Logic and Logical Philosophy
Volume 21 (2012), 33–51
DOI: 10.12775/LLP.2012.003

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A MODAL LOGIC OF INFORMATION*

So much of what we find on the Web has no truth or justification, and one would have to be a fool to believe it, and it is not even clear that anyone would want to claim credit for thinking it.

J. Michael Dunn [2008, 581]

Abstract. We consider modal epistemic and doxastic logics as intuitively inadequate logics of information, and we outline a modal system of the operator *being informed that* which avoids inconsistency with our intuitive concept of information. The system has modal structure of the normal modal logic K4, and is sound and complete on the class of all transitive frames. We compare this logic with Floridi's KTB information logic, and we consider a possibility of extending our system to a dynamic logic.

Keywords: Information; Epistemic Logic; Doxastic Logic; Kripke Model; Completeness; Dynamic Logic.

1. Epistemic and doxastic logics as logics of information

My first fuller acquaintance with the concept of information goes back to the lecture *A Brief History of the Concept of Information* which Professor J. Michael Dunn delivered to the Polish Society of Logic and

* The main ideas of this paper were presented at The Third Conference: *Non-Classical Logic. Theory and Applications*, NCU, Toruń, September 16–18, 2010. I thank Andrzej Indrzejczak for his comments. But first of all my thanks go to the anonymous Referee for his valuable, detailed, and enormously helpful suggestions to earlier versions of this paper. All the suggestions have been implemented by me with great pleasure contributing the beauty to my raw project.

Received September 20, 2010; *Revised* October 20, 2011

 ISSN: 1425-3305

Philosophy of Science in Warsaw several years ago. I had the opportunity to attend that lecture. Professor Dunn described then among many conceptions of information, two complementary conceptions, that of Carnap and Bar-Hillel on the one hand, and Shannon's conception, on the other. It took me a couple of years to formulate my own view on information. It is not a surprise that my conception of information rests on the same intuitive understanding of information which may be found in Dunn [2008], where the information is understood as the content of a sentence which does not need to be true, or justified, or even believed by someone. Here is how one may argue for this conception of information:

> I think it is part of the pragmatics of the word "information" that when one asks for information, one expects to get true information, but it is not part of the semantics, the literal meaning of the term.
>
> [Dunn 2008, p. 582]

This account of information makes information different from knowledge which traditionally is understood as justified true belief. As I argued elsewhere, these three features of knowledge do not form together the sufficient condition of knowledge, and a further necessary condition of knowledge should be added to solve the so called Gettier Problem (see [Misiuna 2010]). Gettier in [Gettier 1963] convincingly argues against the traditional definition of knowledge. His problem is connected with the fact that we are not willing to attribute knowledge to someone whose justified belief is true, but who is unaware of its truth, because the belief turns out to be true by luck or accident. This can happen when one does not have an opportunity to recognize the truth of a proposition based on evidence. Here is one of the Gettier examples. If Smith says "Either Jones owns a Ford, or Brown is in Barcelona", and the disjunction is true, because accidentally Smith says something true about Brown, not being aware of the fact that *Brown is in Barcelona* is a true proposition, then we do not want to say that Smith knows that either Jones owns a Ford, or Brown is in Barcelona, although it is Smith's justified true belief. In this example Smith's justification is his strong evidence for his false belief *that Jones owns a Ford* which has as its logical consequence the proposition *that either Jones owns a Ford, or Brown is in Barcelona*. If logical consequences of justified beliefs are justified, we get justified true belief which is not an instance of knowledge. I argued that to count belief as an instance of knowledge, besides the belief being true and justified, we should have the subject's proper recognition of what makes his/her belief true. This recognition of truth is not the same as the justification of our beliefs. We may have good evidence for justifying our false beliefs, so it happens that the evidence which serves well justification does not serve well recognition of truth.

Thus, on our view, the set of instances of knowledge and the set of instances of information overlap, but do not coincide. Such a relation between knowledge and information seems to be the closest to the philosophical as well as commonsense ideas concerning these two notions. This account contrasts with another widespread account of knowledge and information which goes back to J. Hintikka [1962] where the notion of knowledge and information are used interchangeably, and epistemic and doxastic logics are regarded as theories of information. The recent valuable works concerning dynamic logic by Johan van Benthem have the same philosophical background.

