Logic and Logical Philosophy
Volume 15 (2006), 17–37
DOI: 10.12775/LLP.2006.002

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A LATTICE FOR THE LANGUAGE OF ARISTOTLE'S SYLLOGISTIC AND A LATTICE FOR THE LANGUAGE OF VASIĽÉV'S SYLLOGISTIC

Abstract. In this paper an algebraic system of the new type is proposed (namely, a vectorial lattice). This algebraic system is a lattice for the language of Aristotle's syllogistic and as well as a lattice for the language of Vasiľév's syllogistic. A lattice for the language of Aristotle's syllogistic is called a vectorial lattice on $\cap$-semilattice and a lattice for the language of Vasiľév's syllogistic is called a vectorial lattice on closure $\cap$-semilattice. These constructions are introduced for the first time.

Keywords: Aristotle's syllogistic, Vasiľév's syllogistic, vectorial lattice on $\cap$-semilattice, vectorial lattice on closure $\cap$-semilattice, quartum non datur.

Set up the problem of construction a lattice for the language of Aristotle's syllogistic and as well as a lattice for the language of Vasiľév's syllogistic.

The Aristotle syllogistic (see [15], [1], [7], [8], [14]) is based on propositional logic.

Definition 1. The alphabet of propositional logic is the ordered system $\mathcal{A} = \langle V, L_1, L_2, K\rangle$, where

1. V is the set of propositional variables p, q, r, ...;
2. L_1 is the set of unary propositional connectives consisting of one element $\neg$ called the symbol of negation;

Received November 17, 2005; *Revised* February 7, 2006

ISSN: 1425-3305

3. L_2 is the set of binary propositional connectives consisting of three elements: $\wedge$, $\vee$, $\Rightarrow$ called the symbols of conjunction, disjunction, and implication respectively;

4. K is the set of auxiliary symbols containing two parenthesis: (,).

V, L_1, L_2, K are disjoint sets. The set V is denumerable, and the union of sets L_1 and L_2 isn't empty.

DEFINITION 2. The language of propositional logic is the ordered system $\mathcal{L} = \langle \mathcal{A}, \mathcal{F} \rangle$, where

1. $\mathcal{A}$ is the alphabet of propositional logic;

2. $\mathcal{F}$ is the set of all formulas that are formed by means of symbols in $\mathcal{A}$.

   Notice that elements of $\mathcal{F}$ are defined by induction:

   (a) every propositional variable p, q, r, ... is a formula of propositional logic;

   (b) if α, β are formulas, then expressions $\neg\alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \Rightarrow \beta$ are formulas of propositional logic;

   (c) a finite sequence of symbols is called a formula of propositional logic if that sequence satisfies two above mentioned conditions.

DEFINITION 3. The propositional logic (or propositional calculus) is the ordered system $\mathcal{S} = \langle \mathcal{A}, \mathcal{F}, \mathcal{C} \rangle$, where

1. $\mathcal{A}$ is the alphabet of propositional logic;

2. $\mathcal{F}$ is the set of all formulas formed by means of symbols in $\mathcal{A}$;

3. $\mathcal{C}$ is the inference operation that is the map of formulas in $\mathcal{F}_0 \subseteq \mathcal{F}$ to formulas in $\mathcal{C}(\mathcal{F}_0)$, i.e., to the set of all corollaries from $\mathcal{F}_0$.

The inference rules of propositional logic are as follows:

1. *the substitution rule*, according to that we replace a propositional variable p_j of formula $\alpha(p_1, \ldots, p_n)$, containing propositional variables $p_1, \ldots, p_n$, by a formula $\beta(q_1, \ldots, q_k)$, containing propositional variables $q_1, \ldots, q_k$, and we obtain a new formula $\alpha'(p_1, \ldots, p_{j-1}, \beta(q_1, \ldots, q_k), p_{j+1}, \ldots, p_n)$:

$$\frac{\alpha(p_1, \ldots, p_j, \ldots, p_n)}{\alpha'(p_1, \ldots, p_{j-1}, \beta(q_1, \ldots, q_k), p_{j+1}, \ldots, p_n)};$$

2. *modus ponens*, according to that if two formulas α and $\alpha \Rightarrow \beta$ hold, then we deduce a formula β:
$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta}.$$

The inference operation is inductively defined as follows:

(i) for any set of formulas $\mathcal{F}_0 \subseteq \mathcal{F}$ we get a set $\mathcal{C}(0)$ such that $\mathcal{C}(0) \subset \mathcal{C}(\mathcal{F}_0)$ and $\mathcal{C}(0)$ is called *a set of tautologies* for propositional logic;

(ii) if the set $\mathcal{C}(\mathcal{F}_0)$ contains a set $\mathcal{C}(\alpha)$, then $\mathcal{C}(\mathcal{F}_0)$ contains also a set $\mathcal{C}(\beta)$, where $\alpha, \beta \subset \mathcal{F}_0$ and $\alpha \subseteq \beta$ as $\mathcal{C}(\beta) \not\subset \mathcal{C}(\alpha)$[1];

(iii) $\mathcal{C}(\mathcal{F}_0)$ is the minimal set that satisfies two above mentioned conditions.

The propositional logic has a lot of axiomatization depending on choice of the input set $\mathcal{C}(0)$. We shall use the set of axioms of *Łukasiewicz's propositional calculus* $\mathcal{S}_{PL}$ as the input set $\mathcal{C}(0)$ (see [7]):

$$(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r)), \tag{1}$$

$$(\neg p \Rightarrow p) \Rightarrow p, \tag{2}$$

$$p \Rightarrow (\neg p \Rightarrow q). \tag{3}$$

The implication and complement are given here as basic operations. Other operations are derivable, e.g., the conjunction and disjunction are defined as follows:

$$p \wedge q \rightleftharpoons \neg(p \Rightarrow \neg q), \tag{4}$$

$$p \vee q \rightleftharpoons \neg p \Rightarrow q. \tag{5}$$

Combining axioms (1) – (3) and using inference rules, we obtain all other tautologies of the set $\mathcal{C}(0)$ for the system $\mathcal{S}_{PL}$.

Aristotle's syllogistic is an extension of propositional logic.

Definition 4. The alphabet of Aristotle's syllogistic is the ordered system $\mathcal{A}_{SA} = \langle V, Q, L_1, L_2, L_3, K\rangle$, where

1. V is the set of propositional variables p, q, r, $\ldots$;

[1] By definition, there exists a minimal element α with property $\mathcal{C}(\alpha)$ for any tuple $\langle\alpha, \beta\rangle \in \mathcal{F}_0 \times \mathcal{F}_0$.

