











should be preceded by a reservation “for a certain admissible precisification of the terms used”.

It should be noted that the subvaluation logic does not preserve classical consequence. Hence,

$$\neg(A_1, \dots, A_n \vDash_{\text{Cl}} B \implies A_1, \dots, A_n \vDash_{\text{SbV}} B).$$

The following is valid however:

$$A_1, \dots, A_n \vDash_{\text{Cl}} B \implies A_1 \& \dots \& A_n \vDash_{\text{SbV}} B.$$

There are serious doubts whether or not conjunction (i.e. “&”) used in this logic can still be called “conjunction”, for the principle of adjunction fails:

$$A, B \not\vDash_{\text{SbV}} A \& B,$$

and it may seem that if a connective fails adjunction, then it is not conjunction (cf. [7], p. 159). The failure of adjunction explains the validity of LNC.

$$A, \neg A \not\vDash_{\text{SbV}} A \& \neg A.$$

The explanation of this fact is that although there is a precisification in which  $A$  is true and there is a precisification in which  $\neg A$  is true, there is no precisification in which both  $A$  and  $\neg A$  are true.

In the subvaluation theory “validity” is defined as preservation of truth (as opposed to preservation of determinate truth). An argument is SbV-valid iff whenever the premises are true in some admissible precisification the conclusion is true in some admissible precisification. The precisification in which the premise is true need not be the same precisification in which the conclusion is true. It appears that all the classical tautologies are preserved in the subvaluation logic. On this account it is the principle of the exclusivity of truth-values that must fail. There are sentences which have two opposite truth-values. As has been mentioned LNC is still valid, however.

Unlike supervaluationism, subvaluationism claims that the sorites paradox is not valid (however, see below). It claims that in the sorites argument the paradoxical conclusion does not follow from the premises because the mode of inference, i.e. *modus ponendo ponens* (*MPP*) is not unrestrictedly valid. Consider the sorites paradox again:

- A pile of 100,000 grains is a heap.
- (A)  $\frac{\text{For any } n, \text{ if a pile of } n \text{ grains is a heap, then a pile of } n - 1 \text{ grains is a heap.}}{\text{A pile of 1 grain is a heap.}}$

The subvaluationists' argument to the effect that this reasoning is invalid goes as follows ([4], p. 648):

The sentence "A pile of  $n$  grains is a heap", where a pile of  $n$  grains counts as a borderline case for "heap", is both true and false; so it is true. Since it is also false, the material conditional "If a pile of  $n$  grains is a heap, then a pile of  $n - 1$  grains is a heap" is true by virtue of the falsity of its antecedent. Nonetheless, pile of  $n - 1$  grains might be determinately not a heap, thus making the sentence "A pile of  $n - 1$  grains is a heap" false. So, the conclusion is that *MPP* is not valid for material implication.

Subvaluationists treat vagueness as a species of *ambiguity*. Different precisifications are regarded as different meanings. The statement " $n$  grains can make a heap" is true on some meanings of "heap" and false on some other meanings. The sorites reasoning is invalid because it equivocates between different meanings. One meaning of "heap" is assumed in the minor premise and another meaning is taken into account in the major premise. As Hyde puts it ([4], p. 650):

*Modus ponens* applied to equivocal premises fails to be truth-preserving, but this is hardly news.

The conclusion Hyde draws is that the virtues and vices of the supervaluation theory are mirrored in the subvaluation theory. They use different concepts of truth, but as yet there are no conclusive arguments that would favour one of these concepts over the other. Hence, the same arguments that are used in defence of the supervaluation theory may be used to defend subvaluationism. The former is tenable only if the latter is.

#### 4. Evaluation of the subvaluation theory of vagueness

It is worth remembering that supervaluationism itself faces several serious problems. Subvaluationism is not any better off in addressing those problems. It shares supervaluationism's vices, but in addition it does not share some of supervaluationism's virtues. It has some of its own vices as well.

The main problems connected with supervaluationism are:

- (A) The fact that it is not able to handle higher-order vagueness.
- (B) The claim that vague statements are true prior to precisification is contentious.
- (C) Its solution to the sorites paradox is extremely counterintuitive.

Let us consider these objections in turn and check whether or not subvaluationism can handle them better than supervaluationism does.

