, if x exists, then P). Consider a sentence 'M'' satisfying the biimplication:

(BM′) $\quad M'$ iff nothing necessitates its being the case that M'[14]

Suppose that something necessitates its being the case that M'. By *factualness of necessitation*,[15] M', which, by (BM′), implies that nothing necessitates its being the case that M'. On the supposition that something necessitates its being the case that M', nothing necessitates its

[12] One finds also elsewhere discussions of issues related to Gödelian incompleteness arguments which would seem to [assume that 'is provable' is factive without assuming that it expresses a property different from provability in the background theory] [see e.g. Clark, 2012, pp. 46–47].

[13] Thanks to an anonymous referee for suggestions concerning the material in this section.

[14] Milne [2013, p. 474] actually claims that, given the understanding of 'has a truthmaker' just assumed in the text, the result of substituting 'M' for 'M'' in (BM′) follows from the definition of 'M' given in Section 1. As will become apparent at the end of this section, that claim is however problematic. I think it's useful to distinguish two issues here: one issue is whether, granting that something of the form of (BM′) holds, something like the M_2-argument is a good argument against truthmaker maximalism; the other issue is whether, and how, we can get that something of the form of (BM′) holds in the first place (in particular, whether, and how, we can get that from the definition of 'M' given in Section 1). It is in order not to prejudge the latter issue that I'm *not assuming* that 'M'' (that's a bit rough, but you know what I mean) is *identical* with M (nor, for that matter, am I assuming that 'M'' is *different* from M). Thanks to an anonymous referee for urging this clarification.

[15] Just a fancy name for the tautological principle that, for every P, if [something exists and it is necessary that, if it exists, then P], then P (obviously, so understood, "*factualness*" is for *operators* what "*factivity*" of Section 2 is for *predicates*).

being the case that M'. By *reductio ad absurdum*, nothing necessitates its being the case that M'. But, by (BM′), this implies that M'. Hence M' but nothing necessitates its being the case that M'.

I'm unpersuaded. I'll articulate my reasons for being so in three parts. First, I'll prove that, under widely accepted assumptions about necessity and necessitation, the conclusion of the M_2-argument can be further developed to reach the conclusion that everything necessitates its being the case that M'. Second, even without relying on those assumptions, I'll show that the M_2-argument overreaches, since it can equally well be used to provide an apparent refutation of necessary existence conditions for something to be the case which are much weaker than the condition requiring the existence of a truthmaker and which should indeed be totally uncontroversial. Third, I'll argue that López de Sa and Zardini's worry about the M_1-argument applies just as well to the M_2-argument.[16]

First, if (BM′) is acceptable, it is presumably acceptable that it is necessary.[17] But then the only undischarged assumption of the M_2-argument is necessary, and so, if the M_2-argument is valid, by *closure of necessity under logical consequence* the M_2-argument's conclusions and what in turn follows from them are necessary too. In particular, one such conclusion is that M', from which it in turn follows, by the properties

[16] The M_1-argument has also been criticised on different grounds by Rodríguez-Pereyra [2006], with Barrio and Rodríguez-Pereyra [2015] reacting to the M_2-argument on grounds overall rather different from the ones I offer in this paper (although there are two points of broad convergence on which I'll remark in footnotes 23 and 26, respectively).

[17] For example, one might — mistakenly, as I'll argue in the third last paragraph of this section — think that something like (BM′) is forced by a standard diagonalisation procedure: such a procedure yields an arithmetical proof of the relevant biimplication, and what is *arithmetically provable* is *necessary*. More generally, as Milne [2013, p. 474] himself notes, (BM′) is supposed to hold "[. . .] in virtue of what M' means/says [. . .]" (notation changed in conformity with the one of this paper); since, when a sentence is evaluated at a possible world, *what it means/says should be held fixed*, it presumably follows that (BM′) is supposed to hold *at every possible world*, and so that it is supposed to be *necessary*. (Milne [2013, p. 475] himself would seem sympathetic with the presumption in question, as he writes that "[. . .] the proof assumes only *material* equivalence of M' and the claim that M' has no truthmaker, albeit that my exemplar gives rise to a stronger connection" (notation changed in conformity with the one of this paper).) Thanks to two anonymous referees for discussion of these issues.