In the classical work by J. Hintikka [1962] as well as in the recent monograph by J. van Benthem and M. Martinez [2008], where knowledge and information are extensionally the same, knowledge is paraphrased as "the best of agent's information". Modal epistemic logic, regarded as a logic of information, is a complete system built over a complete system of propositional logic augmented with the modal operators forming the following formulas: $\mathrm{K}_i\,\varphi$ whose intended meaning is: "i knows that φ", while the intended meaning of $\neg\,\mathrm{K}_i\,\neg\varphi$ is: "i considers φ possible". Kripke models for this epistemic language are relational structures consisting of three sets: a set of possible worlds (or states, or situations; we shall make use of the letters: s, t, w for denoting them): W; binary relations between worlds which may be different for different agents: R_i; the valuation V is a function from the set of atomic propositions P into the set of all subsets of W, that is, $V\colon P \to \mathcal{P}(W)$ and extends to compound formulas in the following way:

- $V(p, s) = 1$ iff $s \in V(p)$;
- $V(\neg\varphi, s) = 1$ iff $V(\varphi, s) = 0$;
- $V(\varphi \wedge \psi) = 1$ iff $V(\varphi, s) = 1$ and $V(\psi, s) = 1$;
- $V(\mathrm{K}_i\,\varphi, s) = 1$ iff for every $t \in W$ such that $s\,R_i\,t$, $V(\varphi, t) = 1$.

Worlds or states in these models represent alternative states considered by agents. Thus, $x\,R_i\,y$ may be paraphrased as "in the state (world) x, y is an i-alternative state to x compatible with everything i knows in x." J. Hintikka [1962] defines this relation as a reflexive transitive ordering, that is, a quasi-ordering or a pre-ordering. It is assumed that these relations may be different for different agents who differ as to their information. Formal features of these accessibility relations are important for the epistemic principles which are valid on frames with the appropriate relation. Reflexivity of the accessibility relation is a guarantee that the Principle of Veridicality, which is the epistemic version of the axiom T in the modal system T: $\mathrm{K}\,p \rightarrow p$, is valid on frames with such an accessibility relation. The feature of transitivity guarantees the validity of the Principle of Positive Introspection being an epistemic version of the axiom **4** in the modal system S4: $\mathrm{K}\,p \rightarrow \mathrm{K}\,\mathrm{K}\,p$.

The idea behind the truth-condition for $\mathrm{K}_i\,\varphi$ is that knowledge of φ as the best of agent's information takes place in the state s if and only if φ is true in all i-alternative states which are compatible with everything i knows in s. To give an example, I know in the present state that *Tarski's monograph about the concept of truth was published in 1933* if and only if "Tarski's monograph about the concept of truth was published in 1933" is true in all my alternative states compatible with everything I know in the present state.

As a certain logic of information can serve also doxastic logic having the operator $\mathrm{B}_i\,\varphi$ with the intended meaning: "agent i believes that φ", occurring in the place of the knowledge operator. The truth-condition of the sentence with the belief operator takes the following form: $V(\mathrm{B}_i\,\varphi, s) = 1$ if and only if for every t such that $s\,R_i\,t$, $V(\varphi, t) = 1$.

This condition expresses the idea that our belief is true if the proposition which we believe is true in the alternatives that are most plausible for us. I believe that he will come is true if the proposition that he will come is true in the alternatives which are most plausible for me. If plausibility is understood as subjective probability, as a degree of agent's expectation, the condition above is an account of the common view that our beliefs can be false. This feature of beliefs has some consequences for our understanding of the accessibility relation in Kripke models: Such relation cannot be reflexive, because reflexivity of the accessibility relation assures the validity of the epistemic Principle of Veridicality. The principle is valid on frames whose accessibility relation is reflexive. We shall return to the principles of modal epistemic and doxastic logic below.

2. Intuitive account of information

Claude Shannon's communication theory [1948] is one of the classical information theories. The proper object of his study is the *communication channel* between source and receiver. He posed the question which communication channel is reliable. Shannon suggested that there is an objective measure of information in a message which goes from the source to the receiver. Shannon's theory identifies the amount of information associated with the occurrence of an event with the reduction of uncertainty. Its measure takes into account the inverse of probability, strictly speaking, log to the base 2 of the inverse of the probability of the event. In consequence, the more surprising the event at a source is, the more information it conveys. For example, if we have 8 equally likely possibilities, and we eliminate these possibilities by choosing one of them, then that event has the probability equal 1/8. Thus, Shannon's theory gives us the quantity 3 bits as the measure of the amount of information generated by our choice, because

$$\mathrm{Info}(E) = \log_2 \frac{1}{\mathrm{Prob}(E)} = \log_2 \frac{1}{\frac{1}{8}} = \log_2 2^3 = 3\,\mathrm{bits}.$$

Note that if we have 16 equally likely possibilities, and we eliminate these possibilities by choosing one of them, our event consisting in choosing one possibility has the probability equal 1/16, and in consequence we obtain 4 bits information generated by our choice, since $\log_2 \frac{1}{\frac{1}{16}} = \log_2 2^4 = 4$.