### (A) Higher-order vagueness

**The supervaluation theory** is not able to accommodate vagueness within its framework. It proposes to analyse vague expressions in terms of admissible precisifications. Now, if “admissible precisification” is itself precise, then we end up with precise boundaries after all: one between positive cases and borderline cases and the other between borderline cases and negative cases. Hence, although there is no cut-off point between persons who are bald and ones who are not bald, there exists nevertheless a sharp boundary between clearly bald persons and those that are not clearly bald. This seems implausible, for instead of one sharp cut-off point one is merely offering two such points. Thus, one may argue that the supervaluation did not solve the problem but merely shifted it to another boundary (cf. [2], p. 196).

On the other hand, if “admissible precisification” is itself vague, then supervaluationism must give up the desire for a precise metalanguage. If it is unclear which precisifications are admissible, then it is also unclear which statements are true in all admissible precisifications. Therefore there will be statements which will be neither clearly (super)true nor clearly (super>false.

It is clear that **the subvaluation theory** is equally incapable of handling higher-order vagueness, because this incapability is strictly connected with the whole idea of precisifications. Hence, the higher-order vagueness objection formulated against supervaluationists applies here too.

Recall that the difference between borderline and other cases is defined in terms of precisifications; borderline statements are those that are true in some admissible precisifications and false in some other admissible precisifications, whereas, for instance, positive cases are those that are true in all admissible precisifications. Given that there is a limited number of admissible precisifications and assuming that each precisification is precise one may take the «extreme» precisification and obtain a clear division between borderline and positive cases. In the sorites argument concerning “heap”, in which the consecutive steps differ by a single grain, there must be a last statement which is both true and false, and whose immediate neighbour is determinately true. In a precise meta-language vagueness disappears.

On the other hand the analysis proposed by subvaluationism accords ill with a vague meta-language. In such a language borderline statements would be not only those that are both true and false but also those that are neither



clearly determinately true nor clearly borderline. In other words borderline cases would be not only those that come up true and false in different precisifications, but also those about which it is unclear whether they come up true and false. But then, the whole idea of precisifications is lost.

### (B) Ambiguity

In order to make their solution to the sorites paradox more plausible **supervaluationists** use an analogy with ambiguity (see [3], p. 284). They argue that ambiguity provides examples of disjunctions of the form  $A \vee \neg A$  which are true despite the fact that neither of the disjuncts is true. According to them, an ambiguous sentence is true if each of its disambiguations is true. Consider the ambiguous statement J:

(J) John went to the bank.

Let  $J_1$  and  $J_2$  be its disambiguations, i.e. “John went to the money-bank” and “John went to the river-bank” respectively, and suppose that only  $J_1$  is true. Then neither J nor  $\neg J$  is true, for each has a false disambiguation. However, the disjunction  $J \vee \neg J$  is true, because both its disambiguations, namely  $J_1 \vee \neg J_1$  and  $J_2 \vee \neg J_2$  are true.

This conception has been criticised. It has been argued that the assertion on which this reasoning is based, namely the assertion that an ambiguous sentence is true if each of its disambiguations is true, is itself not true. Tye argues ([10], p. 143) that

[t]he fact that an ambiguous sentence would be true were it to come out true under all its permissible disambiguations is not a good reason to hold that the sentence is, in fact, true, prior to disambiguation.

Tye claims that before the statement “John went to the bank” is disambiguated it is neither true nor false, since prior to disambiguation one cannot know what it means.

The same objection may be raised against the supervaluationists’ claim that

(2) There is an  $n$ , such that a pile of  $n$  grains is a heap, but a pile of  $n - 1$  grains is not a heap,

is true. Tye argues that the fact that (2) would be true, were “heap” made precise in any acceptable way, is not a reason to assert that (2) is true prior to any precisification of “heap” (see [10], p. 143).

It is worth pointing out however, that supervaluationists are by no means forced to treat vagueness as the same phenomenon as ambiguity (cf. [3], p. 282). The solution to the sorites does not hinge on the treatment of ambiguous statements. The analysis was meant merely to illustrate the solution. Hence, potential failure in supervaluationists' analysis of ambiguity is not tantamount to the incorrectness of their analysis of vagueness.