2. Q is the set of syllogistic variables S, P, M, $\ldots$;

3. L_1 is the set of unary propositional connectives consisting of one element $\neg$ called the symbol of negation;

4. L_2 is the set of binary propositional connectives containing three elements: $\wedge$, $\vee$, $\Rightarrow$ called the symbols of conjunction, disjunction, and implication respectively;

5. L_3 is the set of binary syllogistic connectives containing four elements **a**, **e**, **i**, **o** called the functors "every. . . is. . . ", "no . . . is. . . ", "some . . . is. . . ", and "some. . . is not . . . " respectively.

6. K is the set of auxiliary symbols containing two parenthesis: (,).

Here V, Q, L_1, L_2, L_3 are disjoint sets. The sets V and Q are denumerable. The union of sets L_1, L_2, and L_3 isn't empty.

DEFINITION 5. The language of Aristotle's syllogistic is the ordered system $\mathcal{L}_{SA} = \langle \mathcal{A}_{SA}, \mathcal{F}_{SA} \rangle$, where

1. $\mathcal{A}_{SA}$ is the alphabet of Aristotle's syllogistic;

2. $\mathcal{F}_{SA}$ is the set of all formulas formed by means of symbols in $\mathcal{A}_{SA}$; this set $\mathcal{F}_{SA}$ contains all formulas defined by the rules (a), (b), and (c) of definition 2 and by the following rules:

   (d) if S and P are syllogistic variables, then expressions $S\mathbf{a}P$[2], $S\mathbf{e}P$[3], $S\mathbf{i}P$[4], $S\mathbf{o}P$[5] are formulas of Aristotle's syllogistic[6].

   (d′) if α and β are formulas of Aristotle's syllogistic, then expressions $\neg\alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \Rightarrow \beta$ are also formulas of Aristotle's syllogistic;

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[2] The proposition "every S is P" has the following notation in predicate logic: $\forall x(x \in S \Rightarrow x \in P)$ or $\neg\exists x(x \in S \wedge x \notin P)$.

[3] The proposition "no S is P" has the following notation in predicate logic: $\forall x(x \in S \Rightarrow x \notin P)$ or $\neg\exists x(x \in S \wedge x \in P)$.

[4] The proposition "some S is P" has the following notation in predicate logic: $\exists x(x \in S \wedge x \in P)$.

[5] The proposition "some S is not P" has the following notation in predicate logic: $\exists x(x \in S \wedge x \notin P)$.

[6] Nominal constants that we substitute for the variable S are called a subject. Nominal constants that we substitute for the variable P are called a predicate.

Thus, an expression that is derivable by rules of definition 5 is called a formula of Aristotle's syllogistic. Formulas that are defined by rules *(d)* and *(d')* of definition 5 is called *formulas of Aristotle's syllogistic in the restricted sense*.

Definition 6. Aristotle's syllogistic is the ordered system $\mathcal{S}_{SA} = \langle \mathcal{A}_{SA}, \mathcal{F}_{SA}, \mathcal{C} \rangle$, where

1. $\mathcal{A}_{SA}$ is the alphabet of Aristotle's syllogistic;

2. $\mathcal{F}_{SA}$ is the set of all formulas formed by means of symbols in $\mathcal{A}_{SA}$;

3. $\mathcal{C}$ is the inference operation in $\mathcal{F}_{SA}$.

The inference rules of Aristotle's syllogistic are as follows:

1. *the substitution rule*, we replace a propositional variable p_j of formula $\alpha(p_1, \ldots, p_n)$, containing propositional variables $p_1, \ldots, p_n$, by a formula $\beta(q_1, \ldots, q_k)$, containing propositional variables $q_1, \ldots, q_k$ (according as by a formula $\beta(S_l, P_m)$, containing syllogistic variables S_l, P_m), and we obtain a new propositional formula $\alpha'(p_1, \ldots, p_{j-1}, \beta(q_1, \ldots, q_k), p_{j+1}, \ldots, p_n)$ (according as a new syllogistic formula $\alpha'(p_1, \ldots, p_{j-1}, \beta(S_l, P_m), p_{j+1}, \ldots, p_n)$):

$$\frac{\alpha(p_1, \ldots, p_j, \ldots, p_n)}{\alpha'(p_1, \ldots, p_{j-1}, \beta(q_1, \ldots, q_k), p_{j+1}, \ldots, p_n)}$$

or

$$\frac{\alpha(p_1, \ldots, p_j, \ldots, p_n)}{\alpha'(p_1, \ldots, p_{j-1}, \beta(S_l, P_m), p_{j+1}, \ldots, p_n)},$$

In the same way, from any syllogistic formula $\alpha(S_j, P_i)$ follows a new formula $\alpha'(S_k, P_i)$ or $\alpha'(S_j, P_l)$ if we replace a syllogistic variable S_j by a syllogistic variable S_k or P_i by P_l:

$$\frac{\alpha(S_j, P_i)}{\alpha'(S_k, P_i)}$$

or

$$\frac{\alpha(S_j, P_i)}{\alpha'(S_j, P_l)};$$

2. *modus ponens*, according to that if two formulas of Aristotle's syllogistic α and $\alpha \Rightarrow \beta$ hold, then we deduce a formula β:

$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta}.$$

The axioms of Aristotle's syllogistic consist of axioms of propositional logic (e.g., axioms (1), (2), (3) of the propositional system $\mathcal{S}_{PL}$), and of the following expressions:

$$SaS, \tag{6}$$

$$SiS, \tag{7}$$

$$(M\mathbf{a}P \wedge S\mathbf{a}M) \Rightarrow S\mathbf{a}P, \textit{ i.e., Barbara}, \tag{8}$$

$$(M\mathbf{a}P \wedge M\mathbf{i}S) \Rightarrow S\mathbf{i}P, \textit{ i.e., Datisi.} \tag{9}$$

The given axiomatic system was created by Łukasiewicz (see [7]). Here the functors **a** and **i** are basic and two other are defined as follows:

$$S\mathbf{e}P \rightleftharpoons \neg(S\mathbf{i}P), \tag{10}$$

$$S\mathbf{o}P \rightleftharpoons \neg(S\mathbf{a}P). \tag{11}$$

Using axioms (1), (2), (3), (6), (7), (8), (9), and definitions (4), (5), (10), (11), we obtain all tautologies of Aristotle's syllogistic.