The information generated need not be the information transmitted, and Shannon's communication theory makes a quantitative difference between them. This difference is a measure of the reliability of the communication channel connecting source and receiver. The information transmitted is equal to the information generated minus the amount of the statistical independence between events occurring at the source and at the receiver. If these events are absolutely statistically independent then the information transmitted equals zero. We obtain this from the following definition of transmitted information:

$$\mathrm{Info}_t(E) = \mathrm{Info}_g(E) - Eq,$$

where Eq (equivocation) is a measure of the statistical independence between events occurring at the source and the receiver. If we have a trustworthy communication channel, the amount of equivocation is equal to zero, and in consequence all the information generated at the source reaches its destination (the case of maximum communication). But it may happen that $Eq > 0$. If in our example $Eq = 3$ bits, then $\mathrm{Info}_t(E) = 0$. In such a case we have zero communication.

Fred Dretske [2008] makes use of Shannon's communication theory in epistemology. The concept of communication channel may be applied to epistemology if one understands the receiver as knower, while the source of information as known. The question then naturally arises which condition should be satisfied so that the events at the source were *known* to the knower. According to Dretske, only if the amount of information transmitted is the same as that of generated, one can *know* the event at the source.[1] Dretske stresses that any empirical fact can be known only in the case when equivocation equals zero. If we assume that no communication channel is entirely free of equivocation, on this account of knowledge, we must also accept that the information is never communicated, and in consequence that nothing is known. Such an account of knowledge leads immediately to skepticism which is self-defeating for the very account, since on the ground of that conception of knowledge, we cannot explain how we know that $Eq = 0$ is impossible. Dretske mentions briefly how one may avoid skepticism. He suggests a way out consisting in accepting a "more realistic", as he calls it, interpretation of conditional probabilities defining equivocation where the probabilities of background circumstances are ignored in computing equivocation. Dretske [2008, p. 46] illustrates this strategy by an example: Although there is a non-zero probability that there were hallucinatory drugs in my morning coffee, my perception delivers the information needed to know that what I see in the grocery are bananas. This understanding of knowledge may lead to knowledge meant as objective or subjective, depending on our understanding of information, and in particular, on the concept of probability involved in its definition. If the probability is understood objectively as frequency, the information and knowledge based on it are also understood objectively; if we have to do with the subjective interpretation of probability as a degree of our expectation, the information and knowledge are understood subjectively too. In Dretske [1981], an objective interpretation of knowledge is connected with a causal theory

[1] Dretske [2008, p. 44] has written: "In order to know what happened at s you have to receive as much information [...] about s as is generated by the event you believe to have occurred there."

of knowledge: an instance of a belief is an instance of knowledge only if the belief stands in a causal relation to the facts.

The striking feature of Dretske's concept of information is that information is true by definition. That understanding of information makes possible the close relationship between information and knowledge: We seek information to obtain knowledge. Dretske writes:

> If nothing you are told is true, you may leave an information booth with a lot of false beliefs, but you won't leave with knowledge. You won't leave with knowledge because you haven't been given what you need to know: information. [Dretske 2008, p. 30]

Information for Dretske is a necessary condition of knowledge, because he understands instances of information as true contents. This view is opposite to the one mentioned before where information need not be a true proposition. Perhaps the two views are a further consequence of the Carnap-Bar-Hillel distinction between information and content, where the two concepts are regarded as duals. The information is understood as the set of state descriptions, which are conjunctions of atomic sentences and their negations, that make a proposition true, while the content — as the set of state descriptions that make a proposition false. In consequence, the following equations hold for the information and content in the Carnap-Bar-Hillel sense (cf. [Bar-Hillel 1953–55]):

- $\mathrm{Info}(A \wedge B) = \mathrm{Info}(A) \cap \mathrm{Info}(B)$;
- $\mathrm{Info}(A \vee B) = \mathrm{Info}(A) \cup \mathrm{Info}(B)$;
- $\mathrm{Cont}(A \wedge B) = \mathrm{Cont}(A) \cup \mathrm{Cont}(B)$;
- $\mathrm{Cont}(A \vee B) = \mathrm{Cont}(A) \cap \mathrm{Cont}(B)$.

The relation between the information as knowledge and the information as meaningful content needs a further study. At present, we shall assume that the set of instances of information and the set of instances of knowledge overlap, but are not the same. There are instances of knowledge which are instances of information, but there are also instances of knowledge which are not information and instances of information which are not knowledge. I shall only mention some instances of knowledge which are not instances of information, since the other are much less problematic. I include to the set of instances of knowledge *logically true* propositions, but I do not include them in the set of instances of information. Besides, I include among the instances of knowledge the *introspection* propositions, such as “I know that I do not know what the time is now” which do not belong to the instances of information.

