his systems, and he treated the theorems of deductive theories as true propositions in the same sense in which the propositions of the empirical sciences are true. It did not interfered with his being at the same time a radical formalist. Generally, it is possible to construct for sciences the systems of formal logic concordant to the object of study of those sciences (cf. [37, pp. 82–84]).

A few general remarks should be made on constructing the logical systems adequate for certain types of sciences, that will contain intensional connectives beside extensional ones. These systems, called non-classical logics, are said, in the logical literature, to be less formal than the classical logic. Propositional variables in the classical propositional logic represent arbitrary propositions. The aspect of its content is put aside and the only attention is focused on the purely formal moment of truth and falsehood, whereas, non-classical logic, containing for example such temporal connectives as 'and then', 'and next', must respect some ontological presuppositions.

The assumptions of a logic of such temporal connectives may be the following cosmological theses: time is linear, time is circular, time is infinite in the past, etc. The connective 'and then', which was brought forth by way of example, expresses a truth-connection and a certain temporal succession. If we establish that the time continuum which comes into play in a certain real science is linear, then we can accept only such axioms characterizing 'and then', which do not exclude the linearity of time. The field of the neutrality of the meaning of this connective is demarcated by propositions concerning events (or states of affairs etc.) that remain in the appropriate temporal relation with each other and occur in linear time. The connections between such propositional formulas containing the connective 'and then' that do not negate the linearity of time, may already be purely formal. In constructing the logic of the connective 'and then' one thus cannot disregard some theory of time.

The same applies to other temporal connectives. In a system of logic that respects established theory of time, they are neutral in content. The theorems of classical propositional logic are exactly those well-formed formulas that are proven valid by a method of truth-table, while the theorems of the system of the connective 'and then' are moreover the characteristic axioms of the connective 'and then' and consequences of those theorems and axioms. The acceptation of axioms and rules of inference must depend, however, on their compatibility with the ontological and cosmological assumptions regarding time and similarly for systems, based on classical logic, that characterize other non-extensional connectives.

For example, some specific axioms and theorems containing epistemic non-extensional connectives express certain assumptions concerning human knowledge and the human being. It is sometimes remarked that the so-called common epistemic logic does not express assumptions concerning human knowledge or idealized human knowledge, but a partial knowledge of omniscient being.[8] It should also be added that in the latest literature there are attempts at demonstrating that in some contexts where modal terms come into play, one or another system of logic can be applied. It is emphasized that modal systems express various ideas of possibility and necessity (cf. [11]). The choice of an adequate modal system is thus connected with the study of the assumptions of the systems of modal logics.

The assumptions, especially the ontological ones, which should be respected by systems of non-classical logic, can be called 'the descriptive semantics' of respective systems. The apparatus of set theory may be employed to provide also an appropriate formal semantics for these systems. The adequacy of such a formal semantics with respect to a determinate branch of science must be demonstrated already on the grounds of philosophy of science. It seems that in the appropriate course of development for systems of non-classical logic, conformed to determinate types of knowledge, an adequate axiomatic system is to be constructed first and only later a formal semantics strictly corresponding to this system. Such was, among others, the course of development of the temporal propositional logics presented by A. N. Prior. The logical apparatus of model theory by itself, without the appropriately justified adjustments is too restricted from the philosophical point of view. In the philosophico-logical literature warnings may be found against placing excessive hopes in its philosophical applications (cf. [29, p. 56]).

[8] P. Weingartner, among others, called attention to this issue in his lecture *Paradoxes Solved by Simple Relevance Criteria*, given on Oct. 21, 1984, at the 30th Conference of the History of Logic in Cracow.

Part 2. The logic of change

Preliminaries

The problem of the change is one of the oldest problems in philosophy. Changes taking place in the world have been explained from various point of view. It should be noted that in the theory of science it is emphasized that the expression ‘change’ is one of the few basic terms that physics exploits to talk about the world. The great discoveries of twentieth century physics did not left unaffected views underlying natural considerations, i.e. philosophical assumptions of natural science. The concepts of time and space had to undergo some modifications. In the philosophy of science much attention has been dedicated to this issue. However, the concept of change, which associates with the current vision of the deepest ontic level of physical reality more strictly than the concepts of time and space, has not been sufficiently elucidated.

It should be recalled that the precise laws of physical theory are expressed in the language of mathematics. A mathematical image of atomic phenomena has even been discovered. On the basis of this image the twofold nature of radiation and material can be read out. Mathematical models makes possible to calculate certain magnitudes, provided that some other magnitudes are given. There is no doubt, that that the logic of mathematical language is the classical logic calculus.

However, mathematical language is not the only language which professional physicists and philosophers of science need to use. L. de Broglie stated that contemporary physics has resolved to a significant degree the question of mathematical formalism, but it has not resolved the difficult problem of interpretation. According to the French physicist, the resolution of this problem, will require the hard work of those whose chief concern is directed to understanding the nature of the physical world. Thus, the theoretical physicists encounter great difficulties concerning concrete, ontological interpretation of mathematical apparatus that comprises in its forms e.g. the world of atoms, but which renders this world inaccessible to imagination. Then it is difficult to speak of the contemporary physics’ understanding of what we commonly call matter. To render universally intelligible the world of atom, the facts which are there discovered must be expressed in an appropriate language that would be close to natural language.

Physicists emphasize that for several centuries people believed that in natural science the problem of an adequate language does not exist. It seemed that natural language may be easily used to communicate the achieved results. The situation has changed in the 20th century. The experimental discoveries which were analyzed theoretically in the theory of relativity and quantum theory brought about a revision of the foundations of physics. It became problematic to talk about this new areas of research (cf. [7]). Heisenberg maintained even, that the artificial language of mathematical theory of relativity and corresponding language of experimenting physicists, adapted to each other. There emerged in connection with the theory of relativity certain manner of speaking of spatial and temporal relations on a large scale. Language of classical physics, that is close to natural language, fails in describing mutual correlations formulated mathematically in quantum theory. It is difficult to speak univocally about mere particles, that in some experiments seem to be corpuscles, and in other experiments to be waves. The physicist must speak of such particles in an almost natural language if he wishes to understand his own experiments, since every physical experiment must be described in such a language. The language of the experimental physics is today principally a language which corresponds to the mathematical formalism of classical physics. Observational operations have a macroscopic character. In this sense classical physics prevails over contemporary physics. A group of immeasurably important general concepts is to be found in the interpretative language of classical physics. Among those are the following: time, space, causality, mass, force, energy, change, body. These concepts appear also in respective languages of other physical theories, but their meaning in those theories may differ. The language that corresponds to the mathematical formalism of quantum theory cannot be lacking of these fundamental conceptual components of the natural sciences (at least of some of them). Without doubt there appears here the category of change.

In the light of the above considerations, it seems that the logic corresponding to the natural sciences (including physics) must be somehow related to the above mentioned base concepts of natural science. In the language (already existing or just being created) adapted to the mathematical language of physical theories, there appear connectives connected at least with some of the above mentioned categories, e.g. the connective ‘and then’. In this group of logic systems there must be a place for the logic of change. There are many connectives connected with the term ‘change’. Here is the context in which one of these connectives appears: ‘There is a change in the fact that *p*’. The logic of change, valid for physics and other natural sciences, is intended chiefly to investigate the formal properties of this connective. This is not an easy task, for we are not dealing here with a truth-functional connective. Thus the specific axioms characterizing this connective must take into consideration also some contents, they must comply with certain assumptions, especially ontological ones. The principle of the selection of criteria of adequacy which was mentioned in the first part of this article shows in outline whence these contents are to be derived, but does determine to what extent these contents are to be taken under consideration. The problem of how much objective information about a (given) domain a certain formal representation is to afford depends on the goals for which systems of non-classical logics are constructed. Therefore the above mentioned contents and assumptions must be investigated, and this involves also the search for the criteria of adequacy of non-classical logics. It is only on the basis of findings of those investigation, that some substantial comments concerning systems of the logic of change can be made. Analyses of this kind will explain also why basic connectives associated with the category of change must be propositional connectives of propositional arguments.

