



Nicholas J. J. Smith

## A THEORY OF PROPOSITIONS

**Abstract.** In this paper I present a new theory of propositions, according to which propositions are abstract mathematical objects: well-formed formulas together with models. I distinguish the theory from a number of existing views and explain some of its advantages—chief amongst which are the following. On this view, propositions are unified and intrinsically truth-bearing. They are mind- and language-independent and they are governed by logic. The theory of propositions is ontologically innocent. It makes room for an appropriate interface with formal semantics and it does not enforce an overly fine or overly coarse level of granularity.

**Keywords:** propositions; models; well-formed formulas; logic

### 1. What are Propositions— and What are They Good For?

The topic of this paper is the nature of propositions. The aim is to answer the question: what *are* propositions? More precisely, the question is: What should we take propositions to be, *given* the work we want them to do? So what work is that? Well, propositions are an essential component of what I shall call Grand Theory (GT). GT is a cluster of theories, proto-theories and research programmes concerning:

- belief, desire and other attitudes
- language and communication
- rational action.

Core tenets of GT include the following. Persons (and agents more generally) believe things (call these things *Xs*). Logic is concerned with these things (*Xs*) and the logical relations amongst them. Logic thereby provides norms for belief (e.g. consistency). Explanations of action advert to beliefs and desires (and hence to *Xs*): rational action involves

acting in a way that will achieve one's desires if the world is as one believes it to be; rational choice involves maximising expected utility. Language provides a means of expressing and communicating beliefs. Different persons can believe the same things ( $X$ s) and the same things ( $X$ s) can often be expressed in different languages.

$X$ s — or 'propositions' as I shall call them — are the common coin here which link up language, logic, belief and action. Thanks to propositions, these topics can connect up with one another in the over-arching story of GT. There are various roles for propositions in this over-arching story:

1. Propositions are the objects of the attitudes such as belief and desire.
2. Propositions are expressed by sentences uttered in contexts.
3. Further to **1** and **2**: the very same proposition can be expressed in different languages and can be the object of the attitudes of different agents. This is important for the role of propositions in explaining communication. The very same proposition can also be the object of different attitudes of the same agent (e.g. belief and desire). This is important for the role of propositions in explaining rational action.
4. Propositions are the primary bearers of the properties truth and falsity. (It may be that propositions are true or false relative to possible worlds, in which case they are also the bearers of the properties necessary truth and contingent truth.) Other things can also be called 'true' and 'false', but their truth and falsity will be explained in terms of the truth and falsity of propositions: an utterance of a sentence is true if the proposition expressed is true; a state of belief is true if the content of the belief (a proposition) is true; and so on.
5. Propositions are the objects of logic: propositions and/or sets thereof are the bearers of logical properties such as logical truth and satisfiability, and the relata of logical relations such as logical consequence and equivalence. This is important for the role of logic in providing norms for rational thought (e.g. the objects of an agent's beliefs should form a satisfiable set of propositions). It is also important for the role of logic in explaining behaviour (e.g. Bob turned up at 9am at Carol's office because he wanted to speak to Carol and he inferred — from various other beliefs he had — that she would be there then).

So what we want is a theory of what propositions must be — or at least could be — that would enable them to play these roles. In other words, we initially take 'propositions' to be a label for the things that play the

roles just outlined in GT. At this point we know what propositions *do*. We now look for an account of what they could be like in themselves.<sup>1</sup>

### 1.1. Pluralism about Propositions

When faced with a list of roles that one kind of entity is supposed to play, one may wonder whether in fact several kinds of entity are involved, with each kind playing some of the roles. For example, Lewis [38, p. 54] writes: “The conception we associate with the word ‘proposition’ may be something of a jumble of conflicting *desiderata*.” However, in the present context, if one does wish to adopt a pluralist view—a view according to which one kind of thing plays some of the roles for propositions mentioned above, while different kinds of thing play others of the roles—then one is obliged to tell a further story about how these different kinds of thing interact. Otherwise, GT falls apart.<sup>2</sup> Propositions do not simply feature in the various components of GT—logic, semantics, propositional attitude psychology—they furthermore play the role of nexus which allows these components to combine into an over-arching theory. It is part of the story that what you say might be the very thing I believe; that what you believe might be (logically) inconsistent with what she desires; and so on. This is not to say that we could not, in principle, make GT more complex by telling a story about how the propositions that feature in logic connect up with the different propositions that feature in propositional attitude psychology, and so on. Nevertheless, monist views—according to which there is a single notion of proposition that can play all the roles outlined—will certainly have an advantage of simplicity.

In this context it is important to clarify that I am already opening the way to—although not advocating—what would, from a different

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<sup>1</sup> In order to avoid a possible misunderstanding, I should emphasise that what I have outlined are roles for propositions *in GT*—as opposed, for example, to roles that things labelled ‘propositions’ have been taken to play in the (recent) literature. A list of roles of the latter sort would probably include things not on my list (see the discussion in §1.1) and would probably not include role 5 from my list (for the idea that propositions are the objects of logic, while traditionally important, has dropped off the radar in the recent literature on propositions).

<sup>2</sup> Here it is important that (as discussed in n.1) the roles for propositions presented in §1 are those that propositions need to play *for purposes of GT*. If we had instead given a list of roles that things labelled ‘propositions’ have been taken to play in the (recent) literature then there would be no reason, in principle, why the same kind of thing should be expected to play all the roles.

perspective, count as a form of pluralism about ‘propositions’.<sup>3</sup> For note that I have not included the following as roles for propositions:<sup>4</sup>

- Propositions are the compositional semantic values of declarative sentences.
- Propositions are the referents of ‘that’-clauses.

Role 2 above says that propositions are expressed by sentences uttered in contexts. Presumably, the story about *how* a sentence comes to express a proposition when uttered in a context involves compositional mechanisms. Informally, the meanings of the individual words in a sentence, and the way those words are put together to form the sentence — together with facts about the context in which the sentence is uttered — determine the proposition thereby expressed. Slightly more formally, each linguistic expression is associated with an entity: its *compositional semantic value* (csv).<sup>5</sup> The csv of an expression  $X$  is a function of the csv’s of  $X$ ’s component expressions, together with the syntax of  $X$  (the way that the components of  $X$  are combined to form  $X$ ). This is why csv’s are *compositional*. Furthermore, the csv of a *sentence* should — together with facts about the context of utterance — determine the proposition expressed by uttering that sentence in that context (or, as it is sometimes put, should determine *what is said* by that sentence uttered in that context). So there is a constraint on the relationship between accounts of propositions, and theories in formal semantics: propositions should be the kinds of things that can be *determined* by csv’s together with contexts. But

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<sup>3</sup> I put ‘propositions’ in quotation marks here for the following reason. I have stipulated that by ‘proposition’ I mean the things — the  $X$ s — that play the roles in GT outlined in §1. So given what I mean by ‘proposition’, the view to be presented in this paper is monist, not pluralist (i.e. the same kind of entity plays all the roles). Other authors also take ‘propositions’ to be the things that play certain roles — but they include more roles on their lists. From their perspective, my view will be potentially pluralist (i.e. about ‘propositions’ in their sense) — because I only argue that a certain kind of entity can play all the roles on my list. My view is only potentially pluralist because I do not deny that my propositions can also play these further roles — I just leave the matter open.

<sup>4</sup> Contrast e.g. Briggs and Jago [8, §2.2], who present a more inclusive list of roles for ‘propositions’. Contrast also Bealer [4, p. 19] who stipulates that by ‘proposition’ he means the entities referred to by ‘that’-clauses.

<sup>5</sup> The term ‘semantic value’ was coined by Lewis [37], to provide a neutral term, free from the unwanted connotations of existing terms such as ‘meaning’ and ‘sense’. Unfortunately, the term ‘semantic value’ has now joined ‘meaning’, ‘sense’ and so on in being widely used, to mean a variety of different things. I therefore use the term ‘compositional semantic value’ to mean exactly what Lewis meant by ‘semantic value’.

this does not mean that the csv's of sentences have to *be* propositions: that is, the very same things that are the objects of belief, the relata of logical relations, and so on. Of course they might be the same things — but such an identification is not built into my framework from the outset (as it would be if we defined 'propositions' as the things that play certain roles — and included amongst these roles all those mentioned in §1 *and* being the csv's of sentences).<sup>6</sup>

Similar remarks apply to the second bullet point above: the idea that propositions are the referents of 'that'-clauses. It is part of GT that propositions are the objects of the attitudes such as belief; it is also part of GT that propositions can be expressed by uttering sentences in context. Suppose that Bob utters sentence 1 below. I might report this fact by uttering 2. I might furthermore form a belief about what Bob believes, which — were I to make it public — I might express by uttering 3:

1. Mary is in town.
2. Bob said that Mary is in town.
3. Bob believes that Mary is in town.

An attractively simple view prompted by these simple sorts of example is that the proposition Bob expressed by uttering 1 is the referent of the expression 'that Mary is in town' as it features in 2 and 3.<sup>7</sup> In more complex cases, however, things get [...] more complex. In any case, the point here is that we do not need to — and I have not — built into the list of roles for propositions the idea that propositions are the referents of 'that'-clauses. Deciding what the referents of 'that'-clauses are — like working out the best theory of csv's — is a matter for formal semantics. The only constraint imposed by GT concerns the *interface* between formal semantics and areas such as logic and propositional attitude psychology. In particular, if Bob utters 1, then — given certain further assumptions — 2 and 3 should be *true*. It is not mandatory, however, that they get to be true via having a component (the clause 'that Mary is in town') that refers to the proposition expressed by 1. Of course that might be how they get to be true — but it need not be. So as before (in the case of the csv's of sentences), I am not denying that

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<sup>6</sup> Lewis [37] argued that the csv's of sentences are not propositions; for more recent discussion see e.g. Rabern [46] and Weber [66]. As I have made clear, I am not denying that propositions are the csv's of sentences: I am remaining neutral on this controversial issue.