The accessibility relations of frames with respect to which the principles of epistemic logic are valid are given in Table 1 along with the appropriate principle. The Principle of Knowledge Distribution is valid on every frame.

Epistemic Principle	Property of R_i	Name
$\mathrm{K}(p \to q) \to (\mathrm{K}\,p \to \mathrm{K}\,q)$	—	Knowledge Distribution
$\mathrm{K}\,p \to p$	Reflexivity	Veridicality
$\mathrm{K}\,p \to \mathrm{K}\,\mathrm{K}\,p$	Transitivity	Positive Introspection
$\neg\,\mathrm{K}\,p \to \mathrm{K}\,\neg\,\mathrm{K}\,p$	Euclidean Property	Negative Introspection

Table 1. Epistemic principles and their accessibility relations

The epistemic principles listed in the table 1 define the modal system S5, where ‘K’ is meant as a modal operator like ‘$\Box$’. A similar table will list the principles of doxastic logic with the modal operator B, and the formal features of respective accessibility relations of frames on which the principles are valid. The principles listed in the table 2 define the modal system KD45.

Doxastic Principle	Property of R_i	Name
$\mathrm{B}(p \to q) \to (\mathrm{B}\,p \to \mathrm{B}\,q)$	—	Belief Distribution
$\mathrm{B}\,p \to \neg\,\mathrm{B}\,\neg p$	Seriality	Consistency
$\mathrm{B}\,p \to \mathrm{B}\,\mathrm{B}\,p$	Transitivity	Positive Introspection
$\neg\,\mathrm{B}\,p \to \mathrm{B}\,\neg\,\mathrm{B}\,p$	Euclidean Property	Negative Introspection

Table 2. Doxastic principles and their accessibility relations

The question arises if the respective systems of knowledge and belief are acceptable as intuitively adequate systems of information. The system of doxastic logic seems closer to the intuitively adequate system of information than the system of epistemic logic, because of the lack of the Principle of Veridicality. As we have noted earlier, information in our meaning does not need to be true. And indeed how often it is not true!

Does it mean that the doxastic principles which have been shown in Table 2 should be accepted as the principles of information theory? The answer should be in the negative, because intuitively the Principle of Consistency should not be applied to the concept of information. It may happen, and indeed it happens that we have inconsistent information when we *have been told* by the persons A that φ, and by the person B that $\neg\varphi$. Besides, the Principle of Positive Introspection needs a certain comment when applied to information. It is not contentious if it describes the received information of conscious agents, but may be contentious when applied to the information flow in other systems, for example, to the flow of genetic information, and when the received information is understood as what has been told by some means.

Accepting the Principle of Negative Introspection is also unfortunate for the logic of information. The point is that if we do not have the information that φ then in many cases we do *not* have the information that we do *not* have the information that $\neg\varphi$. The Principle of Negative Introspection has as its substitutions the following examples:

- If I do not have the information that there were any birds in this garden last night at midnight, then I have the information that I do not have the information that there were any birds in this garden last night at midnight.
- If I do not have the information that I have abnormality of red blood cells, then I have the information that I do not have the information that I have abnormality of red blood cells.

In fact, we are usually in such circumstances that if we do not have the information that we have abnormality of red blood cells, then we do *not* have the information that we do not have such information. Although one may imagine circumstances in which we have been told that we do not have the information about abnormality of our blood cells. This may happen when our doctor tells us that we need to make the proper test of our blood. But the first example above indicates that we are notoriously in such circumstances that if we do not have a certain kind of information, then we do *not* have the information that we do not have that kind of information. There are indefinitely many things of which we have not been told, and we remain unaware that we have not been told of them. Instead of the Principle of Negative Introspection which applies to our reasoning about knowledge well, in the case of information, for some φ holds the following implication: $\neg\,\mathrm{I}\,\varphi \to \neg\,\mathrm{I}\,\neg\,\mathrm{I}\,\varphi$, where 'I' is the operator of *being informed* or *being aware*. The lack of knowledge is not the same as the lack of information understood as being unaware of something. There is a certain cognitive relation between the subject and the lack of his/her knowledge, while there is no such a relation in the case of information. If we do not *know* how our friend is, because we have not got any message from him recently, we indeed *know* that we do not *know* how our friend is. But if we do not have the information if any birds were in this garden last night at midnight, we do *not* have the information that we do *not* have this information. In the case of information, no cognitive relation holds between the subject and the lacking information. The information about our lack of information may be given to us when we have been told by some means, but it is not implied by our lack of information. A consequence of this is not only the failure of the Principle of the Negative Introspection in the case of empirical information meant as being informed. In the case of being informed (or being aware) a new "principle" holds: If the subject has *not* been informed (has not been aware) that it is so and so, then he/she has not been informed (has not been aware) that it is *not* so and so, or he/she has been aware that it is not so and so, because he/she has been told that it is not so and so, what may be rendered this way: $\neg \mathrm{I}\,\varphi \rightarrow (\neg \mathrm{I} \neg\varphi \vee \mathrm{I} \neg\varphi)$. The point of this "principle" is that we live and act with the incomplete information, although in particular circumstances it may happen that our lack of information that it is so and so implies our being aware that it is not so and so as a result of that we have been told that it is not so and so. The disjunction is illustrated by the two following sentences, which are classically equivalent to the disjunction in the principle.