There are some remarks concerning formal systems, called in the philosophical-logical literature logics of quantum mechanics, to be made before taking up this question There are few fundamental groups of this kind of calculus. Z. Zawirski aimed at constructing a system of many-valued logic conformed to the language of probability theory. The characteristic of this logic in comparison to the classical propositional calculus was that it admitted more truth-values. H. Reichenbach attempted to develop a language for quantum mechanics by making use of a three-valued logic.

The connectives of the new propositional calculus were characterized by means of a logical matrices, in such a way that they maintained affinity with the corresponding tables of two-valued logic. Reichenbach assumed that the language of the new system must also be extensional — its connectives should be truth-functional — even though he undermined the principle of bivalence. The conception of a language was for Reichenbach a principle that influences the choice of criteria of adequacy, concerning the truth-value of physical laws and expressions stating causal anomalies, formulated in the proposed language of three-valued logic. C. F. von Weizsäcker and W. Heisenberg referred to the conception of the logic quantum mechanics initiated by G. Birkhoff and J. von Neumann. The paper in which Heisenberg argues for the need of this logic as rival to classical logic, is neither convincing nor coherent. In Heisenberg's conception, the logic of quantum mechanics is the logic of the language of physics, conformed to the mathematical scheme of quantum theory. Heisenberg observes that this language employs the term 'electron'. He emphasizes, that entities denoted by this term appear in certain experiments as fast-moving electrically charged particles, and as waves, that cannot be treated as particles of small extensions, since they occupy a larger region of space, in other experiments. In the light of this argument, the electron cannot be treated as classically understood body. Subsequently, in the part of the argument serving to refute the law of excluded middle, Heisenberg speaks of the electron as of a certain object in the sense of classical physics. It can be said to complement the first section of his argument, that an electron is an event, it is something which constantly acts and changes energetically. It seems that the ontology of contemporary physics is not sufficiently discussed in the logic of quantum mechanics in the style of von Neumann. Since the electron cannot be imagined after the pattern of a classical object, such sentences as 'There is an electron in the left half of the box' are lacking in sense. The imaginative language should be developed in accordance with the fundamental findings of contemporary physics. It seems that elementary particles can be grasped, as we said above, in judgments that state the altering, the energetic action of something, that is however closer unspecified. It should be added that Heisenberg's principle of the selection of adequacy criteria was essentially connected with the conception of a certain logical language different from the language of classical logic.[9]

It seems that systems of non-classical logic which would be adequate with respect to the natural sciences must be constructed in such a way that they will conform to the, already mentioned, principle of the selection of adequacy criteria in non-classical logic. Other attempts made in this respect, as characterized above, rise a number of objections. It should be said, among other things, that besides the fact that these attempts were associated with one theory, i.e. quantum mechanics, it seem

[9] Heisenberg seems to have been influenced by the cultural milieu of the nineteen-thirties and forties. According to Zinoviev, this milieu aspired to give to all important discoveries the trait of being a revolution not only in people's concepts of one or another area of reality, but a revolution in the logical foundations of science themselves.

not to have pursued a sufficiently clear program for the development of non-classical logics. In connection with the problem of the logic of change, which is necessary for a contemporary physics and other natural sciences, even empirical ones, it must be said that the problem of change in general, and change in the sense of contemporary physics, is an ontological problem, since in a certain way it concerns reality. It can be analyzed and presented only by means of logic. In order to develop a suitable logic of change for contemporary physics and other natural sciences in same way based upon physics, it is necessary to answer, among others, the following question: What is changing in the world of physics? With what kind of changes we are dealing? It must be stressed that in mechanistic physics and in contemporary physics somewhat different answers are given to these questions.[10] Change could be considered in terms of the conception of the world presented by Democritus.

Contemporary natural science is rooted in this conception. Democritus conceded that changes could occur in objects which can be differentiated by the senses, but he regarded them as changes in man's psyche, in his subjective feelings. The only admissible changes in objective reality were changes in the spatial relations between atoms. Descartes' findings in the realm of physics were an attempt to complete the kind of changes which can take place in nature. He put forth a thesis concerning the division and the coming together of elementary particles, that was not accepted by his successors. Descartes claimed also that changes in the mutual position of elementary particles are subject to certain laws and this thesis found acceptance among successors. Newton referred his principles of motion to elementary components and to other objects. Within contemporary natural science that is based on physics, change must be related foremostly with elementary particles, which cannot be treated on a par with the atoms of mechanistic physics. Contemporary physicists assume that among known particles exist only six kinds of particles and corresponding to these, antiparticles, which are stable or relatively stable in a free state. These are protons, neutrons, electrons, photons, electron neutrinos and meson neutrinos. The first four kinds of particles play an immeasurably important role in the structure of matter. All these elements of the deepest ontic level are peculiar constructs in the contemporary physical conception of the world. They should be treated as

[10] I have presented broader analyses concerning the understanding of change in natural science in the above mentioned book [12], pp. 90–116, 188–193.

certain events.[11] Some of them can, under certain conditions, interact energetically.[12] Free electrons should also be mentioned here. By its nature every such electron must emit and absorb photons, i.e. quanta of electro-magnetic radiation, which may be of various magnitudes.[13] Thus we are dealing with change of an energetic type, which is not connected with causal considerations. Change of this type, connected with at least certain kinds of elementary particles, and therefore with the deepest ontic level of contemporary physics, is the most elementary kind of change. The logic of change for natural science must in a certain way consider this moment.[14]

It has also been observed above that elementary particles may interact among them. But then, considering each particle, one can also say that particles change energetically.[15] The existence of particles is connected with the occurrence of energetic interactions. The above

[11] Generally, by event it is meant that some objects or some totalities change in a certain way. The events are said to be happening, to last in time. Although they have some spatial location, they can hardly be, it is difficult to ascribe to them spatial dimensions. Cf. among other, [5, pp. 1–15], [23, p. 51], [25, pp. 101–102], [32, p. 20].

Such conception of event does not entail total adhesion to A. N. Whitehead's position in this question. It seems that the basis of this conception of this English philosopher lies his monistic position on knowledge types. It is also emphasized in the literature that cognitively grasped events may be expressed by means of propositions.

In this paper we take the term event in the sense in which it is used by philosophers of science. Thus we are using this term in a sense somewhat different from that in which R. Ingarden used it. However, it should be added that Ingarden was aware of the various meanings of this word (cf. [9, pp. 216–217]).

It should be still noted that by state of affairs philosophers of science understand the possession by a given thing of a certain characteristic, or rather a group of characteristics, which is in some respects essential (cf. [15, pp. 9–11]).

[12] Properly speaking, in changes of this type one deals with changes of changes. It is emphasized however, that modern science began when people had become accustomed to the idea of changes which themselves undergo change (acceleration, deceleration; cf. [32, p. 18]). Conception of this type is associated with the Bergson's motion of motions and with the ideas of A. N. Whitehead.

[13] Protons and neutrons behave in a similar manner. In the case of the latter not photons, but certain kinds of hadrons will be virtual particles.

[14] Connecting basic logic of change with the microphysics is consistent with L. de Broglie's thesis that one should search for the ultimate secrets of reality in the microscopic area.