of *material implication*,[18] that, if, say, the Eiffel Tower exists, then M'. Therefore, by closure of necessity under logical consequence, it is necessary that, if the Eiffel Tower exists, then M', and so, since the Eiffel Tower does exist, the Eiffel Tower does necessitate its being the case that M'. The argument generalises in the obvious way to every existing object other than the Eiffel Tower. Therefore, far from having *no* truthmaker, 'M'' is such that *every existing object* is a truthmaker for it.[19]

Second, since the previous conclusion that every existing object is a truthmaker for 'M'' relies on the M_2-argument's immediate conclusion that nothing is a truthmaker for 'M'' (plus the other widely accepted assumptions about necessity and necessitation appealed to in the last paragraph), there are clear indications that we've entered a paradoxical area. Even without relying on those assumptions, this impression is reinforced by noticing that the M_2-argument *does not rely on any specific property of truthmaking save for its factualness*, and so that it is *generalisable* to every necessary existence condition for something to be the case as long as this is factual.

Let's go through an example. Say that x *accompanies* its being the case that P iff [x exists and P]. Then, just as we have the maximalist *truthmaker* principle:

(T) For every P, if P, something makes it true that P

[18] Notice that, in the debate on truthmaking, necessitation is indeed typically understood in terms of necessity of *material* implication (as evidenced, for example, by the typical assumption that a reduction of truthmaking to necessitation entails that everything is a truthmaker for a necessary truth).

[19] Eventually, it really doesn't matter much whether the M_2-argument relies on a *necessary* biimplication (like (BM$'$)) or on a *contingent* one (as familiar from *contingent versions of the semantic paradoxes*, see Footnote 24). Take a contingent version of the M_2-argument working with a contingently self-referential sentence M'^* that gives rise to a contingent biimplication (BM$'^*$). For one thing, there's little solace in observing that such version does not conclude to an absurdity, for, essentially by *reductio ad absurdum*, it still concludes that (BM$'^*$) is contingent, and, while that is true, it is not something that should be provable *by logic* (that is similar to how there's little solace in observing, say, that Epimenides' claim that every Cretan is a liar can consistently be taken to be false by concluding that some other Cretan is not a liar). For another thing, there is no reason to doubt that *something x necessitates its being the case that (BM$'^*$) holds* (the whole world would be a good first shot), and so in particular that it is necessary that, if x exists, (BM$'^*$) holds; since, according to the M_2-argument, it is in turn necessary that, if (BM$'^*$) holds, then M'^*, by *transitivity of necessary implication* it follows that it is necessary that, if x exists, then M'^*, and so, since x does exist, x does necessitate its being the case that M'^*.

we also have the maximalist *accompaniment* principle:

(A) For every P, if P, something accompanies its being the case that P

Since something (say, the Eiffel Tower) does exist, (A) should be totally uncontroversial. Yet, the M_2-argument apparently refutes (A) just as well as (T). For consider a sentence 'A' satisfying the biimplication:

(BA) A iff nothing accompanies its being the case that A[20]

Suppose that something accompanies its being the case that A. By *factualness of accompaniment*,[21] A, which, by (BA), implies that nothing accompanies its being the case that A. On the supposition that something accompanies its being the case that A, nothing accompanies its being the case that A. By *reductio ad absurdum*, nothing accompanies its being the case that A. But, by (BA), this implies that A. Hence A but nothing accompanies its being the case that A. And, while one *might* have blithely accepted the M_2-argument's apparent refutation of (T), one *cannot* so blithely accept the M_2-argument's apparent refutation of (A), for, by definition of accompaniment, if A but nothing accompanies its being the case that A, nothing exists.[22]

Third, there is not even need to focus on existence conditions and maximalist principles: the M_2-argument overreaches even more dramatically, to the extent that López de Sa and Zardini's worry about the M_1-argument *applies just as well* to the M_2-argument. For consider a sentence 'S'' satisfying the biimplication:

(BS′) S' iff it is not the case that [S' and 'S'' is short]

[20] For the time being, I assume that, just as Milne infers (BM′) without further justification from the uncontroversial self-reference fact guaranteed by the definition of M given in Section 1, one can analogously infer (BA) (and (BS′) in the next paragraph) from the relevant uncontroversial self-reference facts. I see no reason for doubting that these inferences stand or fall together, and I'll scrutinise at the end of this section under which conditions they do stand. Thanks to an anonymous referee for recommending this comment.

[21] Just a fancy name for the tautological principle that, for every P, if something exists and P, then P.