Definition 7. The function I regarded as the map of formulas of propositional logic $\mathcal{F}_0 \subseteq \mathcal{F}$ to the set $\{\top, \bot\}$ of truth values, where $\top$ is "true" and $\bot$ is "false", is defined as follows:

$$p^I = \begin{cases} \top, \\ \bot, \end{cases}$$

where p is a propositional variable;

$$(\neg\alpha)^I = \begin{cases} \top & \text{if } (\alpha)^I = \bot, \\ \bot & \text{if } (\alpha)^I = \top, \end{cases}$$

where α is a formula of propositional logic;

$$(\alpha \wedge \beta)^I = \begin{cases} \top & \text{if } (\alpha)^I = (\beta)^I = \top, \\ \bot & \text{otherwise,} \end{cases}$$

$$(\alpha \vee \beta)^I = \begin{cases} \top & \text{if } (\alpha)^I = \top \text{ or } (\beta)^I = \top, \\ \bot & \text{otherwise,} \end{cases}$$

$$(\alpha \Rightarrow \beta)^I = \begin{cases} \bot & \text{if } (\alpha)^I = \top \text{ and } (\beta)^I = \bot, \\ \top & \text{otherwise,} \end{cases}$$

Note that metavariables α and β range over all formulas of propositional logic.

Let $\{\vartheta_0, \vartheta_1, \vartheta_2, \ldots, \vartheta_n, \ldots\}$ be any infinite set with a minimal member ϑ_0 and with one operation 'inf' (infimum) defined on all members of this set.

Definition 8. Suppose the set $\mathcal{F}_0$ contains all superpositions of conjunction, disjunction, implication, negation of formulas of the form $S\mathbf{a}P$, $S\mathbf{e}P$, $S\mathbf{i}P$, $S\mathbf{o}P$ and the set $\mathcal{F}_1$ contains all formulas of the form $S\mathbf{a}P$, $S\mathbf{e}P$, $S\mathbf{i}P$, $S\mathbf{o}P$. Then the function I regarded as the map of syllogistic formulas $\mathcal{F}_0 \subseteq \mathcal{F}_{SA}$ to the set $\{\top, \bot\}$ of truth values is defined by rules of definition 7. This function I regarded as the map of syllogistic formulas $\mathcal{F}_1 \subseteq \mathcal{F}_{SA}$ to the set $\{\vartheta_0, \vartheta_1, \vartheta_2, \ldots, \vartheta_n, \ldots\}$ of syllogistic truth values and to the set $\{\top, \bot\}$ of propositional truth values is defined by the following rules:

$$S^I = \begin{cases} \vartheta_0, \\ \vartheta_n > \vartheta_0, \end{cases}$$

where by $(S)^I$ we denote a nominal constant that we substitute for the variable S[7].

$$(S\mathbf{a}P)^I = \begin{cases} \top & \text{if } (S)^I = \vartheta_m, (P)^I = \vartheta_n, \text{ and } \inf(\vartheta_m, \vartheta_n) = \vartheta_m, \\ \bot & \text{otherwise,} \end{cases}$$

$$(S\mathbf{e}P)^I = \begin{cases} \top & \text{if } (S)^I = \vartheta_m, (P)^I = \vartheta_n, \text{ and } \inf(\vartheta_m, \vartheta_n) = \vartheta_0, \\ \bot & \text{otherwise,} \end{cases}$$

$$(S\mathbf{i}P)^I = \begin{cases} \top & \text{if } (S)^I = \vartheta_m, (P)^I = \vartheta_n, \text{ and } \inf(\vartheta_m, \vartheta_n) > \vartheta_0, \\ \bot & \text{otherwise,} \end{cases}$$

[7] Thus, the truth interpretation $(S)^I$ and $(P)^I$ ranges over not the set $\{\top, \bot\}$, but the set $\{\vartheta_0, \vartheta_1, \vartheta_2, \ldots, \vartheta_n, \ldots\}$ of nominal constants.

$$(S\mathbf{o}P)^I = \begin{cases} \top & \text{if } (S)^I = \vartheta_m, (P)^I = \vartheta_n, \text{ and } \inf(\vartheta_m, \vartheta_n) < \vartheta_m, \\ \bot & \text{otherwise,} \end{cases}$$

Since we can define truth values of arbitrary formulas, we have semantics for this language. Usually a lattice is considered as semantics for a formalized language.

DEFINITION 9. A lattice for a formalized language $\mathcal{L}$ is an ordered system $\mathfrak{A} = \langle A, \Omega \rangle$, where

1. A is the set of arbitrary elements;
2. Ω is the set of n-ary relationes ω_A over elements of A, and every n-ary relation ω_A in Ω corresponds to an n-ary formula ω in $\mathcal{L}$.

DEFINITION 10. The lattice for the language of propositional logic is a Boolean algebra, i.e., the ordered system $\mathfrak{B} = \langle B; \cap, \cup, \neg, 1, 0 \rangle$.

It is known that to each logical relation (to each formula) of propositional logic we can assign a relation of Boolean algebra. It is easily shown by induction on length of formula that an intersection is assigned to a conjunction, a union is assigned to a disjunction, a pseudo-complement relative to an element is assigned to an implication, and a complement is assigned to a negation.

The following definition is needed for the sequel.

DEFINITION 11. The lattice for the language of Aristotle's syllogistic is a vectorial lattice on the $\cap$-semilattice. Let $\mathfrak{B} = \langle B; \cap, \cup, \neg, 1, 0 \rangle$ be a Boolean algebra and let $\mathfrak{B}_\cap = \langle B_\cap; \cap, \mathbf{0} \rangle$ be a $\cap$-semilattice, i.e., the ordered system $\mathfrak{B}_\cap$ such that there exist only one binary operation $\cap$ and only one constant $\mathbf{0}$. Further, let λ_k and μ_k[8] be unary operations defined on the set B for any element k of the $\cap$-semilattice $\mathfrak{B}_\cap$. Then the vectorial lattice on the $\cap$-semilattice is the ordered system $\mathfrak{V}_\mathfrak{B} = \langle B; \cap, \cup, \neg, 1, 0; \{\lambda_k \colon k \in B_\cap\}, \{\mu_k \colon k \in B_\cap\} \rangle$, where $\{\lambda_k \colon k \in B_\cap\}$ (according as $\{\mu_k \colon k \in B_\cap\}$) is the set of all λ_k (according as the set of all μ_k) such that k belongs to $B_\cap$. Every element of the set B is called a vector, every element of the set $B_\cap$ is called a scalar.