[15] The transformation of elementary particles into other elementary particles is also a change of energetic type. It is however a change which creates a new, elementary component of matter, which at the present stage of the development of science is not treated as composite, and which has its beginning in time.

mentioned connective 'there is a change in the fact that ...', is a propositional connective of a propositional argument. The expression with this connective can be written in abbreviated form as '$\mathsf{Z}\,p$'.[**] In the light of the above, the variable 'p' can represent propositions concerning elementary particles, that are specific objects of an event-type. It should also be noted that the connective and variable understood in this way can also be used in connection with the language of classical physics, as it (the classical physics) also employs propositions concerning events connected with energy, for example: Body A conveys energy.

It should be stressed that the need to introduce, among others, propositional variables in the fundamental systems of a possible logic of change follows from analysis of certain epistemological, as well as ontological theses. Such an analysis is conducted in this article only in a very abbreviated manner. It seems that philosophical considerations also led Aristotle to introduce in his system of logic a certain type of name variables. It is said that in logic a variable represents an arbitrary expression of some class. Propositional variable represents in classical logic arbitrary proposition. Nothing prevents us from narrowing that class of propositions which are to be represented by propositional variables in some non-classical logic.[16]

As it has already been observed, in classical logic one abstracts from the content of a proposition. In the logic of change that will be constructed here, abstraction does not include not only the purely formal aspect of truth or falsehood of a proposition, but also this contentual aspect that it concerns an event of a certain type.[17]

We have considered so far some particular principles accepted in physics. As we pointed out, these principles influence in a way a system of the logic of change which would be adequate with respect to

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** The letter 'Z' comes from the Polish word 'zmiana' (change). In addition, further used the symbols '**ZI**' and '**ZII**' mean respectively: the first and the second system of the logic of change (see p. 220 and p. 232).

16 When Słupecki gives a more complete formal form to Łukasiewicz's philosophical remarks concerning the foundations of three-valued propositional logic, he assumes that propositional variables in some formulae represent exclusively propositions concerning events (understood in rather specific manner). He also admits that in this same system of logic one can speak of propositions which do not describe events (cf. [34, pp. 186, 190] and [28, pp. 122–127]).

17 The laws of the logic of change understood in this way shall be true, among others, in every non-empty set of precisely defined events which are able to change.

contemporary physics.[18] It should be noted also that physicists and representatives of the various sciences make some assumptions even before they begin their research. Preter-base assumptions and base assumptions are mentioned in this connection. The former concern those paradigms which dictate the manner of practicing the science in a given epoch. In other words, there may be distinguished the internal and the external base of a theory.

Among the elements of the external base one can single out the epistemological and ontological assumptions. In light of the fact that the logic of change attempts to express in some way a physical, natural view of the world, the ontological assumptions of both the above mentioned bases are extremely important.[19] Mention has already been made of particular principles. However the condition for the existence of natural science is the acceptation of two preter-base principles, i.e. the principle of induction and the principle of partial identity (cf. [26, pp. 374–392]).

The first of these principles states the repeatability of the elements in the world in similar conditions The second one states that if a certain element of nature repeats itself, then always some other determinate element repeats itself.[20] The content of the principles of induction and

[18] It should be noted that, according to mechanistic physics, the elementary constituents of matter, elements with no internal structure, were treated in another way. They were treated as invariable, and only spatial relations between them were changing. A logic of change which would be constructed exclusively in association with mechanistic physics would be different.

[19] Such an approach respects the general principle of the selection of adequacy criteria presented in the section of the article. Majewski's book [24] provides many valuable remarks concerning the base assumptions of physical theories.

[20] There can come into play objects, states of affairs, events, properties.

The principle of induction involves what in the most recent literature is sometimes called the principle of homogeneity of time, that states that all moments of time are equivalent with regard to the laws of physics. The question whether the principle of the homogeneity of time applies to the level of the microcosmos, the macrocosmos, and the megacosmos is under discussion. Its applicability on the level of the microcosmos and the macrocosmos remains unquestioned and with these areas is concerned physics, which investigates various systems, while natural cosmology is the science of the universe as a whole (cf. [8, pp. 200–201]).

It is sometimes said that the postulate of the invariance of experiences, of the invariance of influences with respect to time shift, just as every postulate in physics, is of such a type, that it does not lead to any discrepancy with the whole of experience and astronomical observation (cf. [38, p. 21]).

One also speaks of the principles of uniformity of matter and of the uniformity of the laws of nature. At the same time it is stressed that up to the seventeenth century of partial identity makes it possible to make productive use of variables in the language of physics in which time and space co-ordinates play a great role. In the light of the above remarks and of more extensive analyses which could be carried out, it seems possible to find common contents associated with the connective 'there is a change in the fact that …'. This connective can precede at least certain propositions of somewhat different contents. One can, for example, consider a change in the energetic action of an electron, in the energetic action of neutron, of a change in the energetic action of the sun or of a body observed at a short distance be means of the senses. One can consider a change in the transmission of energy through a certain object, etc. It is worth emphasizing that the contents of the above propositions concern the most important magnitude in every department of physics and in the natural sciences in general, i.e. the energy. An energetic interaction, during its realization, can involve various bodies and objects. With regard to the lowest ontic level of physics, change concerns precisely an occurring energetic interaction. Change can also concern events of another type, especially in the case of classical physics. It must be noted here that truth-functional connectives can connect propositions which concern diverse contents. The connective "there is a change in the fact that … " has as its arguments propositions about events. Such events can be said to mark out this connective's field of contentual neutrality. It can also be said that this connective's field of contentual neutrality is narrower than the field of contentual neutrality of truth-functional connectives. It is worth adding that, given the natural science's findings concerning change, one can variously restrict the field of contentual neutrality of the connective 'there is a change in the fact that …'. Its formal properties of this connective would be different in diverse fields. This connective, connected with the term 'change', can have different shades of meaning depending on the context in which it appears. This paper aims at constructing a very general system of the logic of change which can be of use in natural science and other real particular sciences, as well as in the philosophy of science.

It is a fact that in natural science, in physics, which employs abstraction, there is the possibility of considering a body, a group of bodies or certain objects not imaginable closer, from one point of view, without

the matter of the earth was thought to be different from the matter of the moon. The sun was supposed to have been built of yet some other stuff. Given such assumptions astrophysics was impossible. The content of these principles seems to be included in the principles of induction and of partial identity (cf. [44, pp. 109–111]).

taking into account all other properties (cf. [6, p. 71]). For example, one can see certain bodies or those unimaginable objects which are called elementary particles just as changing with respect to their energy supplies or the kinds of energy they possess, i.e., as acting energetically, ignoring at the same time even the fact that we are dealing with the transmission or absorption of energy, or with yet some other instance of energetic action. Nothing prevents us from seeing bodies or those unimaginable objects called elementary particles as transmitting or absorbing energy. They can be seen as transmitting or absorbing energy with greater or lesser than the usual intensity. Nothing stands in the way of seeing bodies as accelerating or decelerating etc. Given the remarks made in the last three sentences, the connective of change 'Z' is connected with the logical calculus **ZI** given below.

It should to be added at this point that in view of the fact that all matter and all radiation is composed of elementary particles, contemporary physicists are intensively searching for the laws concerning such particles because such laws establish the framework of all the regularities of physics. It must be taken into account by the logic of change for the language of physics close to natural language. It seems that there is also the possibility of constructing systems of the logic of change including all and only such theorems which are well formed and true only in some models of change, related to the fundamental ontic level of contemporary physics. Such systems will not be constructed in this article.