[22] Reason: suppose that A but nothing accompanies its being the case that A. Then, by definition of accompaniment, for every object x, it is not the case that [x exists and A]. By universal instantiation, it is not the case that [a exists and A] (where a is arbitrary), and so, since A, by *modus ponendo tollens* a does not exist. By universal generalisation, for every object x, x does not exist — that is, nothing exists. Thanks to an anonymous referee for asking for this explanation.

Suppose that S' and 'S'' is short. By simplification, S', which, by (BS′), implies that it is not the case that [S' and 'S'' is short]. On the supposition that S' and 'S'' is short, it is not the case that [S' and 'S'' is short]. By *reductio ad absurdum*, it is not the case that [S' and 'S'' is short]. But, by (BS′), this implies that S'. Hence, since it is not the case that [S' and 'S'' is short], 'S'' is not short.

López de Sa and Zardini [2006, p. 156] indicated that an adequate solution to the paradoxes of *naive truth* (which we can take to be the equivalence between φ and ''φ' is true') would take care of M and S (cf. Footnote 4). But what's naive truth got to do with the M_2-argument, which, in any of its versions concerning 'M'', 'A' and 'S'', does not overtly involve the notion of truth? A good deal. Start by asking why one should accept something like (BM′). One might have thought that (BM′) is just forced by some *standard diagonalisation procedure*. But that is not so. (BM′) would be forced by some standard diagonalisation procedure if this yielded the *non-standard diagonal lemma* that, for every *context* $\ldots\cdot\ldots$, there is a sentence φ such that 'φ iff $\ldots\varphi\ldots$' holds. But a standard diagonalisation procedure does *not* yield that. Perhaps the most straightforward way to see this is to let $\ldots\cdot\ldots$ be $\neg\cdot$ and observe that, then, the non-standard diagonal lemma would entail that there is a sentence φ such that 'φ iff $\neg\varphi$' holds, which would incredibly mean that even relatively weak classical arithmetical theories (which do allow for standard diagonalisation procedures) are inconsistent.

What a standard diagonalisation procedure does yield is that, letting $\ulcorner\varphi\urcorner$ be the canonical name of φ, for every *predicate* $\Phi(\cdot)$, there is a sentence φ such that 'φ iff $\Phi(\ulcorner\varphi\urcorner)$' holds.[23] Now, clearly, if there were a predicate $\mathrm{T}(\cdot)$ such that φ is *intersubstitutable* with $\mathrm{T}(\ulcorner\varphi\urcorner)$ (a condition which, under natural assumptions we can take for granted for the purposes of this discussion, is biimplied by the condition that $\mathrm{T}(\cdot)$ is a naive-truth predicate), a standard diagonalisation procedure would indeed yield the non-standard diagonal lemma. For, in that case, given any context $\ldots\cdot\ldots$, we could apply the standard diagonalisation procedure to the complex predicate $\ldots\mathrm{T}(\cdot)\ldots$ to get that there is a sentence φ such that 'φ iff $\ldots\mathrm{T}(\ulcorner\varphi\urcorner)\ldots$' holds, which, by intersubstitutability, would then give

[23] The relevance of this fact for the M_2-argument is also briefly noted by Barrio and Rodríguez-Pereyra [2015, pp. 6–7] who however only infer from it that, contrary to Milne's [2013] insistence, the M_2-argument does presuppose *some semantic notion or other*, rather than, as I'm about to do, that it presupposes *something* (*naive truth*) *that it cannot have together with classical logic*.

us that there is a sentence φ such that 'φ iff $\ldots\varphi\ldots$' holds. Therefore, a route to (BM′) is constituted by a standard diagonalisation procedure *plus naive truth* (since, as we've just seen, applying the latter to the former would yield the non-standard diagonal lemma). But that route relies on naive truth, and all the other routes I can think of equally rely on naive truth or on some other equivalent, paradox-breeding ideology.[24]

Obviously, we need to block the M_2-argument at least in its versions concerning 'A' and 'S'', and we can do so by either *rejecting naive truth* (and so rejecting that there are sentences satisfying (BA) and (BS′), see Footnote 4 and Section 2 for more on this option) or *rejecting classical logic* (and so rejecting the reasoning starting from (BA) and (BS′), see Footnote 26 for more on this option). But, on either option, we'll also have blocked the M_2-argument in its version concerning 'M'' (cf. Footnote 4). Therefore, when faced with the M_2-argument, I conclude that one should still stand by López de Sa and Zardini's worry about the M_1-argument: the M_2-argument covertly relies on both naive truth and classical logic, and so cannot be sound for reasons that have little to do with truthmaker maximalism.