In contrast, it seems that the motivation for using **paraconsistent logic** as the analysis of the phenomenon of vagueness depends heavily on treating vagueness as a species of ambiguity. The fact that Jaśkowski regarded vagueness as the same phenomenon as ambiguity is made clear by the following remark of his ([6], p. 144, my emphasis):

Any vagueness of a term  $a$  can result in a contradiction of sentences, because with reference to the same object  $X$  we may say that “ $X$  is  $a$ ” and also “ $X$  is not  $a$ ”, *according to the meaning of the term  $a$  adopted for the moment.*

If we interpret vague properties as overlapping each other then we get simple inconsistency. On such an interpretation there just are patches which both are and are not red. Hence, LNC has to go. The claim that borderline statements are both true and false is consistent with LNC only if we interpret them as ambiguous; i.e. true under one meaning and false under another meaning. The claim that vagueness is a kind of ambiguity is extremely counterintuitive, however. Our intuitions are exactly opposite in fact. Vagueness and ambiguity seem to be two different phenomena. An expression is *ambiguous* if it has two (or more) different meanings. Those meanings can either be vague or precise. Before an ambiguous expression is disambiguated we do not know which meaning is relevant to the given occasion of use. Usually it is the context that does the disambiguating. If someone utters “Jones went to the bank. He withdraw 1000 £ from his account”, we know that it is the money-bank Jones went to. However, if someone said only “Jones went to the bank” we would not know whether Jones went fishing or went to make some cash transactions. Thus, the statement “Jones went to the bank” is not particularly informative. We learn that Jones went either to the money-bank or to the river-bank but we do not know to which one of them. Had the expression “bank” more meanings, we would learn even less. Therefore, it seems that one does not know what the statement “Jones went to the bank” means, until it is disambiguated. For, although one knows that it is one of the two meanings which is correct, one does not know which one.

In contrast, a *vague* expression does not switch between two or more different meanings. It has only one meaning which is not precise. One does not have to disambiguate the meaning of a vague expression “tall” in order to know what the statement “Jones is tall” means. This statement has only one meaning and it concerns Jones’s height. It is not a precise meaning, but nevertheless it is an unambiguous one.

Tye’s attack has been directed towards the supervaluation treatment of vague sentences. But its relevance to subvaluationism is even more evident. If an ambiguous statement cannot be considered true prior to its disambiguation even if *all* its disambiguations are true, then it cannot be deemed true prior to its disambiguation if *one* of its disambiguations is true. The claim that no matter what the speaker’s intentions are the utterance “Jones went to the bank” is true prior to its disambiguation in the case in which Jones went to the money-bank and did not go to the river-bank, seems counterintuitive.

Besides, “truth in a precisification” is a very weak notion. It obviously does not accord with the intuitions attached to the common notion of truth. It does not suffice to know that Jones went fishing in order to claim that someone’s utterance “Jones went to the bank” is true. One has to know in addition what that person meant by “bank”; which of its meanings he had in mind.

### (C) Solution to sorites

The supervaluationists’ solution to the sorites paradox has been heavily criticised for its counterintuitiveness. It seems implausible to accept an existential claim knowing that no answer to the question “What is the witness to it?” is available (see [5], p. 259). The theory of supervaluation forces one to accept both that the existential claim “there is such-and-such  $n$ ” is true, and that “this is such-and-such  $n$ ” is false about every  $n$  ([8], p. 51).

It is not entirely clear what the solution to the sorites paradox proposed by subvaluationists is. On the one hand, Hyde claims that the sorites paradox is not a paradox in fact, for the reasoning used in it is invalid. On the other hand, he argues that the paradox is a mere fallacy of equivocation. Moreover, this latter explanation is taken to be a qualification of the former.

As we have seen, Hyde claims that the sorites paradox does not threaten the subvaluation theory, because the subvaluation logic invalidates it. According to him *MPP* is not unrestrictedly valid in this logic and that is why the sorites conclusion does not follow.

It might seem, that the rejection of *MPP* depends on the assumption that there is no higher-order vagueness. The invalid step has the following

shape:

$$\begin{array}{ccccccc} p & \rightarrow & q, & p & \vDash_{\text{sbv}} & q \\ \text{F} & \text{T} & \text{F} & \text{T} & & \text{F} \end{array}$$

where:  $p$  – a pile of  $n$  grains is a heap;  $q$  – a pile of  $n - 1$  grains is a heap.  $p$  in the first premise is taken to be false and in the second premise — true.