In order to be able to construct the above mentioned system of the logic of change it is necessary to consider the already presented principles of induction and partial identity. The principle of partial identity concerns invariable relations occurring among the elements of nature. It is necessary to present such relations that, modern and contemporary natural science would be impossible without having recognized them. It should be noted that the natural sciences search for theories that would be corroborated by experiment. A theory satisfies this condition, if its hypotheses are verified. Each experimentally verifiable theory refers to the temporal relations, to what is past, present and future. Thus temporal relations may be considered as the fundamental element presupposed by every physical theory. Thus it is no wonder that at least certain professional natural scientists see the need for a logic of temporal propositions.[21] It is also worth noting that concepts of time and space are

[21] J. H. Woodger and C. F. von Weizsačker are such authors, among others.

being analyzed in the aspect of their primordiality. It turns out that some understanding of time intervenes where the critical analysis of the concept of space is undertaken.

The concept of time becomes the most fundamental concept in modern and contemporary physics, since, from the point of view of physics, everything, including changes in all kinds of physical objects, takes place in time. All these observations lead to the conclusion that at least certain connectives of the logic of temporal propositions may be used in the construction of the general and simple system of the logic of change.

There have been developed various systems, which characterize formally different temporal connectives. It was noted in the first part of the paper that the axioms specific to systems of the logic of temporal propositions can respect various ontological, cosmological assumptions. A general system of the logic of change requires the use of such a system of the logic of temporal propositions that respects fundamental conceptions of contemporary physics concerning time and its temporal connective expresses in the simplest way temporal relations important from the point of view of natural science.

It seems that the system "And Then" is just such a system (cf. [41, pp. 1–11] and [42, pp. 208–221]). This system provides the formal characteristics of a proposition-forming connective 'T' of two propositional arguments. This connective has its counterpart in natural language, for one can read it as 'and then'. It is worth to add that the system "And Then" expresses the linearity of time. The use of this connective does not assume that time is discrete, nor does it assume that time is continuous or dense. It should be emphasized moreover that it is claimed nowadays that the model of time in physics is a straight line.[22]

The observations made so far, concerning the assumptions that allow to evaluate the cognitive value and to develop the systems of logic of change for the natural sciences, justify also the use of propositions stating that one object changed into another, especially within the microphysics. Sentences of this type, i.e. referring to change, have the following logical structure: 'there is a change in the fact that p and as a result q'. It is worth noting propositional expressions as: 'the state of matter of the object A changes', 'the energetic action of object A changes' also often occur in the natural sciences. In these expressions the connective

[22] The analyses concerning the problem of the properties of time in physics may be found, among others in: [2], [33, pp. 47–54] and [8, pp. 194–209].

connected with the term 'change' is a propositional connective of the nominal argument. There may be still other connectives connected with the term 'change' occurring in the language of the natural sciences.

The system "And Then"

The general calculus of the logic of change, which will be constructed here, must be based on the above mentioned system "And Then".[23] Von Wright based his system on the axiomatic system of classical propositional calculus. Here is von Wright's axiomatic system. To axioms of Classical Propositional Calculus (CPC) we add the following specific axioms for 'T':

$$(p \vee q \mathsf{T} r \vee s) \equiv (p \mathsf{T} r) \vee (p \mathsf{T} s) \vee (q \mathsf{T} r) \vee (q \mathsf{T} s) \tag{B1}$$

$$(p \mathsf{T} q) \wedge (r \mathsf{T} s) \equiv (p \wedge r \mathsf{T} q \wedge s \vee (q \mathsf{T} s) \vee (s \mathsf{T} q)) \tag{B2}$$

$$p \equiv (p \mathsf{T} q \vee \neg q) \tag{B3}$$

$$\neg(p \mathsf{T} q \wedge \neg q) \tag{B4}$$

It must be noted that in the sequence of symbols: '$\neg$', '$\wedge$', '$\vee$', '$\rightarrow$', '$\equiv$', 'T', each preceding symbol binds more strongly (more shortly) than all the symbols following it.

The primitive rules of the system "And Then" are: the rule of substitution, the rule of detachment and the rule of extensionality which states that if an equivalence is a theorem, then its sides are mutually interchangeable in the theorems of the system.

The Finnish logician outlined the proofs of the following theorems:

$$(p \mathsf{T} q) \vee (p \mathsf{T} \neg q) \vee (\neg p \mathsf{T} q) \vee (\neg p \mathsf{T} \neg q) \tag{T1}$$

$$(p \mathsf{T} p) \vee (p \mathsf{T} \neg p) \vee (\neg p \mathsf{T} p) \vee (\neg p \mathsf{T} \neg p) \tag{T2}$$

$$(p \mathsf{T} q) \rightarrow p \tag{T3}$$

$$\neg(p \wedge \neg p \mathsf{T} q) \tag{T4}$$

$$p \wedge (q \mathsf{T} r) \equiv (p \wedge q \mathsf{T} r) \tag{T5}$$

[23] In the light of the conclusions and analyses carried out in the second part of the paper and belonging to descriptive semantics of non-classical logics connected with natural sciences, it becomes clear that many systems of this kind employs some calculi of the logic of temporal propositions. The order of the construction of certain sections of logic connected with physics is not arbitrary. Undoubtedly, the proper logic of temporal propositions must be developed first and the logic of change must be constructed before the logic of causality.

$$(p \mathsf{T} q) \equiv p \wedge (\tau \mathsf{T} q), \tag{T6}$$

$$p \wedge (q \mathsf{T} r) \to (p \mathsf{T} r) \tag{T7}$$

$$(p \mathsf{T} q \wedge r) \to (p \mathsf{T} q) \tag{T8}$$

$$(p \mathsf{T} q) \wedge (p \mathsf{T} r) \equiv ((p \mathsf{T} q) \mathsf{T} r) \tag{T9}$$

$$((p \mathsf{T} q) \mathsf{T} r) \equiv ((p \mathsf{T} r) \mathsf{T} q) \tag{T10}$$

$$(p \mathsf{T}(q \mathsf{T} r)) \to (p \mathsf{T} r) \tag{T11}$$

$$\neg(\tau \mathsf{T} \neg p) \to (\tau \mathsf{T} p) \tag{T12}$$

where τ stands for any tautology of CPC.

It is possible to base the system "And Then" on the natural deduction system of CPC. Here is the definition of a theorem of a thus modified system of the logic of temporal propositions.

The theorems of order 1 are: (1) formulas for which there exists an indirect proof from assumptions, that employs exclusively the primary rules of the classical propositional calculus of adding new lines to a proof, and (2) the specific axioms of the system "And Then", formulated as theorems of the system.

Theorems of n order are: (1) formulas for which there exists an indirect proof from assumptions employing the theorems of orders lower than n as well as rules of adding new lines to a proof, used in the natural deduction system of classical propositional calculus, and (2) formulas obtained from the theorems of orders lower than n by virtue of the rule of substitution and the rule of extensionality.[24]

Besides the theorems indicated by von Wright, the following theorems are provable in the system "And Then":[25]

$$p \to \neg(\neg p \mathsf{T} q) \tag{a}$$

Proof. 1. $(\neg p \mathsf{T} q) \to \neg p$ substitution in (T3)
2. $p \to \neg(\neg p \mathsf{T} q)$ 1, CPC

[24] As noted above, the original von Wright's system "And Then" is based on the axiomatic system of the classical propositional logic. In connection with the question whether it is possible to use proofs from assumptions in the axiomatic systems based on the classical logic, which, apart from the specific axioms, include specific rules which lead from theorems to theorems, two theorems have been proved, that admit such proceeding (cf. [13, pp. 45–48]). We will understand the term 'proof from assumptions' in the way it was presented in [35, pp. 10–45, 77–117].

[25] Each proof given in this article may be presented as a proof from assumption in the sense introduced above. However, some axiomatic proofs are significantly shorter. In such cases, we adopt shorter form of a proof.