[8] The operations λ and μ take each element k in the set $B_\cap$ to a unique element λ_k and μ_k in the set B.

The operations λ_k and μ_k are defined by induction:

$$\forall a \in B \forall b \in B \forall k \in B_\cap(\lambda_k(a \cap b) = \lambda_k(a) \cap b = \lambda_k(b) \cap a); \tag{12}$$

$$\forall a \in B \forall b \in B \forall k \in B_\cap(\lambda_k(a \cup b) = \lambda_k(a) \cup \lambda_k(b)); \tag{13}$$

$$\forall k \in B_\cap(\lambda_k(0) = 0); \tag{14}$$

$$\forall k \in B_\cap \forall l \in B_\cap(\lambda_k(l) = 0 \text{ if } k = m_0 \cap n = \mathbf{0} \text{ and } l = m_1 \cap n = n); \tag{15}$$

$$\forall k \in B_\cap \forall l \in B_\cap(\lambda_k(l) = 0 \text{ if } k = m_0 \cap n = n \text{ and } l = m_1 \cap n < n); \tag{16}$$

$$\forall k \in B_\cap \forall l \in B_\cap(\lambda_k(l) = 0 \text{ if } k = m_0 \cap n = \mathbf{0} \text{ and } l = m_1 \cap n > \mathbf{0}); \tag{17}$$

$$\forall a \in B \forall b \in B \forall k \in B_\cap(\mu_k(a \cap b) = \mu_k(a) \cap \mu_k(b)); \tag{18}$$

$$\forall a \in B \forall b \in B \forall k \in B_\cap(\mu_k(a \cup b) = \mu_k(a) \cup b = \mu_k(b) \cup a); \tag{19}$$

$$\forall k \in B_\cap(\mu_k(1) = 1); \tag{20}$$

$$\forall k \in B_\cap \forall l \in B_\cap(\mu_k(l) = 1 \text{ if } k = m_0 \cap n > \mathbf{0} \text{ and } l = m_1 \cap n < n); \tag{21}$$

$$\forall k \in B_\cap \forall l \in B_\cap(\mu_k(l) = 1 \text{ if } k = m_0 \cap n = n \text{ and } l = m_1 \cap n < n); \tag{22}$$

$$\forall k \in B_\cap \forall l \in B_\cap(\mu_k(l) = 1 \text{ if } k = m_0 \cap n = \mathbf{0} \text{ and } l = m_1 \cap n > \mathbf{0}). \tag{23}$$

In all expressions $m_0 \cap m_1 = \mathbf{0}$.

We say that an element $\lambda_k(a)$ of vectorial lattice $\mathfrak{V}_{\mathfrak{B}}$ (according as an element $\mu_k(a)$ of vectorial lattice $\mathfrak{V}_{\mathfrak{B}}$) is an intersection of elements k and a (according as a union of elements k and a) and write $k \cap a$ (according as $k \cup a$); notice that $(k \cap a) \in B$ and $(k \cup a) \in B$. Taking into account this interpretation of operations $\lambda_k(a)$, $\mu_k(a)$, we have:

$$\forall a \in B \forall b \in B \forall k \in B_\cap(k \cap (a \cap b) = (k \cap a) \cap b = (k \cap b) \cap a),$$

i.e., the associativity and commutativity of λ_k for an intersection of vectors a and b;

$$\forall a \in B \forall b \in B \forall k \in B_\cap(k \cap (a \cup b) = (k \cap a) \cup (k \cap b)),$$

i.e., the distributivity of λ_k for a union of vectors a and b;

$$\forall k \in B_\cap (k \cap 0 = 0);$$

$$\forall k \in B_\cap \forall l \in B_\cap (k \cap l = 0 \text{ if } k = m_0 \cap n = \mathbf{0} \text{ and } l = m_1 \cap n = n);$$

$$\forall k \in B_\cap \forall l \in B_\cap (k \cap l = 0 \text{ if } k = m_0 \cap n = n \text{ and } l = m_1 \cap n < n);$$

$$\forall k \in B_\cap \forall l \in B_\cap (k \cap l = 0 \text{ if } k = m_0 \cap n = \mathbf{0} \text{ and } l = m_1 \cap n > \mathbf{0});$$

$$\forall a \in B \forall b \in B \forall k \in B_\cap (k \cup (a \cap b) = (k \cup a) \cap (k \cup b)),$$

i.e., the distributivity of μ_k for an intersection of vectors a and b;

$$\forall a \in B \forall b \in B \forall k \in B_\cap (k \cup (a \cup b) = (k \cup a) \cup b = (k \cup b) \cup a),$$

i.e., the associativity and commutativity of μ_k for a union of vectors a and b;

$$\forall k \in B_\cap (k \cup 1 = 1);$$

$$\forall k \in B_\cap \forall l \in B_\cap (k \cup l = 1 \text{ if } k = m_0 \cap n > \mathbf{0} \text{ and } l = m_1 \cap n < n);$$

$$\forall k \in B_\cap \forall l \in B_\cap (k \cup l = 1 \text{ if } k = m_0 \cap n = n \text{ and } l = m_1 \cap n < n);$$

$$\forall k \in B_\cap \forall l \in B_\cap (k \cup l = 1 \text{ if } k = m_0 \cap n = \mathbf{0} \text{ and } l = m_1 \cap n > \mathbf{0}).$$

In all expressions $m_0 \cap m_1 = \mathbf{0}$.