- If I am not informed (I am unaware) that there were any birds in this garden last night at midnight, then I am not informed (I am unaware) that there was no bird in this garden last night at midnight.
- If, having my blood test, I am not informed (I am unaware) that I have abnormality of red blood cells, then I am informed (I am aware) that I do not have abnormality of red blood cells, because I have been told about my blood test.

It should be noted that our new "principle" concerning the concept of being informed that formalized by the operator 'I' has been considered as a concept applying to empirical data. In the case of empirical data, if I am not informed that something has happened, sometimes I have been told that it has not happened. What could we say when *being informed that* applies to conclusions of our reasoning? It is true that rational agents are not aware of logical consequences obtained by long sequences of inferences. It seems that in such cases it is more likely that if we are not informed that φ is a consequence, we are also not informed that $\neg\varphi$ is a consequence. In the case of empirical data, when we have been told about something, we have been told by rational agents, by animals, by artifacts, by computers, or by our sense experiences. If I am not informed that I have a headache at this moment, I am told by my introspection that I do not have a headache at this moment. If I am not informed of the first sentence of Shakespeare's play *Much Ado About Nothing*, I am told this by the content of my visual experience when looking at the published Shakespeare's work. If I am not informed that the English word *strengthening* is pronounced so and so, I am told this by the content of my hearing experience when listening to a native English speaker.

3. The system Σ(K4)

Our logic of information Σ is formed by the principle of Information Distribution, and the principle of Positive Introspection. The language of Σ is formed by propositional variables, logical connectives: $\neg$ and $\wedge$ (with the other logical connectives being defined in terms of those two in the usual way), monadic modal operators: I_i (with their duals being defined in the usual way) and auxiliary symbols. We have in Σ the rules of inference occurring in the modal system K, that is, the Rule of Uniform Substitution, the Rule of *Modus Ponens*, and the counterpart of the Rule of Necessitation (Generalization of Knowledge or Generalization of Information, respectively). Thus, we obtain

Information Principle	Property of R_i	Name
$\mathrm{I}(p \to q) \to (\mathrm{I}\,p \to \mathrm{I}\,q)$	—	Information Distribution
$\mathrm{I}\,p \to \mathrm{I}\,\mathrm{I}\,p$	Transitivity	Positive Introspection

Table 3. The principles of the system Σ and their accessibility relations

Table 3 contains only two modal principles of information, K and 4.These principles are analogues of axioms of modal epistemic and doxastic logics. The specific "principle" $\neg\,\mathrm{I}\,\varphi \to (\neg\,\mathrm{I}\,\neg\varphi \vee \mathrm{I}\,\neg\varphi)$, having the logical form of a classical propositional tautology, is a theorem of the system K. We shall denote this theorem by $\boldsymbol{\pi}$, and we shall call it the theorem of Possibility of Incompleteness. The name of this theorem expresses its meaning which tells us that if we are not informed that p, we are not informed that it is not the case that p, or we are informed that it is not the case that p. Human beings have finite capacities, but reality has infinitely many properties. Thus, we are notoriously in such circumstances in which we are not informed completely about reality.

Our aim is to formalize the logic of discourse about information, then we must articulate the principles of reasoning about information attributions. Our principles for the operator of being informed hold for any rational agent, as well as for any attributor of information. Also the principles are designed to cover the objective flow of information. Thus, the logic of discourse about information is exactly the same as the logic of being informed and the logic of objective flow of information. The theorem of Possibility of Incompleteness tells us that the lack of cognitive relationship between the subject and the contingent proposition implies the lack of cognitive relationship between this subject and the negation of this proposition or the relation of being informed about this negation. But we must stress that being informed is not the same cognitive relationship as having knowledge. For example, justification is not a constitutive property of being informed. Although the instances of information and the instances of knowledge overlap, the property of being informed and the property of having knowledge are distinct. But first, we need to formulate the truth condition for formulas with the monadic information operator 'I' whose intended meaning is: *being informed*.