By substitution in (a) we obtain:

$$p \to \neg(\neg p \,\mathsf{T}\, p) \tag{b}$$

$$p \to \neg(\neg p \,\mathsf{T}\, \neg p) \tag{c}$$

By substitution in (B3) and by (B1) and (CPC) we obtain:

$$p \to (p \,\mathsf{T}\, p) \vee (p \,\mathsf{T}\, \neg p) \tag{d}$$

PROOF. 1. $p \equiv (p \,\mathsf{T}\, p \vee \neg p)$ substitution in (B3)
2. $p \to (p \,\mathsf{T}\, p \vee \neg p)$ 1, CPC
3. $p \to (p \,\mathsf{T}\, p) \vee (p \,\mathsf{T}\, \neg p)$ 2, (B1), CPC

Now, by (d) and CPC, we obtain:

$$p \wedge \neg(p \,\mathsf{T}\, \neg p) \to (p \,\mathsf{T}\, p) \tag{e}$$

$$p \to (\neg(p \,\mathsf{T}\, \neg p) \to (p \,\mathsf{T}\, p)) \tag{f}$$

By (B1) and CPC we have:

$$(p \vee q \,\mathsf{T}\, r) \equiv (p \,\mathsf{T}\, r) \vee (q \,\mathsf{T}\, r) \tag{g}$$

Hence, by CPC, we obtain:

$$(p \,\mathsf{T}\, r) \to (p \vee q \,\mathsf{T}\, r) \tag{h}$$

Finally, notice that, by (T5) and CPC, we obtain:

$$(p \wedge q \,\mathsf{T}\, r) \to (q \,\mathsf{T}\, r) \tag{i}$$

It is worth adding, as has already been emphasized, that all objects, all physical events occurs in time. The time of the duration of objects, of events, can be different. This must be kept in mind in understanding of the corresponding formulas of the logic of change.

The system ZI

The language of ZI. The alphabet of the language of **ZI** consists of the following symbols:

1. the propositional variables: $p, q, r, p_1, q_1, r_1, \ldots$ (representing proposition concerning events);

2. truth-functional connectives: ¬, ∧, ∨, →, ≡ (representing, respectively, connectives of negation, conjunction, disjunction, material conditional and material equivalence);
3. the proposition-forming connective of one propositional argument Z (to be read: "there is a change in the fact that ...");
4. the proposition-forming connective of two propositional arguments T (to be read: "and then");
5. parentheses.

In the sequence of symbols 'Z', '¬', '∧', '∨', '→', '≡, 'T', each preceding symbol binds more strongly than all symbols which occur after it.

The rules of syntax of the language of the system ZI. The following formulas are well formed propositional formulas:

1. propositional variable;
2. formulas composed of the connective 'Z' and its argument, that is a propositional variable, disjunction or conjunction of propositional variables, or a formula logically equivalent to these in the classical propositional logic. It is assumed that the conjunction of two events describes events which consists on both those events occurring. Such an event is a conjunction of events (sometimes it is called a product of events). The disjunction of two events describes events which consists in occurring of at least one of those events. Such an event may be called a disjunction of events (sometimes it is called a sum of events). However, we do not assume that a negation of propositions must describe an event. Such a negation states that the event described by a given proposition does not occur. Or to put it slightly differently, in natural sciences a complement of an event is not necessarily an event. It should be added that treating the complement of event as an event implies the existence of the so-called impossible events (cf. [4, pp. 62, 65]).
3. formulas composed of the above mentioned and the connective 'T' according to the syntax of the calculus "And Then";
4. formulas composed of the above mentioned formulas and the connectives of propositional calculus.

Axioms of the system ZI. Axioms of the system **ZI** are all axioms of the classical propositional calculus, all axioms of the calculus "And Then" and the following specific axioms for 'Z':

$$\mathsf{Z}\,p \to p \tag{A1}$$
$$(p \,\mathsf{T}\, \neg p) \to \mathsf{Z}\,p \tag{A2}$$
$$\mathsf{Z}(p \wedge q) \to \mathsf{Z}\,p \vee \mathsf{Z}\,q \tag{A3}$$
$$\mathsf{Z}\,p \wedge q \to \mathsf{Z}(p \wedge q) \tag{A4}$$
$$\mathsf{Z}(p \vee q) \to \mathsf{Z}\,p \vee \mathsf{Z}\,q \tag{A5}$$
$$\mathsf{Z}\,p \to \mathsf{Z}(p \vee q) \tag{A6}$$

The axioms (A1), (A3)–(A6) can be read as follows:

(A1): Only an event which in fact occurs changes.
(A3): If a conjunction of two events changes, then at least one of its conjuncts changes.
(A4): If some event changes, then the conjunction of this event and any other factually occurring event also changes.
(A5): If the disjunction of two events changes, then at least one of disjuncts changes.
(A6): If a component of the disjunction of two events changes, then the disjunction changes.[26]

The primitive rules of inference. The first rule of substitution allows us to accept correct substitutions of theorems of the classical propositional calculus and of the system “And Then”, as theorems of the system **ZI**.

The second rule of substitution allows us to accept as theorems correct substitutions of the theorems containing the connective ‘Z’. As has been noted, solely propositional variables, their conjunctions or disjunctions and equivalents of those, may be arguments of this connective.

The rule of detachment allows us to accept as theorem of the system the consequent of an implication that is a theorem of the system, inasmuch as its antecedent is also a theorem.

[26] The axiom (A6) may look unlikely since the proposition $p \vee q$ remains true, provided so does q. But, according to the definition of the disjunction of events (see p. 225), if both propositions p and q are true, then the event described by the proposition $p \vee q$ differs from the event described by the same proposition when only one of p and q is true. Hence $\mathsf{Z}(p \vee q)$ is true, provided so is $\mathsf{Z}\,p$, even if $p \vee q$ is now and will always be true. Notice that a disjunction refers to a sum of events, which is itself supposed to be an event. The formula ‘$\mathsf{Z}(p \vee q)$’ claims exactly that the sum of events changes and that there occurs a change in it, without necessarily any change of truth-values of formulas. Notice also that ‘$p \,\mathsf{T}\, \neg p$’ is a sufficient, but not necessary, condition for ‘$\mathsf{Z}\,p$’.

The rule of extensionality states that if an equivalence is a theorem then its both sides are mutually interchangeable in the theorems of the system.

The consistency of system **ZI** may be proved by the method of interpretation. The connective 'Z' is replaced in the translation by the connective of assertion and connective 'T' by the connective of conjunction. After replacements all specific axioms, containing constants 'T' and 'Z' are converted into theorems of the classical propositional calculus. Due to consistency of the classical propositional calculus, this constitutes the proof of the consistency of the system **ZI**.

The system **ZI** can also be based — just as the system "And Then" — on the natural deduction system of the classical propositional calculus. Here is the definition of a theorem in the system **ZI** thus understood:

The theorems of order 1 are: (1) formulas for which there exists an indirect proof from assumptions, that employs exclusively the primary rules of the classical propositional calculus of adding new lines to a proof, and (2) the specific axioms of the system "And Then", formulated as theorems of the system.

Theorems of n order are: (1) formulas for which there exists an indirect proof from assumptions employing the theorems of orders lower than n as well as rules of adding new lines to a proof, used in the natural deduction system of classical propositional calculus, and (2) formulas obtained from the theorems of orders lower than n by virtue of the rule of substitution and the rule of extensionality.