The $\cap$-semilattice $\mathfrak{B}_\cap$ is partially ordered. In other words, elements of the set $B_\cap$ satisfy the following axioms:

(24) $\forall a \in B_\cap\, a \leqslant a$, *i.e., the antireflexiveness condition,*

(25) $\forall a \in B_\cap \forall b \in B_\cap \forall c \in B_\cap\, (a \leqslant b \;\wedge\; b \leqslant c \Rightarrow a \leqslant c)$, *i.e., the transitivity condition,*

(26) $\forall a \in B_\cap \forall b \in B_\cap (a \leqslant b \wedge b \leqslant a \Rightarrow a = b)$, *i.e., the antisymmetry condition.*

The unique binary operation $a \cap b$ is defined in the $\cap$-semilattice so:

$$\forall a \in B_\cap \forall b \in B_\cap (a \leqslant b \Leftrightarrow a \cap b = a).$$

The axioms of the $\cap$-semilattice are as follows:

(27) $\forall a \in B_\cap (a \cap a = a)$, *i.e., the reflexivity condition,*

(28) $\forall a \in B_\cap \forall b \in B_\cap (a \cap b = b \cap a)$, *i.e., the commutativity condition,*

(29)
$$\forall a \in B_\cap \forall b \in B_\cap \forall c \in B_\cap (a \cap (b \cap c) = (a \cap b) \cap c),$$
i.e., the associativity condition,

(30) $\forall a \in B_\cap (a \cap \mathbf{0} = \mathbf{0})$, *i.e., the* $\mathbf{0}$*-boundedness condition.*

There is also the strict order in the $\cap$-semilattice :

$$\forall a \in B_\cap \forall b \in B_\cap (a < b \Leftrightarrow a \leqslant b \wedge a \neq b)$$

It is easy shown that we can assign a relation of the vectorial lattice on the $\cap$-semilattice to each relation (formula) of Aristotle's syllogistic. It can be checked by induction on a length of formula:

1. a complement of a vector α is assigned to a negation $\neg\alpha$;

2. an intersection of vectors α and β is assigned to a conjunction $\alpha \wedge \beta$, a union of vectors α and β is assigned to a disjunction $\alpha \vee \beta$, a pseudo-complement of a vector α relative to a vector β is assigned to an implication $\alpha \Rightarrow \beta$;

3. an intersection of scalars $S \cap P = S$ is assigned to a universal affirmative proposition $S\mathbf{a}P$, an intersection of scalars $S \cap P = \mathbf{0}$ is assigned to a universal negative proposition $S\mathbf{e}P$, an intersection of scalars $S \cap P > \mathbf{0}$ is assigned to a particular affirmative proposition $S\mathbf{i}P$, an intersection of scalars $S \cap P < S$ is assigned to a particular negative proposition $S\mathbf{o}P$. If S, P_0 are fixed for syllogistic propositions $S\mathbf{a}P_0$, $S\mathbf{e}P_0$, $S\mathbf{i}P_0$, $S\mathbf{o}P_0$, then

   (a) in the case $S\mathbf{a}P_0$ is true, we have

   $S \cap P_0 = S$ for $S\mathbf{a}P_0$,

   $S \cap P_1 = \mathbf{0}$ for $S\mathbf{e}P_0$,

   $S \cap P_0 > \mathbf{0}$ for $S\mathbf{i}P_0$,

   $S \cap P_1 < S$ for $S\mathbf{o}P_0$,

   where P_0 and P_1 are mutually disjoint, e.g., the proposition "every man (S) is mortal (P_0)" is true and the proposition "no man (S) is mortal (P_1)" is false, therefore $S \cap P_0 = S$ and $S \cap P_1 = \mathbf{0}$,

(b) in the case $S\mathbf{e}P_0$ is true, we have

$S \cap P_1 = S$ for $S\mathbf{e}P_0$,

$S \cap P_0 = \mathbf{0}$ for $S\mathbf{a}P_0$,

$S \cap P_1 > \mathbf{0}$ for $S\mathbf{o}P_0$,

$S \cap P_0 < S$ for $S\mathbf{i}P_0$,

where P_0 and P_1 are mutually disjoint, e.g., the proposition "every man (S) is dolphin (P_0)" is false and the proposition "no man (S) is dolphin (P_1)" is true, therefore $S \cap P_0 = \mathbf{0}$ and $S \cap P_1 = S$,

(c) in the case $S\mathbf{i}P_0$ is true, we have

$S \cap P_0 > \mathbf{0}$ for $S\mathbf{i}P_0$,

$S \cap P_1 < S$ for $S\mathbf{o}P_0$,

where P_0 and P_1 are mutually disjoint,

(d) in the case $S\mathbf{o}P_0$ is true, we have

$S \cap P_1 > \mathbf{0}$ for $S\mathbf{o}P_0$,

$S \cap P_0 < S$ for $S\mathbf{i}P_0$,

where P_0 and P_1 are mutually disjoint.

As an example we prove general validity of the mood (modus) Barbara in the $\cap$-semilattice.

Example 1. This mood has the following notation in the language of the $\cap$-semilattice:

$$\text{if } M \cap P = M \text{ and } S \cap M = S, \text{ then } S \cap P = S.$$

Substitute an expression $S \cap M$ for S in $S \cap P$. We have $(S \cap M) \cap P$. By associativity, we obtain $S \cap (M \cap P)$. But it is known that $M \cap P = M$. Hence, we deduce S.

Example 2. This mood has the following notation in the language of the vectorial lattice on the $\cap$-semilattice:

$$((M \cap P = M) \cap (S \cap M = S)) \Rightarrow (S \cap P = S).$$

By substitution, we obtain

$$(M \cap S) \Rightarrow S = \neg(M \cap S) \cup S = \neg M \cup \neg S \cup S = \neg M \cup 1 = 1.$$

Note that we have the binary contradictory (contrary) relation in Aristotle's syllogistic. Therefore we can deduce here the law 'tertium non datur' (the law of excluded middle). Now consider a new system of syllogistic in that there is the ternary contradictory relation. Here we can deduce the law "quartum non datur". This system is called Vasiľév's syllogistic (see [20], [21], [2], [6]). It is more simple deductive system, than Aristotle's syllogistic. Recall that N. A. Vasiľév is well-know Russian logician. 1880–1940 were years of his life. He wrote scientific works in 1910–1914. Then he stopped logical investigations because of serious alienation.

Definition 12. The alphabet of Vasiľév's syllogistic is the ordered system $\mathcal{A}_{SV} = \langle V, Q, L_1, L_2, L_3^{\sim}, K\rangle$, where

1. V is the set of proposition variables p, q, r, ...;
2. Q is the set of syllogistic variables S, P, M...;
3. L_1 is the set of unary propositional connectives consisting of one element $\neg$ called the symbol of negation;
4. L_2 is the set of binary propositional connectives containing three elements: $\wedge$, $\vee$, $\Rightarrow$ called the symbols of conjunction, disjunction, and implication respectively;
5. $L_3^{\sim}$ is the set of binary syllogistic connectives containing three elements **a**, **e**, **m** called the functors "every... is...", "no ... is...", and "some, but not every... is ..."[9] respectively.
6. K is the set of auxiliary symbols containing two parenthesis: (,).