<sup>7</sup> Cf. the 'face-value theory' of Schiffer [52].

propositions are the referents of ‘that’-clauses — I am remaining neutral on this controversial issue.

## 1.2. Theory not Analysis

In the previous section, the project of this paper — giving an account of what propositions could be, given the roles they are supposed to play in GT — was distinguished from certain projects in formal semantics (determining the csv’s of various expressions; determining the referents of ‘that’-clauses). Before proceeding, it will be useful to clarify our aims further by pointing out that the project here is not that of analysing a folk notion. Propositions in the sense of interest here play a role (several roles) in GT: they belong to the *theorist* of human thought, language and behaviour — not to the theorised subjects. Of course GT incorporates certain common-sense explanatory strategies — concerning, for example, why Bill turns up in a certain place at a certain time, having heard Ben say ‘Let’s meet at the cinema at 7pm’ and desiring to meet Ben (etc). But systematising, generalising and making precise folk explanatory strategies while incorporating them into a broad over-arching theory is a different project from *analysing* a notion that the folk themselves employ when, for example, they explain each others’ behaviour. I am not supposing that propositions, in the sense of interest here, feature in folk explanations: only that they feature in GT. Hence, in our search for entities to play the roles identified for propositions in GT, there is no constraint that the candidates must be things of a sort that the folk could easily see themselves as getting in touch with whenever they believe or say something.<sup>8</sup>

## 2. The Shape of the Theory

The fundamental guiding idea behind formal or model-theoretic semantics is to use tools and techniques from model theory for formal languages to shed light on natural language semantics. In model theory one considers a formal language and one or more models of the language.<sup>9</sup> Details

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<sup>8</sup> I am thinking here of Bealer’s claims that various theories of propositions are “counterintuitive” and “intuitively implausible” [4, §2].

<sup>9</sup> Models are sometimes called ‘interpretations’. On a different usage — not the one employed in this paper — a ‘model’ of a set of sentences is an interpretation (i.e. a ‘model’ in the sense of this paper) on which the sentences all come out true.

will vary, but the essential thing about a model is that it assigns values to expressions (simple and complex) of the language. In applying this framework to natural language semantics, the standard analogies are as follows:

- well-formed formula (wff) of the formal language  $\approx$  a sentence of natural language
  - values assigned in model  $\approx$  meanings (semantic values) of expressions
- Propositions* are then often taken to be the meanings (semantic values) of entire sentences.

I propose a different analogy. On this view, a wff of the formal language does not correspond to or represent a sentence of natural language.<sup>10</sup> Rather:

- wff of the formal language  $\approx$  (part of) the *proposition* expressed by a sentence of natural language (in some context)
- model (that assigns values to expressions in the wff)  $\approx$  (the remainder of) the proposition

So on this view, a proposition is a *wff together with a model*.

To get a feel for this view, think about the process (as taught for example in introductory logic classes) of representing ordinary claims in the language of first-order logic (FOL).<sup>11</sup> For example:

- claim:  
Jim has read every novel that any of his friends has read.
- glossary:  
 $j$ : Jim  
 $Nx$ :  $x$  is a novel  
 $Rxy$ :  $x$  has read  $y$   
 $Fxy$ :  $y$  is a friend of  $x$
- wff:  
 $\forall x((Nx \wedge \exists y(Fjy \wedge Ryx)) \rightarrow Rjx)$

There are different views about what is going on here. One idea is that we are *translating* the English sentence into a corresponding sentence of the logical language, much as we might translate it into German.

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<sup>10</sup> This is not to say that we cannot represent a sentence of natural language using a wff of some formal language. Of course we can—and indeed should. The point is that there is also a different role to be played by wffs of a formal language. (Whether the same wffs play both roles—or different wffs play each role—is an issue we come to shortly.) It is this other role that I am talking about now.

<sup>11</sup> For simplicity, I shall use ‘FOL’ as an abbreviation of both ‘first-order logic’ and ‘first-order language’.

The glossary is then an English–Logic dictionary: much like an English–German dictionary, only arbitrarily stipulated (not established by the regular usage of speakers) and temporary (later on we might use  $j$ ,  $N$  and so on to mean different things). A significantly different idea – the one I wish to focus on – is that the wff  $\forall x((Nx \wedge \exists y(Fjy \wedge Ryx)) \rightarrow Rjx)$  does not represent the English *sentence* ‘Jim has read every novel that any of his friends has read’: it represents the *proposition expressed* by (some particular utterance of) this sentence. But of course the wff all by itself does not represent this proposition: it is the wff *under the given glossary* that represents the proposition. For the very same wff would represent a completely different proposition, if we provided a different glossary:

$j$ : Jane

$Nx$ :  $x$  is a mountain

$Rxy$ :  $x$  has climbed  $y$

$Fxy$ :  $y$  is a compatriot of  $x$

What does the glossary contribute? Intensions. That is, functions from possible worlds to objects (in the case of names) or to sets of  $n$ -tuples (in the case of  $n$ -ary predicates). In conjunction with the actual world, these intensions determine a model. On this view, then, the proposition expressed by (some particular utterance of) the sentence ‘Jim has read every novel that any of his friends has read’ is (represented by) a wff together with something else: either intensions for the nonlogical symbols, or a model.<sup>12</sup> Depending on exactly what we mean by ‘interpreted’, we could express this by saying that a proposition is (represented by) an *interpreted wff*.<sup>13</sup>

Now of course we might also want to represent the English *sentence* using a wff of a formal language. If we do, the wff we use for *this* purpose may or may not be the same as the wff given above. In any case, one crucial role for the wff  $\forall x((Nx \wedge \exists y(Fjy \wedge Ryx)) \rightarrow Rjx)$  is representing part of a certain proposition (as opposed to some sentence that expresses this proposition).

We have considered the activity of representing claims made in English in FOL. Consider now a second example: doing formal semantics

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<sup>12</sup> I advocate this sort of view in Smith [57]. Readers should consult that work (in particular ch. 11) for complete details: here I have sketched the view in only the barest outline. Note that for present purposes it does not matter whether this is the *correct* view of what is going on when we represent ordinary claims in FOL.

<sup>13</sup> Cf. Smith [55, p. 254].

in the two-step fashion of Montague [43]. We begin with a sentence of English. We then derive a wff in the formal language of *intensional logic* (IL). We then consider models of IL (rather than directly defining models for English). As in the example of FOL considered above, there are different views about what is going on here. Montague himself viewed the process as one of *translating* English into the language IL.<sup>14</sup> But we can also think of things in a different way, that will exhibit the structure of the kind of view that I am proposing. We can see the proposition expressed by a sentence of English not as the value assigned to the wff corresponding to the whole sentence, but as the corresponding wff together with its value (or the values of its components). Crucially, the wff involved here — the one that (on this view) is part of the proposition expressed by the English sentence — is not the same object as the sentence (and nor is it taken to represent the sentence).

My aim in this paper is to present and argue for a view with a certain overall shape — not to settle all the details. My aim is to say what kinds of things propositions might be, given the roles they play in GT. My aim here is not to complete GT — or even to work out in any detail a certain fragment of GT (pertaining, say, to certain kinds of agents in certain kinds of circumstances). But it is only at the stage of detailed working out of GT (or some fragment thereof) that certain of the fine details concerning propositions will get fixed. So the examples just presented are intended to give the shape of the view — not the fine details. The overall shape is this. A proposition should be seen as a wff together with a model (of the fragment of the formal language needed to form that wff). The details that I wish to leave open — the ones that will get filled in as GT is completed — are these:

1. Which formal language provides the wff parts of propositions?
2. Which kind of model theory provides the remaining parts of propositions?
3. Should we use the same formal language that we use for representing propositions, to represent sentences of English or other natural languages?

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<sup>14</sup> Furthermore, he did not view the two-step process as essential (he did not adopt it in all his papers) — and most subsequent work in formal semantics abandons it in favour of a one-step process in which one directly considers models of English (considered as a formal language). So I am certainly not claiming Montague as an adherent of the kind of view I wish to advocate here.

Regarding **1**: For the sake of simplicity and familiarity to the widest possible set of readers, when giving examples below I shall use FOL — but the essential points go through just the same for the more complex (lambda-categorical, typed, higher-order etc.) languages typically employed in formal semantics. Regarding **2**: Again, for the sake of simplicity and familiarity to the widest possible set of readers, when giving examples I shall use a standard classical model theory for FOL — but again, the essential points go through just the same for the more complex (intensional etc.) models typically employed in formal semantics.<sup>15</sup> Regarding **3**: One commitment I do want to make is that we should not use the *very same wff* to represent a sentence and (the wff part of) the proposition it expresses. This is because it should be possible to express the very same proposition using different sentences — indeed using sentences of different languages. This point will be discussed in more detail below (§3.4).

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<sup>15</sup> One thing I should make clear is that a ‘model’, in the sense in which I am using the term (which is standard in logic), is a precisely defined mathematical object. Details will vary depending on what kind of formal language — and what kind of model theory for that language — are in play. Generally, however, a model includes an assignment of a value (of an appropriate sort) to each expression (of a certain sort) of the language (and as there are usually infinitely many such expressions, such an assignment is typically specified recursively). For example: in the case of classical models of propositional logic, a model comprises an assignment of exactly one of the two truth values to each wff (and such an assignment is typically specified by (a) stipulating values for the basic, unstructured wffs and (b) giving truth tables which determine values for complex wffs, given values for their components); and in the case of classical models of FOL, a model includes an assignment of an object to each name, a set of  $n$ -tuples of objects to each  $n$ -place predicate, and exactly one of the two truth values to each closed wff (and such an assignment is typically specified in a recursive way — see e.g. Smith [57, §12.2.1] for details). On the other hand, if (for example) we are employing FOL with two one-place predicates  $P$  and  $Q$ , then we do *not* specify a model of the language (in the sense of interest here) if we just say something like ‘let  $P$  mean dogs and  $Q$  cats’ or ‘let  $P$  be people and  $Q$  horses’ — or if we just give a glossary (of the kind mentioned earlier in this section): for such pronouncements, by themselves, are insufficient to deliver a unique, well-defined assignment of values to expressions and wffs of the language. So, in leaving it open what kind of model theory provides the remaining (i.e. non-wff) parts of propositions, I am *not* leaving open what *kind* of thing I mean by a ‘model’: I always mean a well-defined mathematical object that includes an assignment of a value (of an appropriate sort) to each expression (of a certain sort) of the language. What I am leaving open is what kinds of values are appropriate to what sorts of expressions: for example, whether closed wffs should be assigned truth values, or functions from indices to truth values (and if so, what those indices should be like), and so on.