On the basis of the specified axioms and the primary rules of the system **ZI**, one can prove a number of theorems. Here are some of them:

$$p \wedge \neg \mathsf{Z}\, p \to (p \,\mathsf{T}\, p) \tag{L1}$$

PROOF. 1. $(p \,\mathsf{T} \neg p) \to \mathsf{Z}\, p$ (A2)
2. $\neg \mathsf{Z}\, p \to \neg(p \,\mathsf{T} \neg p)$ 1
3. $p \wedge \neg \mathsf{Z}\, p \to p \wedge \neg(p \,\mathsf{T} \neg p)$ 2
4. $p \wedge \neg \mathsf{Z}\, p \to (p \,\mathsf{T}\, p)$ 3, (e)

$$\neg \mathsf{Z}\, p \wedge \neg \mathsf{Z}\, q \to \neg \mathsf{Z}(p \wedge q) \tag{L2}$$

$$\mathsf{Z}(p \wedge q) \wedge \neg \mathsf{Z}\, p \to \mathsf{Z}\, q \tag{L3}$$

These theorems can be easily proved using (A3) and CPC.

$$p \wedge q \to (\neg \mathsf{Z}(p \wedge q) \to \neg \mathsf{Z}\, p \wedge \neg \mathsf{Z}\, q) \tag{L4}$$

PROOF. 1. $q \to (\mathsf{Z}\, p \to \mathsf{Z}(p \wedge q))$ by (A4)
2. $p \wedge q \to (\neg \mathsf{Z}(p \wedge q) \to \neg \mathsf{Z}\, p)$ 1
3. $p \to (\mathsf{Z}\, q \to \mathsf{Z}(q \wedge p))$ by (A4) and substitution
4. $p \to (\mathsf{Z}\, q \to \mathsf{Z}(p \wedge q))$ 3, the rule of extensionality
5. $p \wedge q \to (\neg \mathsf{Z}(p \wedge q) \to \neg \mathsf{Z}\, q)$ 4
6. $p \wedge q \to (\neg \mathsf{Z}(p \wedge q) \to \neg \mathsf{Z}\, p \wedge \neg \mathsf{Z}\, q)$ 2, 5

$$\mathsf{Z}(p \wedge q \wedge r) \to \mathsf{Z}\, p \vee \mathsf{Z}\, q \vee \mathsf{Z}\, r \tag{L5}$$

PROOF. 1. $\mathsf{Z}(p \wedge q \wedge r)$ assumption
2. $\mathsf{Z}((p \wedge q) \wedge r)$ 1
3. $\mathsf{Z}(p \wedge q) \vee \mathsf{Z}\, r$ 2, (A3)
4. $\mathsf{Z}(p \wedge q) \to \mathsf{Z}\, p \vee \mathsf{Z}\, q$ (A3)
5. $\mathsf{Z}\, p \vee \mathsf{Z}\, q \vee \mathsf{Z}\, r$ 3, 4

$$\mathsf{Z}(p \vee q \vee r) \to \mathsf{Z}\, p \vee \mathsf{Z}\, q \vee \mathsf{Z}\, r \tag{L6}$$

PROOF. 1. $\mathsf{Z}(p \vee q \vee r)$ assumption
2. $\mathsf{Z}((p \vee q) \vee r)$ 1
3. $\mathsf{Z}(p \vee q) \vee \mathsf{Z}\, r$ 2, (A5)
4. $\mathsf{Z}(p \vee q) \to \mathsf{Z}\, p \vee \mathsf{Z}\, q$ (A5)
5. $\mathsf{Z}\, p \vee \mathsf{Z}\, q \vee \mathsf{Z}\, r$ 3, 4

$$\neg p \to \neg \mathsf{Z}(p \wedge q) \tag{L7}$$

PROOF. 1. $\mathsf{Z}(p \wedge q) \to p \wedge q$ substitution in (A1)
2. $\mathsf{Z}(p \wedge q) \to p$ 2, CPC
3. $\neg p \to \neg \mathsf{Z}(p \wedge q)$ 3, CPL

Theorem (L7) may easily be generalized for any n and $i = 1, \ldots, n$, as follows:

$$\neg p_i \to \neg \mathsf{Z}(p_1 \wedge \ldots \wedge p_n)$$

Moreover, by (A3) and (A5), we obtain:

$$\mathsf{Z}(p_1 \wedge q_1 \vee p_2 \wedge q_2) \to \mathsf{Z}\, p_1 \vee \mathsf{Z}\, q_1 \vee \mathsf{Z}\, p_2 \vee \mathsf{Z}\, q_2 \tag{L8}$$

PROOF. 1. $\mathsf{Z}(p_1 \wedge q_1 \vee p_2 \wedge q_2)$ assumption
2. $\mathsf{Z}(p_1 \wedge q_1) \vee \mathsf{Z}(p_2 \wedge q_2)$ 1, (A5)
3. $\mathsf{Z}\, p_1 \vee \mathsf{Z}\, q_1 \vee \mathsf{Z}\, p_2 \vee \mathsf{Z}\, q_2$ 2, (A3), CPC

It is worth emphasizing that (L8) can be respectively generalized.

$$\mathsf{Z}\,p \to \neg(\neg p \mathbin{\mathsf{T}} q) \tag{L9}$$

Theorem (L9) can be easily proved using (A1) and (a).

$$\neg p \to (\neg q \to \neg\,\mathsf{Z}(p \vee q)) \tag{L10}$$

The statement (L10) can be easily proved on the basis of the axiom (A1), from which we obtain '$\neg p \wedge \neg q \to \neg\,\mathsf{Z}(p \vee q)$'.

$$(\neg p \mathbin{\mathsf{T}} q) \to \neg\,\mathsf{Z}(p \wedge q) \tag{L11}$$

Theorem (L11) can be proved using (a) and (L7).

Various extensions of ZI

As it has been already noted in this paper, the language of natural sciences contains other connectives concerning the term 'change', besides the very important connective of one propositional argument. There can be considered the change of objects, of states, and even the change of events, understood as certain individuals and expressed by means of specially constructed names.[27] It seems that at least certain expressions of this kind — used in the language of natural sciences and concerning change — may be defined in the system **ZI**. Having extended system **ZI** by the calculus of predicates, one can formulate the following definitions:[28]

$$Z(x) \equiv \exists P\ \mathsf{Z}\,P(x) \tag{D1}$$

x changes if and only if for some P, the fact that $P(x)$ changes.

$$Z(x, K) \equiv \exists P\,(\mathsf{Z}\,P(x) \wedge K(P)) \tag{D2}$$

x changes with respect to the properties belonging to a type K if and only if x changes with respect to some definite property belonging to K.

For example, the object x changes with respect to the odour if and only if there exists a definite odour such that the fact that x has this odour changes.

[27] Such understood events and expressed as names, are conceptual constructs to a far greater extent than are persons, animals etc. when these are grasped cognitively and expressed by the use of names (cf. [22, p. 444]).

[28] In (D1) and (D2) and in further formulae the existential and the universal quantifier will be symbolized respectively as '∃' and '∀'.