Here V, Q, L_1, L_2, $L_3^{\sim}$ are disjoint sets. The sets V and Q are denumerable. The union of sets L_1, L_2, and $L_3^{\sim}$ isn't empty.

Definition 13. The language of Vasiľév's syllogistic is the ordered system $\mathcal{L}_{SV} = \langle \mathcal{A}_{SV}, \mathcal{F}_{SV}\rangle$, where

1. $\mathcal{A}_{SV}$ is the alphabet of Vasiľév's syllogistic;

[9] By Vasiľév's opinion, there exists a unique particular proposition, namely, particular affirmative negative proposition and its functor is **m**. This proposition can be formulated as *an indifferent statement* ("S is and is not P"), as *a disjunctive statement* ("S is P or is not P"), and as *an accidental statement* ("S can be P").

2. $\mathcal{F}_{SV}$ is the set of all formulas formed by means of symbols in $\mathcal{A}_{SV}$; this set $\mathcal{F}_{SV}$ contains all formulas defined by the rules (a), (b), and (c) of definition 2 and by the following rules:

   (d) if S and P are syllogistic variables, then expressions $S\mathbf{a}P$[10], $S\mathbf{e}P$[11], $S\mathbf{m}P$[12] are formulas of Vasiľév's syllogistic.

   (d′) if α and β are formulas of Vasiľév's syllogistic, then expressions $\neg\alpha$, $\alpha \wedge \beta$, $\alpha \vee \beta$, $\alpha \Rightarrow \beta$ are also formulas of Vasiľév's syllogistic;

Also, an expression that is derivable by rules of definition 13 is called a formula of Vasiľév's syllogistic. Formulas that are defined by rules (d) and (d′) of definition 13 is called *formulas of Vasiľév's syllogistic in the restricted sense*.

DEFINITION 14. Vasiľév's syllogistic is the ordered system $\mathcal{S}_{SV} = \langle \mathcal{A}_{SV}, \mathcal{F}_{SV}, \mathcal{C} \rangle$, where

1. $\mathcal{A}_{SV}$ is the alphabet of Vasiľév's syllogistic;
2. $\mathcal{F}_{SV}$ is the set of all formulas formed by means of symbols in $\mathcal{A}_{SV}$;
3. $\mathcal{C}$ is the inference operation in $\mathcal{F}_{SV}$.

The inference rules of Vasiľév's syllogistic are as follows:

1. *the substitution rule*, we replace a propositional variable p_j of formula $\alpha(p_1, \ldots, p_n)$, containing propositional variables $p_1, \ldots, p_n$, by a formula $\beta(q_1, \ldots, q_k)$, containing propositional variables $q_1, \ldots, q_k$ (according as by a formula $\beta(S_l, P_m)$, containing syllogistic variables S_l, P_m), and we obtain a new propositional formula $\alpha'(p_1, \ldots, p_{j-1}, \beta(q_1, \ldots, q_k), p_{j+1}, \ldots, p_n)$ (according as a new syllogistic formula $\alpha'(p_1, \ldots, p_{j-1}, \beta(S_l, P_m), p_{j+1}, \ldots, p_n)$):

$$\frac{\alpha(p_1, \ldots, p_j, \ldots, p_n)}{\alpha'(p_1, \ldots, p_{j-1}, \beta(q_1, \ldots, q_k), p_{j+1}, \ldots, p_n)}$$

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[10]The proposition of Vasiľév's syllogistic "every S is P" has the following notation in predicate logic: $\forall x(x \in S^+ \Rightarrow x \in P^+)$ or $\neg\exists x(x \in S^+ \wedge x \notin P^+)$, where S^+ and P^+ are closed sets, i.e., $S^+ = \mathbf{C}S$ and $P^+ = \mathbf{C}P$.

[11]The proposition of Vasiľév's syllogistic "no S is P" has the following notation in predicate logic: $\forall x(x \in S^+ \Rightarrow x \notin P^+)$ or $\neg\exists x(x \in S^+ \wedge x \in P^+)$.

[12]The proposition of Vasiľév's syllogistic "some, but not every S is P" has the following notation in predicate logic: $\exists x(x \in S^+ \wedge x \in (P^+ \cap \neg P^+))$.

or

$$\frac{\alpha(p_1,\ldots,p_j,\ldots,p_n)}{\alpha'(p_1,\ldots,p_{j-1},\beta(S_l,P_m),p_{j+1},\ldots,p_n)},$$

For the same reason, from any syllogistic formula $\alpha(S_j,P_i)$ follows a new formula $\alpha'(S_k,P_i)$ or $\alpha'(S_j,P_l)$ if we replace a syllogistic variable S_j by a syllogistic variable S_k or P_i by P_l:

$$\frac{\alpha(S_j,P_i)}{\alpha'(S_k,P_i)}$$

or

$$\frac{\alpha(S_j,P_i)}{\alpha'(S_j,P_l)};$$

2. *modus ponens*, according to that if two formulas of Vasiľév's syllogistic α and $\alpha \Rightarrow \beta$ hold, then we deduce a formula β:

$$\frac{\alpha,\alpha\Rightarrow\beta}{\beta}.$$

The axioms of Vasiľév's syllogistic consist of the axioms of propositional logic (e.g., of (1), (2), (3), (4), (5)), and of the following expressions that I proposed:

(31) $$S\mathbf{a}S,$$

(32) $$(M\mathbf{a}P \wedge S\mathbf{a}M) \Rightarrow S\mathbf{a}P,\ \textit{i.e., Barbara},$$

(33) $$(M\mathbf{e}P \wedge S\mathbf{a}M) \Rightarrow S\mathbf{e}P,\ \textit{i.e., Celarent},$$

(34) $$(M\mathbf{m}P \wedge M\mathbf{a}S) \Rightarrow S\mathbf{m}P,\ \textit{i.e., Disamis-Bocardo},$$

(35) $$S\mathbf{e}P \Rightarrow P\mathbf{e}S,$$

(36) $$S\mathbf{a}P \Rightarrow \neg(S\mathbf{e}P),$$

(37) $$S\mathbf{a}P \Rightarrow \neg(S\mathbf{m}P),$$

(38) $$S\mathbf{m}P \Rightarrow \neg(S\mathbf{e}P).$$

(39) $$(\neg(S\mathbf{a}P) \wedge \neg(S\mathbf{e}P)) \Rightarrow S\mathbf{m}P.$$

Using these axioms, we obtain the following tautologies:

$$S\mathbf{a}P \vee S\mathbf{e}P \vee S\mathbf{m}P, \tag{40}$$

i.e., the law 'quartum non datur',

$$\neg(S\mathbf{a}P \wedge S\mathbf{e}P), \tag{41}$$

$$\neg(S\mathbf{a}P \wedge S\mathbf{m}P), \tag{42}$$

$$\neg(S\mathbf{e}P \wedge S\mathbf{m}P), \tag{43}$$

i.e., the laws of contradiction.