To clarify the view being presented here, it will be helpful to compare it to existing views in the literature. Let us label three kinds of entities: (*A*) sentences of a natural language such as English; (*B*) corresponding wffs of some formal language  $\mathcal{L}$ ; <sup>16</sup> (*C*) models of (fragments of)  $\mathcal{L}$ . <sup>17</sup>

	kind of entity	example
<i>C</i>	model (assignment of values to items at level <i>B</i> )	* •
<i>B</i>	wff of formal language	<i>Rm</i>
<i>A</i>	sentence of natural language	Maisie is barking

Let’s now consider some existing theories of propositions. First, four theories of structured propositions:

1. Russellian propositions. The Russellian proposition expressed by (some utterance of) a sentence at level *A* is a structured entity: its structure matches the structure of the wff at level *B* (which represents the underlying logical structure of the sentence at level *A*) and the places in this structure (i.e. the places which, in the wff at level *B*, are filled by symbols: names and predicates in the example given above) are filled by objects and properties.
2. A regimented version of 1. Propositions, on this view, are just like Russellian propositions except that the places in the structure are filled by *extensions* (objects, sets of objects, sets of *n*-tuples of objects) – that is, by the values assigned at level *C* to the symbols in the wff at level *B*, when at level *C* we have classical model theory for FOL. In the example, \* will be a set of objects (the extension of *R*) and • will be an object (the referent of *m*). So the difference between 1 and 2 is just that – as in classical model theory – *properties* are replaced by *sets*.
3. Fregean propositions. These are like Russellian propositions except that the places in the structure are filled not by objects and properties, but by senses: *modes of presentation* of objects and properties.

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<sup>16</sup> At this point we leave open the precise sense of ‘corresponding’ here: different views will take different stances on the relationship between sentences at level *A* and wffs at level *B*. Regarding the choice of formal language  $\mathcal{L}$ , see the discussion of point 1 above; for purposes of examples we use FOL.

<sup>17</sup> In the example in the diagram, the wff at level *B* comprises two elements: the predicate *R* and the name *m*. At level *C*, \* is the value assigned to *R* and • is the value assigned to *m*. At level *C*, a value will also be assigned to the entire wff *Rm*. This value is not explicitly depicted; as we shall discuss below, on some views it is a structure composed from \* and • while on other views it is not.

4. Carnapian propositions. A regimented version of **3** in which the places in the structure are filled by *intensions* (functions from worlds to extensions). In the example,  $*$  will be a function from worlds to sets of objects (the intension of  $R$ ) and  $\bullet$  will be a function from worlds to objects (the intension of  $m$ ). So the difference between **2** and **4** is that an extensional model theory at level  $C$  is replaced by an intensional one.

Next, a theory of unstructured propositions:

5. Propositions are intensions of sentences (or wffs): functions from worlds to truth values – or equivalently, sets of possible worlds.

On all the views just considered, the ingredients of propositions are all found at level  $C$ . On the view that I am proposing, by contrast, propositions span levels  $B$  and  $C$ . On this view, the proposition expressed by a sentence is a wff together with a model (of the fragment of the language needed to form that wff).

Consider now another class of views:

6. Sententialist (aka lexical) theories. A *sentence* comprises expressions structured in a certain way. According to sententialist theories, propositions are sentence-like structures whose ultimate constituents are not simply expressions, but expressions *together with* semantic values. Examples of sententialist views include the *interpreted logical form* (ILF) view of Larson and Ludlow [35]<sup>18</sup> and the *Russellian annotated matrix* (RAM) view of Richard [47].

The view of propositions being proposed in the present paper bears a structural similarity to sententialist views in that both can be seen as spanning levels  $B$  and  $C$ . However, there is a deep difference. Sententialists can be seen as proposing propositions that span levels  $B$  and  $C$  only if the wff at level  $B$  is taken to be – or to be a representation of – the natural language sentence at level  $A$ . On the view of this paper, by contrast, the wff at level  $B$  is taken to be a quite separate kind of entity, independent of any natural language sentence.<sup>19</sup>

Each of the six kinds of view of propositions just mentioned faces serious problems – for example: **1** and **2** have problems with Frege’s puzzle. So does **4**, assuming that names are rigid designators (in which case coreferential names have not only the same extension but also the

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<sup>18</sup> Cf. also Harman [21], Higginbotham [25], Segal [53], Higginbotham [26] and Larson and Segal [36].

<sup>19</sup> The latter point will be discussed further in §3.4.

same intension). 3 and 6 face ontological worries: what *are* senses exactly? — and what exactly could *expressions* be, that would allow them to play the roles that sententialists need them to play?<sup>20</sup> 5 and 6 run into problems of granularity: 5 is too coarse grained (it has too few propositions to go around: sentences that are true at exactly the same worlds will express the same proposition) and 6 is too fine-grained (it has too many propositions: sentences of different languages, or with different syntactic structures, will express different propositions).

I have outlined a theory of propositions and explained how it differs from these six kinds of view.<sup>21</sup> In the remainder of this paper I shall present the advantages of this new view — which include not succumbing to any of the problems just mentioned.

### 3. Advantages of the Theory

#### 3.1. Ontologically Innocent

A major advantage of the view of propositions as wffs plus models is that it is ontologically innocent: it uses only standard-issue, off-the-shelf materials from logic and model theory. This is in contrast to views of propositions that invent dubious proprietary machinery. If propositions are wffs together with models then the ontology of propositions is just the standard ontology of logic and model theory. We do not need any extra entities at all: we need only entities that already earn their keep as core components of the formal sciences.<sup>22</sup>

Contrast some other recent views of propositions. According to Hanks [20], propositions are *complex actions*, composed of more basic types of actions. According to Soames [61], propositions are cognitive

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<sup>20</sup> On the latter worry for sententialist views, see Cappelen and Dever [9].

<sup>21</sup> Another kind of view from which my view (and the other six kinds of view) differs is the kind that deliberately says *nothing* about what propositions are like in themselves. For example, on the views of Bealer [4, p. 24] and Thomason [65, p. 49], propositions are treated as *primitive* entities.

<sup>22</sup> My point is not that the ontology of mathematics and the formal sciences is ‘lightweight’ in some sense. I am making no claims about the ontology of mathematics and the formal sciences. My point is that *whatever* the correct ontology is, we undoubtedly need it — it is not as if we can *do without* mathematics and the formal sciences — and once we have it, we have all that we need for the account of propositions presented in this paper. Thus, propositions in this sense are ontologically *innocent*.

*acts* or *operations*.<sup>23</sup> Now the ontology of complex actions is far from clear. If we can develop a theory of propositions that steers clear of this problem area then we should do so. Of course, these authors think we cannot: for example Soames [60] thinks that we have to go down his kind of route to get a theory according to which propositions are intrinsically capable (i.e. by their very nature — rather than because they are interpreted in a certain way) of being true or false. However, as we shall see in §3.2, the present theory can explain why propositions have this feature — without the ontological drawbacks.

Perhaps someone might think there is a worry surrounding the question of what a wff is. Cappelen and Dever [9] pose the problem for sententialist theories that no view of what expressions are allows expressions to play the roles that sententialists need them to play. Might there be a similar worry concerning wffs? There is no such worry. The ontology of wffs is straightforward. We begin with a set  $S$  of symbols. The symbols are objects. It actually doesn't matter what objects they are: they could be physical objects or abstract objects. Wffs are then sequences of these symbols — in the mathematical sense of 'sequence'.<sup>24</sup> So wffs are just abstract objects of a kind familiar from mathematics: denizens of the same realm as other entities countenanced in mathematics such as sets, numbers, functions, algebras, metric spaces and probability measures. If there is a problem about having such objects in one's ontology (and I don't think there is), then it is not a special ontological problem for the present view of propositions: it's a general problem for mathematics and all the formal sciences.<sup>25</sup>

<sup>23</sup> See also Hanks [19] and Soames [60]. Cf. also Jubien [29], Moltmann [40] and Moltmann [41, ch. 4].

<sup>24</sup> For further details see Smith [57, §16.7].

<sup>25</sup> What about the claim that wffs are *part of* propositions — that propositions are wffs *together with* models? How are we to understand this claim? Well, once again, no special new notions are required. It is absolutely standard in mathematics and the formal sciences to talk of structures with multiple components — some of which might themselves be structures with multiple components. For example, a *metric space* is a pair  $(S, d)$  where  $S$  is a set and  $d$  is a function, satisfying certain conditions, from pairs of elements of  $S$  to reals; a *Kripke model* of a standard modal language is a triple  $(W, R, V)$  where  $W$  is a set,  $R$  is a binary relation on  $W$ , and  $V$  is a function from pairs comprising a basic proposition of the language and a member of  $W$  to the set of classical truth values; a *bounded integral commutative residuated lattice* is a structure  $(D, \vee, \wedge, \&, \rightarrow, 0, 1)$  where  $(D, \vee, \wedge, 0, 1)$  is a lattice with least element 0 and greatest element 1,  $(D, \&, 1)$  is a commutative monoid, and  $\rightarrow$  is the residuum of  $\&$  (i.e. for all  $x, y, z \in D$ ,  $x \& y \leq z$  iff  $x \leq y \rightarrow z$ ); and the *real numbers* are a structure comprising

The first advantage of the present view of propositions, then, is that it constructs propositions from standard materials that everyone who does any serious work in logic or any of the formal sciences already countenances.