A number of new theorems is provable in the system **ZI** with definitions (D1) and (D2). Here are some examples:

$$\neg\exists P\, K(P) \to \neg Z(x,K) \tag{Z1}$$

PROOF. 1. $\exists P(\mathsf{Z}\, P(x) \wedge K(P)) \equiv Z(x,K)$ (D2)
2. $\neg\exists P(\mathsf{Z}\, P(x) \wedge K(P)) \to \neg Z(x,K)$ 1, CPC
3. $\neg\exists P\, K(P) \to \neg\exists P(\mathsf{Z}\, P(x) \wedge K(P))$ quantifiers
4. $\neg\exists P\, K(P) \to \neg Z(x,K)$ 2, 3

$$\neg\exists P\ \mathsf{Z}\, P(x) \to \neg Z(x,K) \tag{Z2}$$

Theorem (Z2) may be proved analogically to the theorem (Z1).

$$Z(x,K) \to Z(x) \tag{Z3}$$

PROOF. 1. $Z(x,K) \to \exists P\ \mathsf{Z}\, P(x)$ (Z2)
2. $\exists P\ \mathsf{Z}(P(x)) \to Z(x)$ (D1)
3. $Z(x,K) \to Z(x)$ 1, 2

Apart from the above definitions one can still introduce in the system **ZI** extended by the calculus of predicates, the definition of the product and sum of two predicates:[29]

$$P_1 \cap P_2(x) \equiv P_1(x) \wedge P_2(x) \tag{D3}$$
$$P_1 \cup P_2(x) \equiv P_1(x) \vee P_2(x) \tag{D4}$$

A number of new theorems is provable in the system **ZI** with definitions (D3) and (D4). Here are some examples:

$$\mathsf{Z}\, P_1(x) \wedge P_2(x) \to \mathsf{Z}\, P_1 \cap P_2(x) \tag{Z4}$$

PROOF. 1. $\mathsf{Z}\, P_1(x) \wedge P_2(x) \to \mathsf{Z}(P_1(x) \wedge P_2(x))$ substitution in (A4)
2. $\mathsf{Z}\, P_1(x) \wedge P_2(x) \to \mathsf{Z}\, P_1 \cap P_2(x)$ 1, (D3)

$$\mathsf{Z}\, P_1(x) \to \mathsf{Z}\, P_1 \cup P_2(x) \tag{Z5}$$

PROOF. 1. $\mathsf{Z}\, P_1(x) \to \mathsf{Z}(P_1(x) \vee P_2(x))$ substitution in (A6)
2. $\mathsf{Z}\, P_1(x) \to \mathsf{Z}\, P_1 \cup P_2(x)$ 1, (D4)

[29] The definitions (D3) and (D4) can be respectively generalized.

It must be added further, that the definitions analogical to (D1) and (D2) can be introduced in the system **ZI** extended by Leśniewski's ontology:

$$Z(A) \equiv A \,\varepsilon\, V \wedge \exists\phi \;\mathsf{Z}\, \phi(A) \tag{DO1}$$

A changes if and only if A is an object and for some ϕ the fact that $\phi(A)$ changes.

$$Z(A, K) \equiv A \,\varepsilon\, V \wedge \exists\phi \,\varepsilon\, K \;\mathsf{Z}\, \phi(A) \tag{DO2}$$

A changes with respect to the properties belonging to a type K if and only if A is an object and for some $\phi \,\varepsilon\, K$ there is a change in the fact that $\phi(A)$.

The definitions of some terms concerning change can be formulated in **ZI** extended by Leśniewski's ontology in a way closer to that, in which they are used in natural language, since the language of Leśniewski's ontology contains a great variety of syntactic categories and is well correlated with natural language. In such a natural language, used also in the particular sciences, among others natural science, there is often talk about change of the properties of objects, of their states, etc. For example, instead of the definition (DO2), one can introduce the following definition:

$$Z(C\langle A\rangle) \equiv A \,\varepsilon\, V \wedge \exists\phi \,\varepsilon\, C\langle A\rangle \;\mathsf{Z}\, \phi(A) \tag{DO2$'$}$$

Property of a type C, pertaining to the object A changes if and only if A is an object, and for some ϕ, which is a definite property of the type C and pertaining to the object A, the fact that $\phi(A)$ changes.

For example, the colour of the object A changes $\equiv$ A is an object and for some ϕ, which is a definite colour of the object A, the fact that $\phi(A)$ changes. The energetic action of object A changes $\equiv$ A is an object and for some ϕ which is a definite way the object A acts energetically, there is a change in the fact that $\phi(A)$.

It seems that in order to define in the logic of change certain concepts concerning change, as for example concepts of continuous and discontinuous change, it would be useful, and perhaps even necessary, to base the axioms containing the connective 'Z' on some richer system of the logic of temporal propositions, for example on the metric temporal logic or on the temporal logic containing formulas of the form '$U_t p$' ("at the time t it is the case that p") and the term denoting the relation of temporal precedence between the points in time.

The system ZII

Hitherto attempts to construct a rather general calculus of the logic of change, as well as brief outline of its possible development, concerned the connective that is very basic in view of the conclusions of the descriptive semantics, i.e. the connective of one propositional argument ‘there is a change in the fact that . . . ’. As it has been already noted, it is necessary to provide also the formal characteristic of the connective occurring in the propositional expression ‘there is a change in the fact that p and as a result q’. Undoubtedly, this connective may be characterized axiomatically, respecting the intuitions concerning the appropriate types of events. For a very general conception of change, it may be characterized in the calculus based upon the system “And Then” with quantifiers binding propositional variables. This calculus will be represented by the symbol **ZII**. It will include moreover the following general definitions:[30]

$$p \,\mathsf{Z}^{\star}\, q \equiv (p \,\mathsf{T}\, q) \wedge (q \to \neg p) \tag{D1*}$$

There is a change in the fact that p and as a result q if and only if p and then q, and if q then it is not the case that p.

$$\mathsf{Z}\, p \equiv \exists q \; p \,\mathsf{Z}^{\star}\, q \tag{D2*}$$

There is a change in the fact that p if and only if for some q there is a change in the fact that p and as a result q.

System **ZII** may be also formulated as a natural deduction system. The theorems for such a system would be defined as follows:

The theorems of order 1 are: (1) formulas for which there exists an indirect proof from assumptions, that employs exclusively the primary rules of the classical propositional calculus and the rules for quantifiers of adding new lines to a proof (RD, AI, AE, KI, KE, EI, EE, ∀I, ∀E, ∃I,

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[30] The definitions (D1⋆) and (D2⋆) respect somehow the fundamental assumptions discussed in this paper, that allow to evaluate the cognitive value and to develop the systems of logic of change that would be cognitively valuable for the natural sciences. The definition (D2⋆) grasps also the intuitions which K. Ajdukiewicz expressed in his article [1, p. 104]. Ajdukiewicz states that all change consists in transition from state A to state non-A, and that the term ‘state non-A’ is not an individual term, but a general term that comprises all states different from A. We assume that propositional variables represent propositions about events, and that change consists in a transition from a given event to a definite event following it and different from it.

∃E),[31] (2) the specific axioms of the system "And Then", formulated as theorems of the system and (3) definitions (D1*), (D2*).

Theorems of n order are: (1) formulas for which there exists an indirect proof from assumptions employing the theorems of orders lower than n as well as rules of adding new lines to a proof, used in the natural deduction system of classical propositional calculus, and (2) formulas obtained from the theorems of orders lower than n by virtue of the rule of substitution and the rule of extensionality.[32]

All the axioms, and therefore all the theorems of the system **ZI**, may be proved in the calculus "And Then" with quantifiers binding propositional variables and definitions (D1*), (D2*). Here are the proofs of the axioms of the calculus **ZI**:

$$\mathsf{Z}\, p \to p \tag{A1}$$

PROOF. 1. $\mathsf{Z}\, p$ — assumption
2. $\exists q\; p\, \mathsf{Z}^\star\, q$ — 1, (D2*)
3. $p\, \mathsf{Z}^\star\, q_1$ — 2, ∃E
4. $p\, \mathsf{T}\, q_1$ — 3, (D1*)
5. p — 4, (T3)

$$p\, \mathsf{T} \neg p \to \mathsf{Z}\, p \tag{A2}$$

PROOF. 1. $p\, \mathsf{T} \neg p$ — assumption
2. $(p\, \mathsf{T} \neg p) \wedge (\neg p \to \neg p)$ — 1
3. $\exists q (p\, \mathsf{T}\, q \wedge (q \to \neg p))$ — 2, ∃I
4. $\exists q\; p\, \mathsf{Z}^\star\, q$ — 3, (D1*)
5. $\mathsf{Z}\, p$ — 4, (D2*)

$$\mathsf{Z}(p \wedge q) \to \mathsf{Z}\, p \vee \mathsf{Z}\, q \tag{A3}$$

[31] The symbols used in this sentence should be understood as follows: RD – the rule of detachment (modus ponens), AI – the rule of disjunction introduction, AE – the rule of disjunction elimination, KI – the rule of conjunction introduction, KE – the rule conjunction elimination, EI – equivalence introduction, EE – equivalence elimination, ∀I – the rule of introduction of universal quantifier, ∀E – the rule of elimination a universal quantifier, ∃I – the rule of existential quantifier introduction, ∃E – the rule of existential quantifier elimination.