Let $\{\vartheta_0, \vartheta_1, \vartheta_2, \ldots, \vartheta_n, \ldots\}$ be any infinite set with a minimal member ϑ_0 and with one operation 'inf' (infimum) defined on all members of this set.

DEFINITION 15. Suppose the set $\mathcal{F}_0$ contains all superpositions of conjunction, disjunction, implication, negation of formulas of the form $S\mathbf{a}P$, $S\mathbf{e}P$, $S\mathbf{m}P$ and the set $\mathcal{F}_1$ contains all formulas of the form $S\mathbf{a}P$, $S\mathbf{e}P$, $S\mathbf{m}P$. Then the function I regarded as the map of syllogistic formulas $\mathcal{F}_0 \subseteq \mathcal{F}_{SA}$ to the set $\{\top, \bot\}$ of truth values is defined by rules of definition 7. This function I regarded as the map of syllogistic formulas $\mathcal{F}_1 \subseteq \mathcal{F}_{SA}$ to the set $\{\vartheta_0, \vartheta_1, \vartheta_2, \ldots, \vartheta_n, \ldots\}$ of syllogistic truth values and to the set $\{\top, \bot\}$ of propositional truth values is defined by the following rules:

$$S^I = \begin{cases} \vartheta_0, \\ \vartheta_n > \vartheta_0, \end{cases}$$

where by $(S)^I \in \{\vartheta_0, \vartheta_1, \vartheta_2, \ldots, \vartheta_n, \ldots\}$ we denote a nominal constant that we replace by the variable S.

$$(S\mathbf{a}P)^I = \begin{cases} \top & \text{if } (S)^I = \vartheta_m, (P)^I = \vartheta_n, \text{ and } \inf(\vartheta_m, \vartheta_n) = \vartheta_m, \\ \bot & \text{otherwise,} \end{cases}$$

$$(S\mathbf{e}P)^I = \begin{cases} \top & \text{if } (S)^I = \vartheta_m, (P)^I = \vartheta_n, \text{ and } \inf(\vartheta_m, \vartheta_n) = \vartheta_0, \\ \bot & \text{otherwise,} \end{cases}$$

$$(S\mathbf{m}P)^I = \begin{cases} \top & \text{if } (S)^I = \vartheta_m, (P)^I = \vartheta_n, \inf(\vartheta_m, \vartheta_n) > \vartheta_0, \\ & \qquad \text{and } \inf(\vartheta_m, \vartheta_n) < \vartheta_m, \\ \bot & \text{otherwise,} \end{cases}$$

Definition 16. The $\cap$-semilattice $\mathfrak{B}_\cap = \langle B_\cap; \cap, \mathbf{0}\rangle$ is linear if we have the new axiom:

$$\forall a \in B_\cap \forall b \in B_\cap (a > b \vee b > a).$$

Only universal affirmative propositions $S\mathbf{a}P$ hold in the linear $\cap$-semilattice.

Definition 17. The $\cap$-semilattice $\mathfrak{B}_\cap = \langle B_\cap; \cap, \mathbf{0}\rangle$ is semilinear if the following proposition holds:

$$\forall a \in B_\cap \forall b \in B_\cap ((a > b) \vee (b > a) \vee (a \cap b = \mathbf{0})).$$

Only universal affirmative propositions $S\mathbf{a}P$ and universal negative propositions $S\mathbf{e}P$ hold in the semilinear $\cap$-semilattice.

Let us remember that a set S is called closed if $S = \mathbf{C}S$, where $\mathbf{C}$ is a closure operator:

$$\mathbf{C}(S \cup P) = \mathbf{C}S \cup \mathbf{C}P; \tag{44}$$

$$S \subset \mathbf{C}S; \tag{45}$$

$$\mathbf{CC}S = \mathbf{C}S; \tag{46}$$

$$\mathbf{C}0 = 0. \tag{47}$$

By S^+ denote a closed set S. Notice that $S^+ \cap \neg S^+ \neq \emptyset$.

Definition 18. The $\cap$-semilattice $\mathfrak{B} = \langle B; \cap, \mathbf{0}\rangle$ is called the closure $\cap$-semilattice $\mathfrak{B}^+ = \langle B^+; \cap, \mathbf{0}^+\rangle$ if all members of B are closed, i.e., we have the following axioms:

$$\forall a^+ \in B^+ (a^+ \cap a^+ = a^+), \tag{48}$$

$$\forall a^+ \in B^+ \forall b^+ \in B^+ (a^+ \cap b^+ = b^+ \cap a^+), \tag{49}$$

$$\forall a^+ \in B^+ \forall b^+ \in B^+ \forall c^+ \in B^+ (a^+ \cap (b^+ \cap c^+) = (a^+ \cap b^+) \cap c^+), \tag{50}$$

$$\forall a^+ \in B^+ (a^+ \cap \mathbf{0}^+ = \mathbf{0}^+), \tag{51}$$

$$\forall a^+ \in B^+ \forall b^+ \in B^+ ((a^+ > b^+) \vee (b^+ > a^+) \vee (a^+ \cap b^+ \geqslant \mathbf{0}^+)), \tag{52}$$

$$\begin{gathered}\forall a \in B \forall b \in B ((a \cap b = \mathbf{0} \wedge \neg(a = \mathbf{0} \vee b = \mathbf{0})) \Rightarrow \\ \forall a^+ \in B^+ \forall b^+ \in B^+ (a^+ \cap b^+ \geqslant \mathbf{0}^+)),\end{gathered} \tag{53}$$

where $a^+ = \mathbf{C}a$ and $b^+ = \mathbf{C}b$.