### 3.2. Intrinsically Truth Bearing

Recall the fourth role for propositions in GT: they are the primary bearers of truth and falsity. Recently, a number of authors have expressed scepticism about whether anything *could* – as propositions are supposed to (traditionally and according to role 4 in GT) – possess a truth value (or truth conditions) *in and of itself*: that is, without being *interpreted* by agents. For example, King [33, pp. 258–61] writes:

*Unity Question 2* (UQ2): How does the ‘structured complex’ that is the proposition that Dara swims manage to have truth conditions and so represent Dara as possessing the property of swimming? [...] there is one sort of answer to this question that, though it has probably been given (if only implicitly) by everyone who believes in structured propositions except me and Soames,<sup>26</sup> I cannot accept. The sort of answer I have in mind is any answer according to which propositions *by their very natures and independently of all minds and languages* represent the world as being a certain way and so have truth conditions. Though this is part of how propositions have been classically conceived, I cannot accept that propositions are like this [...] I can’t see how a proposition, by its very nature and independently of minds and languages, could have truth conditions and so represent something as being the case [...] any answer to UQ2 according to which propositions represent things as being a certain way and so have truth conditions in virtue of their very natures and independently of minds and languages is in the end completely mysterious and so unacceptable.

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a set of objects together with an ordering, certain algebraic operations, a metric and so on – all satisfying certain conditions. Now whatever the correct account is of how the components of these structures hang together, it carries over to the question of how the wff and the model hang together to form a proposition in the sense of this paper – for propositions in this sense *just are* one more example of mathematical structures with multiple components. The central point of the present section is that the account of propositions presented in this paper requires no ontological machinery beyond what is already needed to make sense of mathematics and the formal sciences.

<sup>26</sup> Here King refers to an unpublished ms of Soames from 2008.

King and others take this sort of worry — the *representation problem* — as a motivation for views of propositions according to which propositions are *not* mind- and language-independent. For example, King continues: “I’ll claim that it is something we speakers of languages do that results in propositions representing things as being a certain way and so having truth conditions. This is the most provocative and novel feature of the view of propositions defended in [32].” I shall argue in §3.4 that this sort of approach is not simply provocative: it is unacceptable. However the point for now is that the view of propositions presented in this paper straightforwardly solves the representation problem. If we take a proposition to be a wff together with a model, then — if we consider models in which the kinds of values assigned to wffs are truth values — it is clear how the proposition can (all by itself) determine a truth value. For a model is precisely something that assigns values to expressions (recall n.15). If the kind of value assigned to a wff is a truth value, then the proposition (wff plus model) will contain in itself — entirely due to its own inner constitution, without outside assistance — a truth value: the truth value of the wff on the model.<sup>27</sup>

Now someone might worry that it isn’t the entire proposition getting a truth value: it is the wff part of the proposition that gets a truth value — and it gets it relative to the model that is the other part of the proposition. But this worry isn’t well-taken. Although things are usually phrased in terms of the ‘proposition having [or bearing] a truth value’ — which suggests that the entire proposition possesses a truth value — all that is actually required for purposes of GT is that propositions determine truth values. That is, once we have a proposition, we do not need anything else to get a truth value. This requirement is met if a proposition comprises two parts, one of which determines a truth value for the other. As a whole, the proposition does then carry a truth value with it — as required by GT.

We have just shown how a proposition conceived as a wff plus model could carry within itself (with no outside help) a truth value. But sometimes in the literature it is said that propositions should have (in and of themselves) not truth values but truth *conditions*. A truth condition

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<sup>27</sup> Recall (§2 and n.15) that we left it open what kinds of models we are dealing with: that is, while models always include assignments of values to expressions, we did not make a ruling on what kinds of values get assigned to what kinds of expressions. We have just considered the case where the kinds of values assigned to wffs are *truth values*; other options will be considered in the paragraph after next.

specifies how things must be for the proposition to be true; together with a ‘way things could be’ it determines a truth value. So on this conception, propositions do not (by themselves) determine truth values: by themselves they determine truth conditions, and a truth condition *together with* a ‘way things could be’ determines a truth value. This idea can also be accommodated in the present framework: it simply depends on the kind of language and model theory we employ. For example, if we use classical FOL, a model assigns a truth value to each wff, and so a proposition (wff plus model) will determine a truth value. If we use instead a system of intensional model theory, then wffs will be assigned intensions by models: functions from indices to truth values. Hence, a proposition (wff plus model) will not determine a truth value: it will determine an intension, that is a truth condition. Together with an index, this intension will determine a truth value. The present framework is, then, quite flexible: it does not foreclose on the decision whether propositions should determine truth values or truth conditions. Furthermore, if the latter, the framework does not foreclose on what needs to be added to a truth condition to determine a truth value: should it be a possible world — or something else? Different options can be accommodated by adopting different systems of intensional model theory with different indices.

Recall the second, parenthesised sentence in the fourth role for propositions in GT: ‘It may be that propositions are true or false relative to possible worlds, in which case they are also the bearers of the properties necessary truth and contingent truth.’ We have just seen how the idea that a proposition is true or false relative to a world can be accommodated within the current approach to propositions: we use an intensional model theory with worlds as indices. If we do so, then propositions can (in and of themselves) possess properties such as necessary truth or contingent truth. If a model assigns a wff an intension that sends every index to truth, then the proposition comprising that model and that wff will — in and of itself, without outside help or interference — determine the property of necessary truth; similarly for other properties and relations defined in terms of intensions.

In sum: it is indeed hard to see how a structure could interpret itself. The present view of propositions solves this problem by seeing propositions as comprising two elements — a wff and a model — one of which interprets the other. This suffices for purposes of GT. We do not actually need a self-interpreting thing: we just need something that has a truth value (or truth conditions) built-in — something that determines

a truth value (or truth conditions). Propositions — on the conception proposed here — do have this desired feature.

Now someone may want to object: ‘But all you have given us is *more structure!* We still need an agent to apply the model part to the wff part. Otherwise all we have is simply more inert machinery. It takes an agent to breathe life into the machinery: to *make* the model interpret the wff.’ My response to this is that it misunderstands the way models work. A model is exactly what we need to add to a wff to determine a truth value (or truth conditions). It is a precise, formally well-defined replacement for the intuitive notion of interpreting a string of symbols. It replaces this vague intuitive notion and does not need to be supplemented by it. Furthermore, if the present objection were a good one, it would not simply count against my view of propositions: it would count against uses of model theory throughout the formal sciences, in which it is understood that models determine values for wffs — by themselves, without need of animation by an act of interpretation or application. So there is a problem here for my view of propositions only if there is also a problem for the whole way that the notion of ‘interpretation’ has been formalised in logic and model theory. But there is no problem: a model is not like a golem.

There is one further issue to discuss before we move on. In the quotation above, King talks of propositions *having truth conditions* and of propositions *representing the world* as being a certain way. He seems to use these ways of talking more or less interchangeably: sometimes he talks of a proposition having truth conditions *and so* representing and sometimes he talks of a proposition representing *and so* having truth conditions. However, at this point in my argument, someone might try to drive a wedge here. They might accept that a wff plus a model determines (all by itself) a truth value or truth conditions and yet still think that a wff plus a model cannot (all by itself) *represent* the world as being some way. Genuine representation (they might say) requires interpretation by an agent: no abstract object (all by itself) can represent the world as being some way. My response to this is that — whether or not this claim about representation is true — it is beside the point: whether determining a truth value or truth conditions suffices for ‘genuinely representing the world as being some way’ does not matter here. The fourth role for propositions in GT is that they are the primary bearers of truth and falsity. What is required for GT is that propositions have built-in truth values or truth conditions; it is not required that they represent the world in any stronger sense than that.

### 3.3. Unified

The problem of the ‘unity of the proposition’ is a venerable one, going back at least to Frege and Russell. King [33] usefully distinguishes three questions under this heading. We have already encountered one of them (UQ2) in §3.2. The other two are as follows [33, p. 258]:

*Unity Question 1* (UQ1): What holds the constituents Dara and the property of swimming together and imposes structure on them in the proposition that Dara swims?

*Unity Question 3* (UQ3): Why does it at least seem as though some constituents can be combined to form a proposition (Dara and the property of swimming), whereas others cannot be (George W. Bush and Dick Cheney)?

Both of these questions are readily answered given the theory of propositions presented in this paper. Let’s discuss them in turn.

UQ1. Here we may distinguish two questions: What holds the wff together? What holds the wff and the model together? We have already discussed the second question: nothing mysterious is required to *apply* the model to the wff. As for the first question, we can *again* distinguish two questions. The first is: What stops the wff falling apart into a bunch of separate constituents — that is, how does the wff stay together *at all*? The response is that if there were a problem about how wffs manage to hold together it would not just be a problem for my view of propositions: it would be a problem for all of the formal sciences. Now the reader may be getting tired of this kind of response — but in fact the ability to deploy this kind of response is one of the great advantages of the present view of propositions. Once again, what we are seeing here are the benefits of using tried-and-tested, off-the-shelf materials to construct propositions. The second question is: What makes the wff stay together *in the right way*? For example, in  $Pa$ , what makes the first constituent the part that picks out a certain property and the second the part that picks out an individual, in such a way that the proposition as a whole is true iff the individual has the property? We have essentially already answered this question in the previous section. It is the way the parts of the wff are treated by the *model* that ensures these things. For example, in  $Pa$ , what makes  $P$  the predicate (the part that picks out a property) and  $a$  the name (the part that picks out an individual) is the role each plays in determining a truth value for  $Pa$  in a model.