Definition 3.1. Let (W, R, V) be a model, where V is a function of valuation. Then for any $s \in W$ and any formulae φ, ψ:

(P) either $V(\varphi, s) = 1$ or $V(\varphi, s) = 0$, if $\varphi \in P$;
($\neg$) $V(\neg\varphi, s) = 1$ iff $V(\varphi, s) = 0$;
($\wedge$) $V(\varphi \wedge \psi, s) = 1$ iff $V(\varphi, s) = 1$ and $V(\psi, s) = 1$;
(I) $V(I_i\varphi, s) = 1$ iff for all t with $s\, R_i\, t$, $V(\varphi, t) = 1$.

Our possible worlds are states. Thus, the condition (I) says that being informed that φ is (considered as) true in the state s if and only if in all states which are compatible with agent's information in the state s, φ is (considered as) true. We shall make use of the above definition to prove that the theorem of Possible Incompleteness is valid on every frame, that is, $V(\boldsymbol{\pi}, s) = 1$ in any model (W, R, V) such that (W, R) is any frame, and s is a member of W.

Proposition 3.2. *The theorem* $\boldsymbol{\pi}$: $\neg\, \mathrm{I}\, p \to \neg\, \mathrm{I} \neg p \vee \mathrm{I} \neg p$ *is valid on every frame.*

PROOF. The proof is straightforward. Let (W, R, V) be any Kripke model based on any frame. Suppose that there is a state s such that $V(\neg \mathrm{I}\, p \to \neg \mathrm{I} \neg p \vee \mathrm{I} \neg p, s) = 0$. Then $V(\neg \mathrm{I}\, p, s) = 1$, but $V(\neg I \neg p \vee \mathrm{I} \neg p, s) = 0$. This is the case if $V(\neg \mathrm{I} \neg p, s) = 0$ and $V(\mathrm{I} \neg p, s) = 0$. But the latter condition implies $V(\mathrm{I} \neg p, s) = 1$ and $V(I \neg p, s) = 0$. Contradiction! ⊣

The correspondence between the principle 4 with the operator of necessity and transitivity of the accessibility relation is well known. Since the operator 'I' in our principle 4: $\mathrm{I}\, p \to \mathrm{I}\,\mathrm{I}\, p$ is interpreted exactly as the box operator, therefore, the correspondence holds in Σ too.

Since the principle of Positive Introspection 4 is valid on every transitive frame, and the principle K is valid on every frame, then the system Σ is valid on every transitive frame.[2] Hence Σ is *sound* with respect to the class of all transitive frames.

The system Σ as a system of modal logic is an *extension* of the system K, then it belongs to the class of systems which are familiar *normal* systems. This means that our system is a class of modal formulas which contains all propositional calculus valid formulas, and the principle K with the rules of inference such as *Modus Ponens*, Uniform Substitution, and Necessitation. If it is so then there is the canonical model of Σ which has the property that a formula is valid in the canonical model of Σ if and only if this formula is a theorem of Σ. In canonical models states or worlds are sets of formulas what enables us to think of a formula as true in a state if and only if that formula is in the set of formulas which constitutes that state. To say that the system Σ is *complete* with respect to a class of frames is to say that every formula valid on this class of frames, that is, valid on every frame in that class, is a theorem of Σ. Since not all modal logics are complete, even if they are normal, we shall prove the following proposition.

PROPOSITION 3.3. *Σ is complete with respect to the class of all transitive frames.*

PROOF. To prove that Σ is complete with respect to the class of all transitive frames we must prove that the canonical model of Σ is based

[2] We make use here of the following metatheorem: If S is any set of modal formulas and (W, R) is a frame on which each formula in S is valid, then every theorem of K+S obtained by the rules *Modus Ponens*, Uniform Substitution and Necessitation is valid on (W, R).

on a frame which is transitive. To prove transitivity, suppose that wRw' and $w'Rw''$. To show that wRw'' we must show that for any formula φ if $\mathrm{I}\,\varphi \in w$ then $\varphi \in w''$. Since $\mathrm{I}\,\varphi \to \Pi$, φ is a theorem of Σ, and if $\mathrm{I}\,\varphi \in w$, then $\mathrm{I}\,\mathrm{I}\,\varphi \in w$, and since wRw', then $\mathrm{I}\,\varphi \in w'$. And since $w'Rw''$, then $\varphi \in w''$. ⊣

4. Comparison with the system IL(KTB) of Luciano Floridi

After the Section 3 of the present paper had been completed, I had the opportunity to read the article on the logic of information by Floridi [2006]. Floridi's aim is very close to the aim of the present article, that is, an intuitively satisfactory formal system of the logic of the operator *being informed*. I realized that Floridi like me is dissatisfied with attempts of making use of an epistemic or doxastic logic for formalizing the relation of being informed, and that he argues that being informed has properties which in order to be modeled adequately require a logic different from any epistemic or doxastic logic. However, Floridi's logic is definitely different than the logic defined by the system Σ. It turns out that Floridi has different underlying intuitions concerning the concept of information, and this fact is decisive for his choice of axioms of his system. The modal axioms of Floridi's system IL has been displayed in the table 4 below. They have been added by Floridi to propositional axioms based on the classical connectives: '¬' and '→' which have been omitted in Table 4.