4. Against a defence of an argument against truthmaker maximalism

Gołosz [2015] has replied to López de Sa and Zardini's worry in an interestingly different way from Milne [2013], by sticking to the original, overtly truth-involving M_1-argument and taking issue instead with the claims made by López de Sa and Zardini that "that S is not short is

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[24] For example, recalling the case of *Epimenides*, one might try to achieve the required kind of self-reference *via* the *exploitation of certain contingent circumstances* rather than *via* a *formal diagonalisation proof.* In more detail, one might try to do so by considering, say, the claim that [nothing necessitates the first bracketed sentence in Footnote 24 of this paper]. But, as — following Milne — we've been understanding it, necessitation takes a *clausal* rather than *objectual* complement in its second argument, so that the attempt just considered is *ungrammatical.* We can make it grammatical by considering the claim that [nothing necessitates its being the case that the second bracketed sentence in Footnote 24 of this paper is true] — thereby relying on *naive truth* — or by considering the claim that, [for every P, if the third bracketed sentence considered in Footnote 24 of this paper says that P, nothing necessitates its being the case that P] — thereby relying on the ideologies of *impredicative higher-order quantification* and *naive saying-that*, which are well-known jointly to breed paradoxes analogous to those of naive truth [see e.g. Zardini, 2008, pp. 563–566].

inconsistent with the deliverances of our senses, but, unlike the Liar sentence, S itself gives rise to no inconsistency when treated as an ordinary sentence and subject to the usual rules of logic" [López de Sa and Zardini, 2006, p. 156, henceforth 'claim LSZ_1'] and that the M_1-argument "[...] could be used to establish (the negation of) just about anything you please" [López de Sa and Zardini, 2006, p. 154, henceforth 'claim LSZ_2']. As I understand him, Gołosz offers basically two arguments against these claims. In essence, the first argument (henceforth 'the G_1-argument') goes thus. Claims LSZ_1 and LSZ_2 crucially rely on the unresolved vagueness of 'is short'. If such vagueness is resolved so that 'is short' is supposed to mean the same as 'contains at most 10 words', S is logically inconsistent, contrary to claim LSZ_1. If instead such vagueness is resolved so that 'is short' is supposed to mean the same as 'contains at most 5 words', S is unproblematically true, contrary to claim LSZ_2. Moving on to the second argument (henceforth 'the G_2-argument'), in essence it goes thus. With G not as in Footnote 6, consider the sentence:

G: G is not [true and F]

Either G is F or it is not. If it is, G holds iff it does not, and so it is logically inconsistent, contrary to claim LSZ_1. If it is not, G is unproblematically true, contrary to claim LSZ_2.

Again, I'm unpersuaded. Again, I'll articulate my reasons for being so in three parts. First, I'll argue that the G_1-argument is misled by the vagueness of 'is short'. Second, I'll show that, in spite of its trappings, the G_2-argument actually proves claim LSZ_2. Third, I'll contest Gołosz' final attempt at identifying a disanalogy between M and S.

First, when, in giving the G_1-argument, Gołosz claims that, if the vagueness of 'is short' is resolved so that 'is short' is supposed to mean the same as 'contains at most 10 words', S is logically inconsistent, he must be assuming, not only that it is *false* (as everyone agrees), but that it is indeed *logically inconsistent* for S to contain more than 10 words—after all, under the present reading of 'is short', if S contained more than 10 words, S would hold.[25] Let's grant Gołosz a broad enough

[25] Quite generally, under the assumption that φ entails ψ, if φ is logically consistent, ψ is logically consistent. Now, let φ be 'S contains more than 10 words' and ψ be 'S is not [true and such as to contain at most 10 words]', so that that assumption is satisfied. Then, by contraposition, if 'S is not [true and such as to contain at most 10 words]' is logically inconsistent, 'S contains more than 10 words' is logically inconsistent. Thanks to an anonymous referee for asking for this explanation.

notion of logical inconsistency so that that assumption is indeed correct (in spite of Gołosz [2015, p. 107] also apparently thinking that it is "[...] an empirical fact" that S contains at most 10 words). Still, contrary to what Gołosz surmises, López de Sa and Zardini's worry does not crucially rely on the unresolved vagueness of 'is short': López de Sa and Zardini could have raised *exactly the same worry* by using, instead of 'is short', a predicate that is *not vague* (for example, 'is absent from the first draft of [Tarski, 1933]', where we may assume that every sentence is either determinately present in the first draft of [Tarski, 1933] or determinately absent from the first draft of [Tarski, 1933], so that there is no relevant vagueness in the application of the predicate), and (to continue with that example) considering the sentence:

S'': S'' is not [true and absent from the first draft of [Tarski, 1933]]

Suppose that S'' is true and absent from the first draft of [Tarski, 1933]. Then it is true. So what it says is the case is the case. Hence S'' is not [true and absent from the first draft of [Tarski, 1933]]. On the supposition that S'' is true and absent from the first draft of [Tarski, 1933], it is not [true and absent from the first draft of [Tarski, 1933]]. By *reductio ad absurdum*, S'' is not [true and absent from the first draft of [Tarski, 1933]]. But this is just what S'' says. Hence S'' is true. Hence, since it is not [true and absent from the first draft of [Tarski, 1933]], it is not absent from the first draft of [Tarski, 1933]. There is no unresolved vagueness in S'', and it cannot be reasonably claimed that S'' is logically inconsistent — after all, if S'' were present in the first draft of [Tarski, 1933], S'' would hold, and whether S'' is present or not in the first draft of [Tarski, 1933] is not a matter of *logic*, but a matter of *history* (so that, in particular, it is logically consistent for S'' to be present in the first draft of [Tarski, 1933]): as per Footnote 25, it follows from these two facts that S'' too is logically consistent. Should we then accept this argument and conclude to the bold historiographical claim that S'' is present in the first draft of [Tarski, 1933]?

Second, even if it might be thought that the G_2-argument improves on the G_1-argument in that it does not appeal to a red herring such as vagueness, far from *refuting* claim LSZ_2 (or the conjunction of claim LSZ_1 and claim LSZ_2), the G_2-argument actually *proves* it. If the assumption that G is F leads to the conclusion that G holds iff it does not, that assumption leads to an absurdity and as such, essentially by *reductio ad absurdum*, we must reject it in favour of its negation (when Gołosz

[2015, p. 107] writes that "[...] nothing can be proved by logically inconsistent sentences", he might mean to say something that contradicts this basic principle of classical logic: the conclusion that [a sentence holds iff it does not] does prove, in classical logic, that whatever entails such conclusion does not hold). That is, we must infer *under no assumptions* that G is not F. But that is by no means a conclusion one can live with. For 'is F' is *arbitrary*, so that, for any P, it can be replaced by 'is such that P'. Therefore, for any P, we can derive that something is not such that P, and so, given that [P iff everything is such that P], *for any P, we can conclude that it is not the case that P*. In other words, employing essentially the same resources as those employed in the M_1-argument, the G_2-argument "[...] could be used to establish (the negation of) just about anything you please", which is precisely claim LSZ_2.[26]

[26] Since we obviously can't establish (the negation of) just about anything you please, the G_2-argument (as I've developed it) must fail. Since the argument essentially relies on naive truth and classical logic, that means that either of these must fail; in either case, that will also block the M_1-argument (and the M_2-argument), as this employs essentially the same resources (cf. Footnote 4). Zooming in on the option of rejecting classical logic, it is indeed the case that some of the steps of the G_2-argument are problematic on many *non-classical logics proposed for the semantic paradoxes*, but then *essentially the same steps* are required in the M_1-argument (and in the M_2-argument). Both Milne [2005, p. 222] and Gołosz [2015, p. 106] correctly remark that *reductio ad absurdum* (from φ's entailing $\neg\varphi$ infer $\neg\varphi$) is valid in *minimal logic* [Kolmogorov, 1925], with both Milne [2013, p. 475] and Gołosz [2015, p. 106] adding that it is also valid in *LP* [Asenjo, 1966]. But, if such remarks are supposed to *legitimate the reasoning of the M_1-argument by the lights of non-classical approaches to the semantic paradoxes*—as they apparently are—they fail to do so (even setting aside the glaring fact that *reductio ad absurdum is indeed invalid in the vast majority of non-classical logics proposed for the semantic paradoxes*). To begin with, minimal logic is irrelevant, as it is a *non-starter* as a solution to the semantic paradoxes (and, basically, it is such precisely because it validates *reductio ad absurdum*: think Curry...). Moreover, while LP is relevant, it is such precisely in the opposite sense of *undermining the M_1-argument*: for, on an LP-approach, to accept [that M does not have a truthmaker] on the basis of *reductio ad absurdum* by no means prevents one from *also* accepting that M has a truthmaker! More generally, it is crucial for a non-classical approach to the semantic paradoxes, for some sentence φ, to accept the equivalence between φ and $\neg\varphi$ (for example in the case of a Liar sentence), but it's easy to see that, *on virtually no logic on which some such equivalence is acceptable*, one can use *reductio ad absurdum* on the left-to-right direction of the equivalence to accept $\neg\varphi$ in a sense that rules out φ (which is what the M_1-argument attempts to do). For, given one's acceptance of $\neg\varphi$, the right-to-left direction of the equivalence forces one to accept φ just as well [see Zardini, 2015a, pp. 469–485 for more discussion of *reductio ad absurdum* in the context of the semantic paradoxes]. Even more generally,