UQ2. Whatever formal language we are using, only some combinations of symbols constitute wffs. There may therefore be groups of symbols such that *no* combination of them is well-formed. In FOL, for example, one can form a wff from a name and a predicate, but not from two names.

### 3.4. Language- and Mind-Independent

We mentioned in §3.1 that Soames was driven to the view that propositions are cognitive acts — and in §3.2 that King was driven to the unorthodox view that propositions are *not* mind- and language-independent — by worries about how propositions could be capable intrinsically (by their very nature — rather than because they are interpreted in a certain way) of having truth values or truth conditions. We have also seen how propositions on the present proposal — wffs plus models — avoid this worry and manage to carry within themselves (without assistance from external acts of interpretation) truth conditions or truth values. It is now time to clarify that propositions on the present proposal *are* mind- and language-independent and to explain why this is a desirable feature in a theory of propositions.

Propositions on the present proposal comprise two things: a wff and a model. Both are taken straight off the shelf — without modification — from the equipment repository of logic and the formal sciences. As we have already noted, they are abstract objects: denizens of the same realm as other entities countenanced in mathematics such as sets, numbers, functions, algebras, metric spaces and probability measures. Note that some of these things might have concrete objects built into them: for example sets with urelements, probability measures over a population, or models that assign Spot, Rover and Tangles as referents of certain names. Nevertheless they are all abstract objects: the set containing two persons is a third object but it is not a third concrete object; the function sending each person to his or her biological mother is another thing in addition to the persons in question but not another physical thing; and so on. Propositions, then — on the present conception — are mathematical objects and are no more mind- or language-dependent than any other such objects.

Of course there are positions in the philosophy of mathematics according to which all mathematical objects are mind- or language-dependent. (There are also views according to which mathematical objects

such as sets are concrete objects.) It is beyond the scope of this paper to argue against such views — both in the sense that it would take too long and in the sense that it is unnecessary: it is enough for purposes of the present paper to locate propositions with apparently paradigmatic mind- and language-independent things such as numbers and sets, as opposed to paradigmatic mind- or language-dependent things such as cognitive acts and sentences of natural languages. The key point is that propositions belong with mathematical entities, not with things like cognitive acts or bits of natural languages.

Why is mind- and language-independence (in at least this relative sense) a good thing? The answer relates to role 3 for propositions in GT. Propositions are supposed to provide a neutral bridge — of common content — between natural languages, between attitudes (of the same agent and of different agents) and between languages and attitudes. Propositions should be potentially common to all public languages and to all languages of thought — to persons, animals, computers and in general any agent whose behaviour might usefully be explained by GT. According to role 3, propositions should be *common currency*. Now if propositions incorporate parts of natural language, or parts of a language of thought, or types of cognitive act — in general, if they are not mind- and language-independent — then they cannot play this role in GT of being the neutral bridge between minds and languages of all kinds.<sup>28</sup>

A couple of clarifications need to be made at this point. The first is that we are arguing that propositions need to be mind- and language-independent if they are to serve the purposes of GT. If we want ‘propositions’ for some other purpose — for example, giving a formal semantic analysis of attitude reports (cf. §1.1) — then the present point might not apply. (Of course, sententialist approaches to the analysis of attitude reports face other well known problems.)<sup>29</sup> The second is that there is a

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<sup>28</sup> In §3.5 we shall see that propositions should be the objects of logic. But as Frege taught us, logic is not mind-dependent. (See e.g. Frege’s Introduction to *Grundgesetze* [17]. For more detailed discussions of and references to Frege’s anti-psychologism see Smith [54, §II] and [56, §3.3].) This gives us another reason for thinking that propositions must be mind-independent. Another reason for thinking that propositions must be language-independent stems from the idea that the association of expressions with meanings is *conventional*: as Cresswell [12, p. 9] puts it, expressions and meanings “must be mutually independent things (whatever their nature), which, in a given language, happen to be correlated in some particular way.”

<sup>29</sup> Cf. e.g. Church [10], Salmon [51], Soames [59, ch. 7] and Higginbotham [28, §2]; and Montague [42] and Thomason [64].

range of sententialist views and the present point does not count against all of them. Some take propositions to be sentences of public natural languages. Others take propositions to be (interpreted) LFs (logical forms), where LFs are theoretical entities posited in (certain areas of) linguistics. Within the latter camp, different views can again be distinguished. Some take LFs to be abstract objects: objects of the kind I have taken wffs to be. Others take them to be mental representations.<sup>30</sup> The points above about mind- and language-independence count against versions of sententialism that take propositions to be sentences of public languages or mental entities. They do not count against versions of sententialism that take propositions to be LFs where these are thought of as abstract entities on a par (ontologically) with wffs. Such versions of sententialism face a different problem, however, which is that they *enforce* an overly fine level of granularity. If two natural language sentences have different LFs—according to best linguistic theory—then they cannot express the same proposition. As I shall argue in §3.7, we do not want a theory of propositions to enforce any such fineness of grain (or coarseness of grain). This is a reason against identifying the wff part of the proposition expressed by a sentence with a representation of the *syntax* of that sentence itself—whether the surface syntax or an underlying logical form.

One issue that we should discuss here is the worry—which someone might have at this point—that if propositions are abstract objects on a par ontologically with other mathematical entities then we face a version of the problem for platonism posed by Benacerraf [6] and sharpened by Field [14]. In essence, the problem for platonism is to explain how we could know any mathematical truths or have reliable beliefs about mathematical entities given that these entities are mind- and language-independent, non-physical and non-spatio-temporal. However, two points prevent this problem transferring to the present view of propositions. The first is that I have not committed to platonism. I have simply said that propositions on my view are ontologically on a par with other mathematical entities such as sets, numbers, functions, algebras, metric spaces and probability measures. What the correct position on the ontology of these entities *is* is a question for philosophy of mathematics and not one on which I need to take a stand in this paper. The second point is that the role played by propositions in GT is very

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<sup>30</sup> For discussion see e.g. Katz and Postal [31], Higginbotham [27] and Collins [11].

different from the role played by mathematical entities in an account of our mathematical practice. In the mathematical case, part of what needs explaining is how we know mathematical truths. If the things that make these truths true are utterly isolated from us then there is an at least apparent worry about how we can know these truths. The case of propositions has a quite different structure. GT is an explanatory theory but the phenomena to be explained do *not* include agents knowing things *about propositions*. Propositions feature in an account in which agents reason, know and communicate. The agent knowing that Bob is in Stockholm (say) is modelled in terms of a relation between the agent and a proposition. This is different from saying that the agent knows some fact *about this proposition*. Hence no worry looms about *how* the agent could possibly know what she knows.

At this point a different worry might arise. How could it possibly be *useful* to model an agent's knowing that Bob is in Stockholm (say) in terms of a relation between the agent and a proposition? This is an interesting question but it falls well outside the scope of the present paper. The dialectic is as follows. GT is a useful explanatory theory. (It is useful because it allows us to explain and predict the behaviour of agents to a degree that we could not possibly hope to achieve by other known means—for example, by viewing them as physical entities and applying the laws of physics. If you want to know where Bob will be tomorrow morning given what he just said to you and what you have observed of his behaviour in the past, do not try applying fundamental physical laws—or biological or chemical laws for that matter. Your best shot is to use the resources of GT.) Now we want to know what propositions could be for purposes of GT. I have proposed an answer and am in the process of describing the virtues of this account of propositions. Now someone wants to know: How does GT manage to be useful? How *could* a theory be useful for explaining the behaviour of certain things (in this case, the behaviour of agents when they reason, communicate and act) when the theory involves further theoretical entities, beyond the ones whose behaviour we originally wanted to explain? This is a general question about the explanatory success of theories. It is a central topic in philosophy of science. But (as noted) there is no serious prospect of useful explanations in purely physical terms of the phenomena that we seek to predict and explain using GT. Hence even in the absence of an answer to the question as to why GT works and how the theoretical entities it involves relate to physical entities, we are entitled to go on

using and developing GT. In general, we are entitled to go on using and developing theories that have explanatory power even in the absence of answers to fundamental questions in the philosophy of science. It is in the spirit of developing GT that I am proposing an account of propositions for purposes of GT. The question of why and how GT is successful is one for another occasion.

In this light, consider role **1** for propositions in GT:

Propositions are the objects of the attitudes such as belief and desire.

One might wonder how propositions could be the objects of the attitudes if propositions are abstract entities (as opposed to e.g. cognitive acts). This will indeed start to seem mysterious if we picture Ed's believing that he is walking his dog to involve Ed's being attached by some kind of ultrafine string to a proposition in Plato's heaven or Frege's third realm in something like the way he is attached to his dog by the dog's lead. But this is just a misleading picture. GT is an explanatory theory that posits certain entities and relations to them in order (partly) to derive predictions about and explanations of behaviour (e.g. Ed's walking his dog now rather than at his usual time, having just looked at a newspaper warning of a torrential downpour later in the day). There is a question about how different levels of explanation coexist: of how the explanation of Ed's movements in terms of his beliefs relates to an explanation in terms of physical forces. But this kind of question is quite general and poses no special problem for explanatory theories that invoke propositions understood as abstract objects. The reason why we should accept that Ed stands in a certain relation to a proposition is that our best explanatory theory posits such an entity and a relation between it and Ed.

### 3.5. The Role of Logic

One of the roles for propositions mentioned in §1 has been relatively neglected in the literature: the role of propositions as the objects of logic. This role is of crucial importance if logic is to provide norms for belief—and as we shall see, the present view of propositions is uniquely well placed to explain how propositions could be the objects of logic.