Axiom of IL	Name
$\mathrm{I}(p \to q) \to (\mathrm{I}\,p \to \mathrm{I}\,q)$	Information Distribution Axiom
$\mathrm{I}\,p \to p$	The Axiom of Veridicality
$p \to \mathrm{I}\,\neg\,\mathrm{I}\,\neg p$	Brouwerian Axiom
$\mathrm{I}\,p \to \neg\,\mathrm{I}\,\neg p$	The Axiom of Consistency
$\mathrm{I}(p \to q) \to (\mathrm{I}(q \to r) \to \mathrm{I}(p \to r))$	The Axiom of Single Agent Transmission
$\mathrm{I}_x\,\mathrm{I}_y\,p \to \mathrm{I}_x\,p$	Hintikka's Axiom of Multiagent Transmission

Table 4. Modal axioms satisfied by Floridi's system IL

Floridi calls his system a KTB-based information logic, and claims that this normal modal logic, known also as B or Brouwer's system, is well suited to formalize the relation of being informed, and that his system IL is an "informational reading" of the KTB system. Since Floridi considers artificial and biological agents in a large sense, 4 is excluded from his system, while this axiom is present in Σ, because the intuitive meaning of being informed, which is formalized by Σ, is synonymous with being aware. For similar reasons, also 5 is not in Floridi's system, because his intuitive conception of information is based on the following observation:

> A dog is informed (holds the information) that a stranger is actually approaching the house, yet this does not imply that the dog is (or can even ever be) informed that he is informed that a stranger is approaching the house. [Floridi 2006, p. 444]

Floridi like Dretske is convinced that false information is not a kind of information, because information for him is true by definition. For that reason, among the axioms of his system there is the Axiom of Veridicality, which is lacking in the system Σ. On this understanding, veridicality, like in the case of knowledge, is a necessary condition of information. So in consequence, if the agent A is informed that p, then p is true. That account of information is contentious. Convincing arguments against Floridi's conception of information may be found in Fetzer [2004]. Among the critics of Floridi, one may also include Dunn whose standpoint has been quoted above. We agree with the critics, because our view is that there are two kinds of the instances of information: one kind is such that the instances of information are identical with the instances of knowledge, and the other kind when the instances of information are not instances of knowledge. Only in the first case veridicality could be a property of information.

As far as the Axiom of Consistency is concerned, this axiom follows from the Axiom of Veridicality, and it may be considered as redundant in IL. This formula is not a theorem of the system Σ. Floridi accepts its normative interpretation. This is the interpretation which this axiom has in doxastic logics: If the agent A has the information that the train leaves at 10.30 am., then the agent A should not have the information that the same train does not leave at 10.30 am. It is a constraint concerning the concept of information that agents with inconsistent information are not taken into account.

As we have already mentioned, Floridi does not include to his system IL the axiom of Positive Introspection 4 which is in the system Σ. Our possible worlds semantics confines the concept of information to the conscious beings. Floridi rightly ascribes information also to animals (for example dogs) and unconscious objects. However, including the Brouwerian Axiom to IL makes the applicability of his axioms confined to creatures endowed with minds, because the Brouwerian Axiom assumes the ability of introspection.

The two last axioms in Table 4 do not belong to the system Σ. They describe the transmission of information, and one may add to our system the Axiom of Single Agent Transmission without inconsistency, although this axiom is a consequence of the Axiom of Information Distribution, since classical logic and the necessitation rule are included in our system. Thus it may be omitted as redundant. On the other hand, the Axiom of Multiagent Transmission is not intuitively adequate formalization of the operator of being informed which is synonymous with being aware. To illustrate that it is the case, consider the following example. Suppose that I am informed (I am aware) that you have been informed that P = NP.[3] Does this implies that I am informed (I am aware) that P = NP? It seems that it is not so. At most, I am informed that you are informed that P = NP. Being informed is dependent on the current state of mind of a given agent, and the current state of minds of different agents differ with respect to their knowledge, past experiences, and interests.

5. The system Σ extended to a dynamic logic

The concept of being informed is closely related to the concepts of communication and information flow whose formal development may be found in dynamic logic. I consider also a possibility of the extension of the system Σ to a dynamic logic.

In everyday life, we have to do with the phenomenon of communication and information flow when we ask questions and receive answers, as in the following conversational scenario:

- A asks the question 'ψ?',
- B gives the true answer 'Yes'.