Third, Gołosz tries once more to chisel apart M and S by claiming:

> [. . .] sentences of the type of S confuse object-language and meta-language assigning truth-value to themselves, which results in similar consequences to the case of the Liar Sentence. Unlike S, the sentence M, which is used in Milne's argument, does not assign a truth-value to itself and is not logically inconsistent. [Gołosz, 2015, p. 108]

As far as I can tell, there are two thoughts here. One thought is the idea that there is a *difference in logical consistency* between S and M, which I've already argued against. The other thought is the idea that S is problematically close to paradox *because it assigns a truth value to itself*, whereas M isn't because it doesn't. But, setting aside the fact that assigning a truth value to oneself is not *sufficient* for being problematically close to paradox (think e.g. of the sentence saying of itself that it is [false and such that everything is true]) and focussing on whether it is *necessary*, a quick survey of the *paradoxes of self-reference* reveals that paradox is not at all avoided by a sentence simply because it does not assign a truth value to itself. For example, the *Knower paradox* arises from a sentence that, instead of assigning a truth value to itself, says of itself that it is not known ([Kaplan and Montague, 1960]; in

without going into such details as *reductio ad absurdum* etc., the reasoning of the M_1-argument is basically the same as the reasoning purporting to show that S is true but is not [true and short]; since, putting dialetheic approaches aside (for which see the above observation), the latter reasoning *has got to fail in some way or other on virtually every non-classical approach to the semantic paradoxes*, so must the former reasoning. (Milne [2013] displays appreciation that LP-approaches do not exhaust the range of non-classical approaches to the semantic paradoxes, but then claims at p. 479 that "[. . .] it seems it is indeed Contraction that must be restricted": while I myself do favour a non-contractive approach [see e.g. Zardini, 2011], that claim oddly disregards many other viable non-classical approaches.) Therefore, contrary to Milne's and Gołosz' hints to the contrary, the M_1-argument exemplifies a pattern of reasoning that is *illegitimate on virtually every non-classical approach to the semantic paradoxes*. (I take the material in this footnote to be broadly congenial to the point made by Barrio and Rodríguez-Pereyra [2015, p. 5], who however focus on what seems to me an infelicitous non-classical approach to the semantic paradoxes, on which a sentence like M is *not true* (as well as not false): since it is certainly the case that, if a sentence is not true, it does not have a truthmaker, such approach would seem committed to accepting 'M does not have a truthmaker', which would be a catastrophic commitment given that that sentence is identical with M and given that, on the kind of approach in question, if one accepts a sentence, one also accepts that the sentence is true.) Thanks to an anonymous referee for feedback on some of the material in this footnote.

the framework of the approach mentioned in footnotes 1 and 26, I offer my own treatment of the Knower and kindred paradoxes in [Zardini, 2016b]). As remarked by López de Sa and Zardini [2006, p. 156], the problem common to many paradoxes of self-reference arguably consists in the fact that a certain sentence *involves a suitable self-referential use of an apparently factive predicate*: 'is true' is apparently factive, and thus gives rise to the Liar paradox; 'is known' is also apparently factive, and thus gives rise to the Knower paradox. Given that it relies on S's involving a suitable self-referential use of 'is true and short' and on the apparent factivity of that predicate, there is thus every reason to distrust the reasoning purporting to show that S is not short. But, *just so*, given that it relies on M's involving a suitable self-referential use of 'has a truthmaker' and on the apparent factivity of that predicate, there is every reason to distrust the M_1-argument. No disanalogy between M and S has thus been made out which gives us reason to think that M is less problematically close to paradox than S is. I conclude that truthmaker maximalism has not been proven false — not by M, anyway.

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