DEFINITION 19. The lattice for the language of Vasiľév's syllogistic is a vectorial lattice on the closure $\cap$-semilattice. Let $\mathfrak{B} = \langle B; \cap, \cup, \neg, 1, 0\rangle$ be a Boolean algebra and let $\mathfrak{B}^+ = \langle B^+; \cap, \mathbf{0}^+\rangle$ be a closure $\cap$-semilattice. Suppose λ_k^+ and μ_k^+ are unary operations defined on the set B for any element k^+ of the closure $\cap$-semilattice $\mathfrak{B}^+$. The ordered system $\mathfrak{V}_{\mathfrak{B}} = \langle B; \cap, \cup, \neg, 1, 0; \{\lambda_k^+ : k^+ \in B^+\}, \{\mu_k^+ : k^+ \in B^+\}\rangle$ is called the vectorial lattice on the closure $\cap$-semilattice, where $\{\lambda_k^+ : k^+ \in B^+\}$ (according as $\{\mu_k^+ : k^+ \in B^+\}$) is the set of all λ_k^+ (according as the set of all μ_k^+) such that k^+ belongs to B^+. Every element of the set B is called a vector, every element of the set B^+ is called a scalar.

The operations λ_k^+ and μ_k^+ are defined by induction:

$$\forall a \in B \forall b \in B \forall k^+ \in B^+(\lambda_k^+(a \cap b) = \lambda_k^+(a) \cap b = \lambda_k^+(b) \cap a); \tag{54}$$

$$\forall a \in B \forall b \in B \forall k^+ \in B^+(\lambda_k^+(a \cup b) = \lambda_k^+(a) \cup \lambda_k^+(b)); \tag{55}$$

$$\forall k^+ \in B^+(\lambda_k^+(0) = 0); \tag{56}$$

$$\begin{gathered}\forall k^+ \in B^+ \forall l^+ \in B^+(\lambda_k^+(l^+) = 0 \text{ if } k^+ = i_0^+ \cap j^+ = j^+,\\ l^+ = ((i_0^+ \cap i_1^+) \cap j^+) > \mathbf{0}^+, \text{ and } l^+ = ((i_0^+ \cap i_1^+) \cap j^+) < j^+;\end{gathered} \tag{57}$$

$$\begin{gathered}\forall k^+ \in B^+ \forall l^+ \in B^+(\lambda_k^+(l^+) = 0 \text{ if } k^+ = i_0^+ \cap j^+ = j^+ \text{ and}\\ l^+ = i_1^+ \cap j^+ = \mathbf{0}^+);\end{gathered} \tag{58}$$

$$\begin{gathered}\forall k^+ \in B^+ \forall l^+ \in B^+(\lambda_k^+(l^+) = 0 \text{ if } k^+ = ((i_0^+ \cap i_1^+) \cap j^+) < j^+,\\ k^+ = ((i_0^+ \cap i_1^+) \cap j^+) > \mathbf{0}^+, \text{ and } l^+ = i_1^+ \cap j^+ = \mathbf{0}^+);\end{gathered} \tag{59}$$

$$\forall a \in B \forall b \in B \forall k^+ \in B^+(\mu_k^+(a \cap b) = \mu_k^+(a) \cap \mu_k^+(b)); \tag{60}$$

$$\forall a \in B \forall b \in B \forall k^+ \in B^+(\mu_k^+(a \cup b) = \mu_k^+(a) \cup b = \mu_k^+(b) \cup a); \tag{61}$$

$$\forall k^+ \in B^+(\mu_k^+(1) = 1); \tag{62}$$

$$\begin{gathered}\forall k^+ \in B^+ \forall l^+ \in B^+ \forall n^+ \in B^+(\mu_k^+(l^+ \cup n^+) =\\ \mu_l^+(k^+ \cup n^+) = \mu_n^+(k^+ \cup l^+) = 1\\ \text{if } k^+ = i_0^+ \cap j^+ = j^+, \mathbf{0}^+ < l^+ = ((i_0^+ \cap i_1^+) \cap j^+) < j^+\\ \text{and } n^+ = i_1^+ \cap j^+ = \mathbf{0}^+).\end{gathered} \tag{63}$$

In all expressions i_0, i_1 are mutually disjoint for the given i_0^+, i_1^+ such that $i_0^+ \cap i_1^+ \neq \mathbf{0}^+$.

It is easy shown that we can assign a relation of the vectorial lattice on the closure $\cap$-semilattice to each relation (formula) of Vasiľév's syllogistic. It can be checked by induction on a length of formula:

1. a complement of a vector α is assigned to a negation $\neg\alpha$;
2. an intersection of vectors α and β is assigned to a conjunction $\alpha \wedge \beta$, a union of vectors α and β is assigned to a disjunction $\alpha \vee \beta$, a pseudo-complement of a vector α relative to a vector β is assigned to an implication $\alpha \Rightarrow \beta$;
3. an intersection of scalars $S^+ \cap P^+ = S^+$ is assigned to a universal affirmative proposition $S\mathbf{a}P$, an intersection of scalars $S^+ \cap P^+ = \mathbf{0}^+$ is assigned to a universal negative proposition $S\mathbf{e}P$, an intersection of scalars $\mathbf{0}^+ < S^+ \cap P^+ < S^+$ is assigned to a particular affirmative negative proposition $S\mathbf{m}P$. If S, P_0 are fixed for syllogistic propositions $S\mathbf{a}P_0$, $S\mathbf{e}P_0$, $S\mathbf{m}P_0$, then
   (a) in the case $S\mathbf{a}P_0$ is true, we have
   $S^+ \cap P_0^+ = S^+$ for $S\mathbf{a}P_0$,
   $S^+ \cap P_1^+ = \mathbf{0}^+$ for $S\mathbf{e}P_0$,
   where P_0, P_1 are mutually disjoint and $P_0^+ \cap P_1^+ \neq \mathbf{0}^+$,
   (b) in the case $S\mathbf{e}P_0$ is true, we have
   $S^+ \cap P_1^+ = S^+$ for $S\mathbf{e}P_0$,
   $S^+ \cap P_0^+ = \mathbf{0}^+$ for $S\mathbf{a}P_0$,
   where P_0, P_1 are mutually disjoint and $P_0^+ \cap P_1^+ \neq \mathbf{0}^+$,
   (c) in the case $S\mathbf{m}P_0$ is true, we have
   $S^+ > S^+ \cap (P_0^+ \cap P_1^+) > \mathbf{0}^+$ for $S\mathbf{m}P_0$,
   where P_0, P_1 are mutually disjoint and $P_0^+ \cap P_1^+ \neq \mathbf{0}^+$,

Also, the lattice of the language of Aristotle's syllogistic is the vectorial lattice on the $\cap$-semilattice. The lattice of the language of Vasiľév's syllogistic is the vectorial lattice on the closure $\cap$-semilattice.

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