This example is a simple example of the familiar phenomenon of common knowledge that ψ, because the two agents A and B know that ψ if the true answer is given by the agent B. We may generalize this phenomenon including the case where B gives an answer which may be false. Assuming that both uses are taken into account, we shall call the phenomenon

[3] I mention the most famous problem of the theory of computational complexity.

exemplified in our scenario *common information*. Now, let us return to the agent B who provides A with information. In this respect, the act of B is an example of the fact ψ which will be denoted by $!\psi$.

We may also generalize the notion of public announcement to the case where the statement ψ is false. Every public announcement potentially changes in some way my state of information. We shall describe this formally by saying that the model (M, s) with the actual world s changes into its sub-model $(M \mid \psi, s)$ whose domain is a new set: either restricted to those worlds where the statement ψ is true, or restricted to those worlds where the statement ψ is false. In this way, we enter into a certain public announcement logic with a properly extended language of our system Σ. Thus, we have now the formulas as below: $[!\psi]\varphi$, with the following intended meaning: *after ψ has been announced, φ is true at the current world*. Let us imagine three players: A, B and C. Let each of them have one card, respectively, from the suits hearts, clubs, and spades. Each of them is informed that they have at their disposal those three cards, but none of them is informed which cards have the other two. Next, B makes the true announcement that she does not have hearts. Thus, after B's announcement that she does not have hearts, C is informed that B has clubs. Also, after B's announcement that she does not have hearts, C is informed that A has hearts. Let us now imagine that B makes the false announcement that she has hearts. Then, in the simplest case, after B's announcement that she has hearts, A is informed that she (that is: A) has hearts.[4] In the case of atomic proposition p, the following equivalence holds: $[!\psi]p \equiv (\psi \to p)$. The effect of a public announcement by B that she does not have hearts is the restriction of the epistemic state to all worlds where the proposition that B does not have hearts is true, or the restriction of the epistemic state to those worlds where the proposition that B has harts is true. The truth-conditions of the formulas $[!\psi]\varphi$ take the following form[5]:

Definition 5.1. Let (W, R, V) be any model, and let (W', R', V') be the restriction of the model (W, R, V) by the condition that ψ is true in (W, R, V), and let (W'', R'', V'') be the restriction of the model (W, R, V) by the condition that $\neg\psi$ is true in (W, R, V), that is, that ψ is false in (W, R, V). Then for any formula $[!\psi]\varphi$ the following conditions hold:

[4] Something like that can happen if A disbelieves or mistrusts B.

[5] Cf. [van Benthem 2009, p. 135] who considers only the case where $V(\psi, s) = 1$.

- $V([!\psi]\varphi, s) = 1$ iff $V(\psi, s) = 1$ implies $V'(\varphi, s) = 1$,
- $V([!\psi]\varphi, s) = 1$ iff $V(\psi, s) = 0$ implies $V''(\varphi, s) = 1$,
- $V([!\psi]\varphi, s) = 0$ iff $V(\psi, s) = 1$ implies $V'(\varphi, s) = 0$,
- $V([!\psi]\varphi, s) = 0$ iff $V(\psi, s) = 0$ implies $V''(\varphi, s) = 0$.

The first condition says that after ψ has been announced, φ is true at the world (mental state) s if ψ is true at s then φ is *true* at s in the model restricted by the condition that ψ is true in it. This is not the only possibility when $[!\psi]\varphi$ is true at s. The other possibility is expresses by our second condition. The condition says that after ψ has been announced, φ is true at the world (mental state) s if ψ is false at s, then φ is *true* at s in the model restricted by the condition that ψ is false in it. This truth condition holds in the case of false announcement. The last two conditions say when $[!\psi]\varphi$ is false at the state s. One case is when ψ is true at s, then φ is *false* at s in the model restricted by the condition that ψ is true in it. The other case is when ψ is false at s, then φ is *false* at s in the model restricted by the condition that ψ is false in it.

6. Conclusions

Modal epistemic and doxastic logics are based on certain assumptions which seem unintuitive when applied to such a concept of information which we use in contemporary everyday life. This is the concept of information which has been succinctly characterized by the motto of this article. Its meaning is then in the opposition to the concept of knowledge meant as true, justified belief, satisfying also some other conditions which enable us to avoid the Gettier Problem. We argue for such a modal logic of the operator *being informed* which respects our intuitive understanding of information. Our system Σ has been designed in a way which answers these needs. What is characteristic of the system Σ is its theorem called the theorem of Possible Incompleteness. The proof of completeness of the system Σ shows that the system is complete with respect the class of all transitive frames. From a formal point of view then, our system is elegant enough, and on the other hand, the system Σ is more realistic, because it incorporates the essential features of the concept of being informed applied to human agents. The system may be extended to a dynamic logic if we drop the assumption that announcements are always true.

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