Before continuing, I should clarify that I am not suggesting that the primary business of logic is providing norms for belief. I am also not

suggesting that norms for belief can be derived in a *simple* way from logical laws—for example, the validity of modus ponens does *not* generate the norm ‘If you believe  $\alpha$  and  $\alpha \rightarrow \beta$  then you should believe  $\beta$ ’. These points have been well made by Harman and others.<sup>31</sup> Nevertheless, even if the route is indirect and complex, logic is a source of norms for belief: in managing one’s doxastic affairs, logical considerations are indeed relevant. Furthermore, we can extend our knowledge using logical deduction. If the objects of the attitudes are propositions then none of this will make sense unless we can explain how logic gets a grip on propositions.

The objects of the logical properties—logical truth, satisfiability and so on—and the relata of the logical relations—equivalence, logical consequence and so on—are wffs and sets (or sequences etc.) of wffs. This is so whether one takes the fundamental definitions of these properties and relations to be model-theoretic (e.g. logical truth is truth on all models; satisfiability is joint truth on some model; logical consequence is truth of the conclusion on every model on which all the premisses are true; and so on) or proof-theoretic (e.g. logical truth is provability from no assumptions; logical consequence is derivability of the conclusion from the premisses taken as assumptions; and so on). By putting wffs at the heart of propositions, the view of propositions presented in this paper can therefore allow propositions to play role 5 on the list of roles for propositions in GT: the role of connecting up *logic* with the objects of *belief*. If the objects of belief are propositions and propositions are wffs plus models, then it is evident how logic can provide norms for beliefs—for example, one should not believe an unsatisfiable set of propositions (or more precisely, a set of propositions whose wff components form an unsatisfiable set)—and how logical inferences can be used to derive further beliefs from existing beliefs.

Note that it is not part of the present view that propositions are the objects of logic (hence the parenthesised remark in the previous sentence): the objects of logic are wffs, and propositions are wffs *plus models*. The situation is similar to the one encountered in §3.2, where we saw that propositions as a whole do not have to bear truth values in order for them to play role 4 on the list of roles for propositions in GT. Here too, it is enough for propositions to play role 5 that they have parts—the wff parts—that bear logical properties and stand in logical relations.

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<sup>31</sup> See e.g. Harman [22], Field [15] and Harman [23].

By getting a direct grip on the wff parts of propositions, logic gets an indirect grip on propositions as a whole — and this is sufficient for role 5.

Other views of propositions, by contrast, cannot allow that (parts of) propositions are the objects of logic. First, consider the four theories of structured propositions introduced in §2. Recall the three kinds of entity distinguished there: (A) sentences of a natural language such as English; (B) corresponding wffs of some formal language  $\mathcal{L}$ ; (C) models of (fragments of)  $\mathcal{L}$ . As we noted, the theories of structured propositions all locate propositions at level C. But the objects of logic are the things that get assigned values — not the values assigned. This is so even on views that take the proof-theoretic definitions of the logical concepts to be primary. Even if it is not the fundamental fact about logical truth (say) that logical truths are assigned the value true on all models, it is still a fact: the thing that is proven — a wff — is the same thing that is assigned values in models. The logical truth is the thing to which values can be assigned: it is not one of the values. The same goes for the other logical properties and relations: the things that bear them and stand in them are the things to which values get assigned. If these things are parts of propositions — as they are on the present view — then logic gets a grip on propositions. But if propositions are just made up of the values assigned — and not the things that get assigned these values — then logic does not get a grip on propositions.

Let's turn now to the next view of propositions considered in §2: the view of propositions as sets of possible worlds. This view too fails to allow logic to get a grip on propositions. Admittedly the view gives us something (not absolutely nothing) in this area: a set  $\Gamma$  of propositions can be said to entail a proposition  $\alpha$  iff the intersection of all the propositions in  $\Gamma$  is a subset of  $\alpha$ ; a set  $\Gamma$  of propositions can be said to be satisfiable iff the intersection of all the propositions in  $\Gamma$  is nonempty; and so on. However this only gives us Boolean properties of, and relations between, propositions — and furthermore even in this limited realm what we are getting here isn't *formal logic* and does not plausibly provide norms of rationality. Although it is common in introductory logic textbooks nowadays to define logical consequence (aka validity) as necessary truth preservation, this property is not in fact something of which formal logic provides a theory. Formal logic gives a theory of necessary truth preservation *in virtue of form*.<sup>32</sup> That is, a logically valid argument is necessar-

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<sup>32</sup> For a more detailed discussion of these issues see Smith [57, §1.4].

ily truth preserving — it is impossible for the premisses to be true while the conclusion is false — but that is not all: a logically valid argument is furthermore necessarily truth preserving *in virtue of its form*. It is not something special about the *subject matter* of the argument that ensures that it is necessarily truth preserving — for example, the premisses talk about water and the conclusion talks about H<sub>2</sub>O — rather, it is simply *the way the argument is put together* that ensures that the premisses cannot be true and the conclusion false. Despite the recent tendency to introduce validity in terms of necessary truth preservation (alone), historically at least it was always clear that the notion of logical validity required something more than this: it required that the argument be necessarily truth preserving *thanks to* its form or structure. For example, this view can be found in Tarski’s seminal discussion of logical consequence, where it is presented as the traditional, intuitive conception.<sup>33</sup>

I emphasize [...] that the proposed treatment of the concept of consequence makes no very high claim to complete originality. The ideas involved in this treatment will certainly seem to be something well known [...]. Certain considerations of an intuitive nature will form our starting-point. Consider any class *K* of sentences and a sentence *X* which follows from the sentences of this class. From an intuitive standpoint it can never happen that both the class *K* consists only of true sentences and the sentence *X* is false.<sup>34</sup> Moreover, since we are concerned here with the concept of logical, i.e. *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence *X* or the sentences of the class *K* refer<sup>35</sup> [...]. The two circumstances just indicated<sup>36</sup> [...] seem to be very characteristic and essential for the proper concept of consequence [...]. [62, pp. 414–415]

Indeed, the idea goes back to Aristotle [2], who begins by saying:

A deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. [§1]

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<sup>33</sup> The footnotes to the quotation are mine, not Tarski’s.

<sup>34</sup> This is the idea that the argument should be necessarily truth preserving.

<sup>35</sup> This is the idea that the argument should be necessarily truth preserving *in virtue of its form*.

<sup>36</sup> That is: (i) necessarily truth preserving; (ii) guaranteed by form.

This is the idea of necessary truth preservation. Then, when discussing arguments, Aristotle first presents an argument form in an abstract way, with schematic letters in place of particular terms — for example:

Every C is B.

No B is A.

Therefore no C is A.

He then derives specific arguments by putting particular terms in place of the letters — for example:

Every swan is white.

No white thing is a raven.

Therefore no swan is a raven.

The reasoning that shows the argument to be necessarily truth preserving is carried out at the level of the argument form (i.e. in terms of A's, B's and C's, not ravens, white things and swans): it is thus clear that Aristotle is interested in those arguments that are necessarily truth preserving *in virtue of their form*.

Now the view of propositions as sets of possible worlds can reconstruct a relation of entailment between propositions — but this relation only captures the idea of necessary truth preservation: it misses the idea that logical validity depends on form. Of course it must do so, because propositions are unstructured on this view. The point now is that this is a problem: it renders this view of propositions unable to say that logic is concerned with (parts of) propositions — because, as we have seen, logic is essentially concerned with structured entities, with entities that have forms. This is not just a curiosity: it matters for GT. Logical consequence (and other logical properties and relations) yield norms for belief; mere necessary truth preservation (and other properties and relations definable in terms of sets of possible worlds) do not — thanks to the existence of a posteriori necessities. Someone who believes  $\forall xRx$  and  $\neg Ra$  is irrational; someone who believes that the glass contains water and does not contain H<sub>2</sub>O might not be irrational — he might simply not have learned chemistry.

The upshot so far is that if we want propositions for purposes of GT, then we need them to be governed by formal logic, which yields norms of rationality. Views of propositions as unstructured sets of worlds cannot allow this, and nor can views of propositions as structured entities comprising things like the *values* assigned in models to expressions of a logical language — rather than the expressions themselves (which are the things that bear logical properties and stand in logical relations).

We turn now to the final kind of view considered in §2: sententialism. Sententialist views face no essential structural problem of the sort just discussed. The structure of the sententialist view is the same as the structure of the view presented in this paper: propositions comprise a structured sentence-like part together with values for the expressions occurring therein. In theory, then, logic can get a grip on propositions, for they contain a part which is both structured and comprises things that get assigned values (rather than the values themselves). The problem for sententialism is that logic is not concerned with anything as parochial as the sentences of a natural language. If anything is mind- and language-independent, logic is. Well, let's be a bit more subtle. Of course there are many logics, used for all sorts of purposes, and taking all kinds of things as their objects. But for purposes of GT, 'logic' is supposed to be an independent arbiter. In GT, we do not want one logic for each language: we want one logic for all propositions. Hence we do not want logic to be tied specifically to natural language.

One point requires discussion before we move on. It is often claimed that the objects of logic — the bearers of the logical properties and the relata of the logical relations — are sentences (and sets thereof) as opposed to propositions.<sup>37</sup> This claim can be understood in several different ways. On one reading the claim is that traditional (Fregean or Russellian) propositions are not the objects of logic; rather, the objects of logic are sentence-like structures where the objects occupying the positions in these structures are expressions of some sort (as opposed to values that might be assigned to expressions in a semantic system). I have already argued for this claim. On another reading the claim is that the objects of logic are natural language sentences. My response to such a claim is as follows. Whenever we have a language meeting certain syntactic constraints — for example, all its sentences are generated from a stock of basic symbols using a finite number of syntactic operations — we can define logics (whether proof-theoretically or model-theoretically) that take as their objects the sentences of that language. However, as already mentioned, 'logic' in the sense required for GT is supposed to be an independent arbiter: a supplier of universal norms that apply equally to any creature or entity capable of believing or expressing propositions

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<sup>37</sup> For a recent example see Russell [49, n.1]. For detailed discussion and references see Smith [58]. Sometimes it is said that the objects of logic are sentences and/or sentence *schemata*.

by whatever means. So logic in the sense needed for GT cannot take as its objects the sentences of a natural language.