[32] It should be added that the rule of substitution for theorems is here derivative from the rule ∀I and ∀E.

PROOF. 1. $\mathsf{Z}(p \wedge q)$ assumption

2. $\exists r((p \wedge q)\, \mathsf{Z}^\star\, r)$ 1, (D2*)

3. $(p \wedge q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg(p \wedge q))$ 2, (D1*), ∃E

4. $(p \wedge q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p \vee \neg q)$ 3, CPC

5. $(p \wedge q\, \mathsf{T}\, r_1) \wedge ((r_1 \to \neg p) \vee (r_1 \to \neg q))$ 4, CPC

6. $(p \wedge q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p) \vee (p \wedge q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg q)$ 5, CPC

7. $(p \wedge q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p) \to (p\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p)$ (T5), (T7)

8. $(p \wedge q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg q) \to (q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg q)$ (T5), (i)

9. $(p\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p) \vee (q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg q)$ 6, 7, 8, CPC

10. $p\, \mathsf{Z}^\star\, r_1 \vee q\, \mathsf{Z}^\star\, r_1$ 9, (D1*)

11. $\exists r(p\, \mathsf{Z}^\star\, r \vee q\, \mathsf{Z}^\star\, r)$ 10, ∃I

12. $\exists r\; p\, \mathsf{Z}^\star\, r \vee \exists r\; q\, \mathsf{Z}^\star\, r$ 11

13. $\mathsf{Z}\, p \vee \mathsf{Z}\, q$ 12, (D2*)

$$\mathsf{Z}\, p \wedge q \to \mathsf{Z}(p \wedge q) \tag{A4}$$

PROOF. 1. $\mathsf{Z}\, p$ assumption

2. q assumption

3. $\exists r\; p\, \mathsf{Z}^\star\, r$ 1, (D2*)

4. $p\, \mathsf{T}\, r_1 \wedge (r_1 \to \neg p)$ 3, (D1*), ∃E

5. $p \wedge q\, \mathsf{T}\, r_1 \wedge (r_1 \to \neg(p \wedge q))$ (T5), 2, 4, CPC

6. $\mathsf{Z}(p \wedge q)$ 5, (D1*), ∃I, (D2*)

$$\mathsf{Z}(p \vee q) \to \mathsf{Z}\, p \vee \mathsf{Z}\, q \tag{A5}$$

PROOF. 1. $\mathsf{Z}(p \vee q)$ assumption

2. $(p \vee q)\, \mathsf{Z}^\star\, r_1$ 1, (D2*), ∃E

3. $(p \vee q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p \wedge \neg q)$ 2, (D1*), CPC

4. $(p\, \mathsf{T}\, r_1 \vee q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p \wedge \neg q)$ 3, (g)

5. $p\, \mathsf{T}\, r_1 \wedge (r_1 \to \neg p \wedge \neg q) \vee (q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg p \wedge \neg q)$ 4, CPC

6. $p\, \mathsf{T}\, r_1 \wedge (r_1 \to \neg p) \vee (q\, \mathsf{T}\, r_1) \wedge (r_1 \to \neg q)$ 5, CPC

7. $\mathsf{Z}\, p \vee \mathsf{Z}\, q$ 5, (D1*), ∃I, (D2*)

$$\mathsf{Z}\, p \to \mathsf{Z}(p \vee q) \tag{A6}$$

PROOF. 1. $\mathsf{Z}\, p$ assumption

2. $\neg\, \mathsf{Z}(p \vee q)$ assumption of indirect proof

3. $\exists r\; p\, \mathsf{Z}^\star\, r$ 1, (D2*)

4. $p\, \mathsf{T}\, r_1 \wedge (r_1 \to \neg p)$ 3, ∃E, (D1*)

5.	$\neg\exists r\ (p \vee q)\ \mathsf{Z}^{\star}\ r$	2, (D2*)
6.	$\neg\exists r((p \vee q \mathsf{T} r) \wedge (r \to \neg(p \vee q)))$	5, (D1*)
7.	$\forall r((p \vee q \mathsf{T} r) \to r \wedge (p \vee q))$	6, QC, CPC
8.	$(p \vee q \mathsf{T} r_1) \to r_1 \wedge (p \vee q)$	7
9.	$(p \mathsf{T} r_1) \to (p \vee q \mathsf{T} r_1)$	(h)
10.	r_1	4, 9, 8, CPC
11.	$\neg p$	4, 10, CPC
12.	p	4, (T3)
13.	contradiction	11, 12

Therefore the calculus **ZI** is a subsystem the very general system **ZII**.

The consistency of the system **ZII** may be proved by interpreting the connective 'T' as a symbol of conjunction and the connectives '$\mathsf{Z}^{\star}$' and 'Z', respectively as a binary and unary falsum.

We have discussed systems of the logic of change which can provide a language suitable for consolidating, preserving and communicating knowledge concerning change in the natural sciences. Most attention was paid to the system **ZI**, which, as it treats change in a manner more fundamental, can have more applications. The axioms of the system of the logic of change **ZI** should be satisfied in the model of change assumed in the natural sciences. The connective 'Z' may be used, among others, in the formal characteristic of a corresponding causal implication. It is also worth adding that on the basis of the principles for evaluating the suitability of logics of change, outlined briefly in this paper, an answer can be given to the question whether existing systems of logic of change are adequate with respect to contemporary physics and the natural sciences which are based upon it.

The following would be a brief response to this question: L. S. Rogowski's system of directional logic, which is basically a system of the logic of change, possesses scientific value rather in connection with a certain type of philosophy than with natural science (cf. [30]). The same can be said of S. V. Šešić systems (cf. [39]). Some works of von Wright, that introduced the term 'the logic of change' are also devoted to the logic of change (cf. [40, 43]) Polish authors also refer to these works.[33] It should, however, be noted, that these works of the Finnish logician and the publications of his Polish followers are joined with the logic of actions and norms. In the polish literature, among others, in the

[33] Here are some works by Polish logicians on this subject: T. Kubiński [18, 19, 20], S. J. Rudziński [31], Z. Ziemba [45].

works of Kubiński we find essential formal development of the logic of change delineated by von Wright. However, it was not done in view of the descriptive semantics connected with the natural sciences. The logic of change was related to the domain of descriptive semantics concerned with human behaviors, with the logic of actions.[34] We should stress the formal virtuosity of methods of the formal semantics. It should also be added that A. Zinoviev's logic of change fulfills a scientific function, in relation to certain domains of physics, as a certain type of methodology for the natural sciences.[35]

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[34] It seems that Rudziński's in-passing suggestion to connect the logic of change for natural sciences with the context of the type 'factor x causes change T' does not consider sufficiently the ontology of the fundamental discipline of contemporary natural science

[35] I present a more extensive discussion of the best known systems of the logic of change and I evaluate these calculi from the point of view of their adequacy with respect to the natural sciences in my above cited book [12, pp. 197–220].

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