### 3.6. The Formal Semantics Interface

Recall role 2 for propositions in GT:

Propositions are expressed by sentences uttered in contexts.

We express propositions by uttering sentences — and we recover the proposition expressed by an utterance by computing on the meanings (csv's) of the words used, the syntax of the sentence and facts about the context of utterance. But as we have already remarked (§1.1), none of this *requires* that propositions *be* the csv's of sentences. Propositions do not have to feature in formal semantics at all. Role 2 imposes a requirement on the *interface* between GT and formal semantics. The requirement is that sentences uttered in contexts must be able to determine propositions. On the present view of propositions, there is in principle no problem in this area.<sup>38</sup> Formal semantics will deal with formulas of some sort, whether sentences of natural language, LFs or wffs of IL (to mention a few possibilities). It will also assign csv's to the components of such formulas. All we need is that such a formula together with csv's determine a (different) wff together with a model.<sup>39</sup> There is no reason to foresee a problem here. Of course if the language of formal semantics was very simple and the language of propositions was a different and more complex language — or if we used an extensional formal semantics but wanted intensional models at the level of propositions — then there would be potential problems. But such considerations only feed into the particular choice of formal languages and models for semantics and for

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<sup>38</sup> Of course the exact details will depend on the details of our formal semantic theory and on the details of our theory of propositions — i.e. from exactly what kind of formal language the wffs are drawn and exactly what kind of models are in play. As mentioned in §2, it is not the purpose of this paper to settle such matters of precise detail.

<sup>39</sup> We gave reasons in §3.4 why, in general, the very same formula should not be used both to represent a sentence of natural language and to represent the wff part of the proposition expressed by that sentence in a context. We left it open however (§2) whether the language of formal semantics — the language from which the formulas that represent sentences of natural language are drawn — should be the same as or distinct from the language of propositions — the language from which the wff parts of propositions are drawn.

propositions — they do not in any way pose a general problem for the present conception of propositions.

### 3.7. Granularity

All the other theories of propositions that we have mentioned force us in certain circumstances to identify or to distinguish propositions in ways that lead to problems of granularity. The view of propositions proposed in this paper, by contrast, does not face a granularity problem in any of these situations.

Let's begin with the most famous problem in this area: Frege's puzzle [16]. Consider a theory of propositions according to which a sentence  $S$  expresses a structured proposition  $P$  such that the structure of  $P$  corresponds to the structure of  $S$ , and where  $S$  contains a name the corresponding element of  $P$  is the referent of that name. Such a theory is forced to say that 'Hesperus is Hesperus' and 'Hesperus is Phosphorus' express the same proposition.<sup>40</sup> It then becomes hard to see how it could be that someone might find the latter informative but not the former or how someone might believe the former but not the latter. Now of course I am not saying that this problem is insuperable for the kind of view of propositions that we have just mentioned: various responses have been proposed.<sup>41</sup> My point here is just that the view of propositions presented in this paper does not face this problem because it does not force us to identify the propositions expressed by 'Hesperus is Hesperus' and 'Hesperus is Phosphorus'. It allows us to say that the wff part of the proposition expressed by 'Hesperus is Hesperus' is something like  $h = h$  while the wff part of the proposition expressed by 'Hesperus is Phosphorus' is something like  $h = p$ . Thus even if  $h$  and  $p$  get assigned the same values in the model parts of these two propositions, still the propositions are distinct. (Another advantage of this view is that we can say that the former proposition is logically true while the latter is not — even if both are necessarily true. Cf. §3.5.)

Next let's consider the view to which Frege was led by his own puzzle, according to which propositions are again structured entities but where

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<sup>40</sup> 'Hesperus' is a name given to the evening star: the first object visible in the sky as night falls. 'Phosphorus' is a name given to the morning star: the last object visible in the sky as dawn approaches. It turns out that the evening star and the morning star are both the planet Venus.

<sup>41</sup> See e.g. Salmon [50], Braun [7] and Soames [59].

the part of the proposition corresponding to a singular term is not the referent of that term but a mode of presentation of the referent [16]. Suppose Bill says ‘I am fond of oranges’ and Ben hears him and — we want to say — believes what he says. The problem is that the sense of ‘I’ as Bill used it seems to be a special first person mode of presentation of Bill to which Ben does not have access: hence Ben cannot believe a proposition that has this mode of presentation as a component. When Ben says ‘Bill is fond of oranges’ the proposition he expresses contains a different mode of presentation of Bill. Hence the Fregean seems forced to distinguish two propositions here in a way that threatens to make communication problematic.<sup>42</sup> The view of propositions presented in this paper does not face any such problem because it does not force us to distinguish the propositions expressed by ‘I am fond of oranges’ (said by Bill) and ‘Bill is fond of oranges’ (said by Ben). There is no reason at all on this view why both sentences cannot be taken to express the very same proposition.

Let’s consider another case in which a view of propositions is forced to distinguish propositions in a way that seems problematic. Any sententialist view which takes the proposition expressed by a sentence to incorporate the sentence itself must distinguish the propositions expressed by utterances of distinct sentences (as opposed to two utterances of the same sentence). Thus a version of sententialism that takes sentences of natural languages in something like the ordinary sense to be parts of propositions will be forced to deny that sentences of different languages can express the same proposition while a version of sententialism that takes LF’s in the sense of some syntactic theory in linguistics to be parts of propositions — while it *can* hold that sentences of different natural languages might have the same LF — will nevertheless be held hostage by that syntactic theory and will be forced to distinguish the propositions expressed by sentences with different LF’s. In particular, both kinds of view will be forced to deny that all of the following sentences can express the same proposition:<sup>43</sup>

- Snow is white
- Schnee ist weiss (German)
- Snö är vitt (Swedish)

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<sup>42</sup> Again, I do not mean to suggest that the objection is insuperable: responses have been proposed. For the objection and some responses see e.g. Perry [44], Kaplan [30], Perry [45], Evans [13] and Heck [24].

<sup>43</sup> These examples are from Ripley [48, pp. 12–3].

- La nieve es blanca (Spanish)
- Yuki-wa shiroi-des (Japanese)
- Ha-shelleg lavan (Hebrew)
- Nix nivea est (Latin)

The view of this paper, by contrast, faces no such problem: on this view we are never *forced* to distinguish two propositions based on syntactic properties of the sentences used to express them.

Let's now consider another case of the kind that arose in connection with Frege's puzzle: a case where a view of propositions forces us to identify certain propositions. Views of propositions as sets of possible worlds force us to identify the propositions expressed by sentences true at exactly the same worlds. Thus, for example, all necessarily true sentences will express the same proposition. The present view, once again, enforces no such identification.<sup>44</sup>

Let's turn now to Kripke's puzzle [34]. Pierre, living in France, sincerely asserts 'Londres est Jolie'. Later, living in London, he sincerely asserts 'London is not pretty'. Intuitively Pierre is not illogical or irrational: he simply does not realise that the city in which he now lives is the very city he once referred to as 'Londres'. The problem is that certain views of propositions force us to the conclusion that Pierre is illogical: that he believes a proposition  $P$  (the one he expresses by saying 'Londres est Jolie') and also its negation (which he expresses by saying 'London is not pretty'). The present view of propositions, however, does not force us to identify the proposition that Pierre expresses by uttering 'London is not pretty' with the negation of the proposition that he expresses by uttering 'Londres est Jolie'. We are free to represent the wff parts of the propositions in question as follows (using FOL as the language from which the wff parts of propositions are drawn, for the sake of illustration):

- Londres est jolie:  $J_s$
- London is not pretty:  $\neg P_n$

Now we may suppose that Pierre believes propositions whose wff components are as follows (the third corresponds to his knowledge that 'pretty' translates 'jolie'):<sup>45</sup>

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<sup>44</sup> Note that even if we adopt an intensional model theory, the present view of propositions can avoid the problem of hyperintensionality. We are not forced to identify propositions whose model parts assign the same intensions to corresponding wff components – because the wff parts themselves might be different.

<sup>45</sup> Alternatively we could forget about the third wff altogether and just represent 'Londres est jolie' as  $J_s$  and 'London is not pretty' as  $\neg J_n$ .

- $J_s$
- $\neg P_n$
- $\forall x(Px \leftrightarrow Jx)$

This is a consistent set of wffs. If we add the wff  $s = n$  — which is something *we* believe — then we get an inconsistent set. But the point of the story is that Pierre does not believe a proposition with wff component  $s = n$ . This is how it can be the case that Pierre is not illogical or irrational.<sup>46</sup>

Kripke's Paderewski case can be handled in a similar fashion. Not realising that the Polish pianist and composer called 'Paderewski' is the same man as the Polish Prime Minister called 'Paderewski', Peter sincerely asserts 'Paderewski had musical talent' (thinking of the composer) and later 'Paderewski had no musical talent' (thinking of the politician). Intuitively Peter is not illogical or irrational: he simply fails to realise that there is only one man called 'Paderewski' in question. The kind of view of propositions I am proposing allows the situation to be handled straightforwardly. It allows us to say that the wff component of the proposition that Peter expresses when he says 'Paderewski had musical talent' is something like  $Tm$  while the wff component of the proposition that Peter expresses when he says 'Paderewski had no musical talent' is something like  $\neg Ts$  — and Peter does not believe any proposition with wff component  $m = s$ . Hence the wff components of his beliefs form a consistent set and he is not illogical or irrational — he simply lacks some knowledge.<sup>47</sup>

### 3.7.1. Anything Goes?

The upshot of §3.7 is that the present view of propositions does not *enforce* any level of granularity, be it fine or coarse. We can see quite different sentences (sentences of different languages, and sentences with different LFs) as expressing the same proposition and we can see the very same sentence as expressing different propositions on different occasions

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<sup>46</sup> If Pierre does learn that Londres is London — i.e.  $s = n$  — then in order to maintain consistency he will need to reject one of  $J_s$  and  $\neg P_n$  (assuming he still believes  $\forall x(Px \leftrightarrow Jx)$ ). Presumably he will do so! If he does not, then intuitively he *is* irrational.

<sup>47</sup> Sententialists can handle the Londres/London case in the same kind of way I do because there are two different words in play ('Londres' and 'London'). Sententialists have more trouble with the Paderewski case because there seems to be only one word in play ('Paderewski').

of use. Now this level of flexibility might engender the worry that there is too much freedom: that propositions are underdetermined. This worry is out of place. Recall the dialectic. We are trying to say what propositions could be for purposes of GT. Sometimes in GT we want fine-grained propositions (e.g. we do not want to be forced to say that Pierre or Peter is irrational and we do not want to say that someone who believes that  $2 + 2 = 4$  automatically believes that water is  $H_2O$ ) and sometimes we want coarse-grained propositions (e.g. we want to be able to allow that the proposition expressed by Bill when he says ‘I am fond of oranges’ might be the same as the proposition expressed by Ben when he says ‘Bill is fond of oranges’ and we want to be able to allow that the same proposition can be expressed by sentences in different languages with different syntactic structures). The present view of propositions *allows* what we want, while other views *enforce* overly fine or overly coarse levels of granularity in certain situations. This is a major advantage of the present view.

Now someone might object that what we want from a theory of propositions is a deterministic or algorithmic theory that tells us exactly which proposition is expressed by which sentence in which context, which proposition is believed by which agent in which situation, and so on. But that was never my task in this paper. The aim was to make available the resources needed for GT: to say what propositions might be for purposes of GT. The aim was not to complete GT, or even a particular fragment of GT. Distinguish two tasks:

1. Saying what kinds of things propositions are.
2. Giving a theory that associates specific propositions with specific utterances.

Now consider Frege’s puzzle. A theory at level 1 (i.e. a theory that tries to perform the first of the two tasks just mentioned) just tells us what kind of thing a proposition is. A theory at level 2 tells us exactly which proposition is expressed by an utterance of ‘Hesperus is Phosphorus’ in some context  $C$ , which proposition is expressed by an utterance of ‘Hesperus is Hesperus’ in  $C$ , and whether they are the same proposition. Now I have taken the problem to be that certain theories at level 1 *enforce* certain bad answers at level 2. And the advantage of the kind of theory I have offered at level 1 is that it *allows* the kinds of answers we intuitively want to give at level 2. But I have not given a theory at level 2 at all. I take giving such a theory to be part of completing GT and/or part of the project of formal semantics (which, as discussed

earlier, has an interface with GT). I am not claiming to have a theory (at level 2) that *generates* the results we intuitively want about Frege's puzzle, Kripke's puzzle and so on. I claim only that the theory I have given (at level 1) does not — unlike other theories (at level 1) — preclude giving a theory at level 2 with the intuitively correct results. Actually giving such a theory at level 2 was never on the agenda in this paper.

Still, there might be a residual worry here, which is that the price of not enforcing bad answers at level 2 is the opening up of *too many* possibilities in a way that makes the task of giving a theory at level 2 highly problematic. The worry is that no theory at level 2 will ever be able to say that sentence  $S$  in context  $C$  expresses proposition  $P$  because there will always be other equally good candidate propositions besides  $P$  — differing from  $P$  in their wff components — and nothing to decide between them. For example, I have claimed that my theory of propositions — at level 1 — allows for theories at level 2 according to which the wff part of the proposition expressed by 'Hesperus is Hesperus' is something like  $h = h$  while the wff part of the proposition expressed by 'Hesperus is Phosphorus' is something like  $h = p$ . The objection now is that no theory at level 2 which made such claims about the propositions expressed by these sentences could ever be warranted because there would always be equally good rival theories that identified the wff parts of these propositions differently.

One reason one might think this has to do with the worry, raised by Benacerraf [5], that we cannot identify the numbers with any particular bunch of set-theoretic entities (e.g. the von Neumann ordinals) because there are always other candidate targets for the reduction and nothing to favour one of these bunches of set-theoretic entities over the others. A problem of this sort arises for anyone who countenances abstract objects of any sort. The question arises whether these abstract objects can be identified with set-theoretic entities. If they can be identified — in one way — then the problem arises that they can also be identified in other ways and there seems to be nothing to favour one identification over the others. This is a quite general problem and I shall not propose any solution to it here. If I can show that there is a reasonable prospect of favouring some theory at level 2 that associates a proposition  $P$  with a certain sentence  $S$  over rival theories (that associate with  $S$  propositions that differ from  $P$  over their wff components) then I shall take my work to be done: the problem that the wff component of  $P$  (which is an abstract object) could still then be identified with many different set-

theoretic entities — that is, the Benacerraf problem — is a quite general problem and one for another day.

Setting aside now the Benacerraf problem of how one might identify a given wff — an abstract object — with some set-theoretic entity, still there are reasons for thinking that no theory at level 2 that associates a certain sentence  $S$  with a proposition  $P$  could ever be preferred over rival theories that associate  $S$  with propositions that differ from  $P$  over their wff components. First, there is the issue of choosing the formal language from which the wffs are drawn. Won't there be a limitless number of equally good alternatives and hence won't the choice of one formal language be completely arbitrary? I don't think so. There are serious constraints here. For example, the formal language should be such as to support an appropriate logic and it should also be such as to support a suitable interface with formal semantics. Now in the literature on logics of belief revision (for example) and in the literature on formal semantics one finds debates about the appropriate underlying formal language. However, one does not find a ridiculously large number of live alternatives. It is a deep and interesting question — and one that is (as I have already mentioned) beyond the scope of this paper — what kind of formal language will work best here: but I see no reason to think that there will be a vast number of equally good alternatives. Of course there may be more than one viable alternative with no single absolutely clear best choice: but this is not any new kind of problem. Recall that — on the approach to propositions taken in this paper — our reason for believing in propositions at all is that they play a role (several roles) in a successful explanatory theory: GT. Empirically equivalent theories are ubiquitous. We should not expect that theories involving propositions — in this case, GT — will magically be immune from having empirically equivalent alternatives when it is well known that theories in many other areas have such alternatives. Our goal here cannot be to show that there will be just one correct formulation of GT: it can only be to show that any theoretical indeterminacy here will be of a familiar sort and at familiar levels.

OK, so suppose we have fixed on a formal language from which the wff components of propositions are to be drawn. For the sake of example, let's suppose it's FOL. Further problems loom. In order not to deem Peter illogical we need to suppose that his utterances of sentences involving the name 'Paderewski' fall into two groups: some of them express propositions whose wff parts feature one name and others express

propositions whose wff parts feature a different name. But now focus on the utterances in just one of these groups. If we can get an empirically adequate theory by associating them all with propositions whose wff parts feature a single name (say  $p$ ) then can't we get an equally empirically adequate theory by associating them with propositions whose wff parts feature *different* names ( $p_1, p_2, \dots, p_n$ ) and supposing that Peter also believes propositions with wff parts  $p_1 = p_2, p_2 = p_3, \dots, p_{n-1} = p_n$ ? But then what is to decide between these theories? It seems to become completely arbitrary whether we think Peter expresses two propositions involving the same name or two propositions involving different names, when he makes two utterances of sentences involving the name 'Paderewski'. There is a straightforward response to this problem: simplicity. In developing GT, we should endeavour to find the simplest theory that fits the phenomena. Of course there may sometimes be ties and trade-offs, but this is — once again — a familiar issue with empirical theories, not some special new problem facing the theory of propositions presented in this paper. Once again, our goal here cannot be to show that there will be just one correct formulation of GT: it can only be to show that any theoretical indeterminacy here will be of a familiar sort and at familiar levels.

Consider a third and final kind of problem case. For any implementation of GT in a particular area — say, in an explanation of the behaviour of some agents — there will be a different, empirically equivalent implementation that differs from the first by uniform substitution of symbols in the language of propositions. For example, we could take Pierre to believe propositions whose wff components are  $Js, \neg Pn$  and  $\forall x(Px \leftrightarrow Jx)$  — or  $Kt, \neg Qo$  and  $\forall y(Qy \leftrightarrow Ky)$ . As long as the substitution is uniform, this will make no difference. But then what is to decide between these theories? It seems to become completely arbitrary whether Pierre believes a proposition with wff component  $Js$  or a proposition with wff component  $Kt$ . My response to this is that these are not two distinct theories at all: they are notational variants of the same theory. The content of this theory is that Pierre believes propositions with wff parts where the same symbol occurs here and here and here, a different symbol occurs here and here, a different symbol again occurs only here, and so on. We can express this by labelling the first symbol just mentioned  $J$ , the second one  $s$ , and so on — or we can express it by labelling the first symbol just mentioned  $K$ , the second one  $t$ , and so on. But either way, we are expressing the same theory. And once

again, it is a familiar fact — rather than a special problem facing the theory of propositions presented in this paper — that there can be different notational variants of a single empirical theory.

#### 4. Conclusion

In this paper I have presented a new theory of propositions, according to which propositions are abstract mathematical objects: wffs together with models. I have distinguished the theory from a number of existing views and explained some of its advantages — chief amongst which are the following. On this view, propositions are unified and intrinsically truth-bearing. They are mind- and language-independent in the way required by GT and they are governed by logic. The theory of propositions is ontologically innocent. It makes room for an appropriate interface with formal semantics and it does not enforce an overly fine or overly coarse level of granularity.

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NICHOLAS J. J. SMITH  
Department of Philosophy  
University of Sydney  
NSW 2006 Australia  
[njjsmith@sydney.edu.au](mailto:njjsmith@sydney.edu.au)