In order to get invalidity two conditions have to be fulfilled:

- (a)  $p$  has to be a borderline statement and hence be both true and false;
- (b)  $q$  has to be a determinately false statement (for if it were borderline, it would be both true and false; hence true).

Notice that the heaps described in  $p$  and  $q$  differ merely by one grain. Hence the conditions (a) and (b) are tantamount to the claim that there is a sharp boundary between borderline and negative cases of “heap”.  $n$  grains is the last borderline case and  $n - 1$  grains is the first negative case. Hence, it seems that the phenomenon of higher-order vagueness does not exist. In order to avoid that conclusion the proponents of suvaluationism have to argue that although the boundary between borderline and negative cases exists, it is not boundary lies between  $n$  and  $n - 1$ .

Secondly, it has been argued that there are different formulations of the sorites reasoning possible, and the use of *MPP* is not always necessary (see [9], p. 468). Even in the formulation cited by Hyde one uses induction, rather than *MPP*. And in the formulation:

$$\begin{array}{l} 10000 \text{ grains make a heap.} \\ 1 \text{ grain does not make a heap.} \\ \hline \exists x(x \text{ grains make a heap \& } x - 1 \text{ grains do not make a heap}). \end{array}$$

neither *MPP* nor induction is used.

If this were the case the claim that *MPP* is invalid would not suffice to solve the sorites paradox and subvaluationists would still owe an account of how to deal with those other formulations. It is unclear however, whether there are any formulations that do not use *MPP* at all.<sup>2</sup> Mathematical induction is not different from *MPP* in finite cases and it seems that even in the above formulation *MPP* has to be used in order to derive the conclusion.

<sup>2</sup> Another well-known formulation of the sorites paradox is the following: 1 grain does not make a heap, Not: 1 grain does not make a heap and 2 grains make a heap, [...]. Hence 10000 grains do not make a heap. This formulation does not use *MPP*, but it uses *modus tollendo ponens* instead. Hence, subvaluationists’ solution could be applied here as well.

Thirdly, it seems that even in the formulation (A) it is not *MPP* that is the culprit. Given the definition of validity formulated above, *MPP* is invalid, because it does not preserve subvaluationists' truth. Because of the existence of statements that are both true and false, one can start with true premises and arrive at a false conclusion. However, this explanation seems at least insufficient. It is true that there are in this theory statements which are both true and false, but they are true and false in their different meanings. The use of *MPP* in the sorites reasoning equivocates between different meanings of  $p$ . To prove that a given form of reasoning is not valid in the case of equivocation is not to prove that it is not valid *simpliciter*. Blaming *MPP* for the invalidity of reasoning: "Jones went to the money-bank"; and "If Jones went to the river-bank, then Jones withdrew some money"; hence "Jones withdrew some money" seems utterly unjustified. It is not the mode of reasoning which is to be blamed. The reasoning contains a defect: it commits the fallacy of equivocation.

If the reasoning  $p \rightarrow q, p \vDash_{\text{SBV}} q$  equivocates between two different meanings of  $p$ , then it has the following form in fact:  $p \rightarrow q, r \vDash_{\text{SBV}} q$ . No wonder that it does not work. On the face of it *MPP* is invalid, but after closer investigation it appears that it has been simply misapplied. Thus, in the subvaluation theory the rejection of *MPP* lacks motivation.

There is another way in which **subvaluationism could provide a solution** to the sorites paradox, however. Consider the formulation (A) again. The minor premise is true in all admissible precisifications. So, it is determinately true. In contrast, the conclusion is false in all admissible precisifications. Hence, it is determinately false. What about the major premise? It seems that it is also false in all precisifications. Recall that in each precisification there is an  $n$  such that although  $n$  grains do make a heap,  $n - 1$  grains do not. It follows then, that the premise "For any  $n$ , if a pile of  $n$  grains is a heap, then a pile of  $n - 1$  grains is a heap" is determinately false on the subvaluationists' account. Hence, the subvaluation proposes the same sort of solution to the sorites paradox as supervaluation did: it claims that the major premise is false. Moreover, in order to justify this claim subvaluationists are forced to use the same arguments. For in order to claim that the major premise is determinately false, they have to argue that the statement:

$$\exists n(n \text{ grains make a heap} \ \& \ n - 1 \text{ grains do not make a heap})$$

is true. This statement is in fact true in all precisifications, so it is determinately true. Now, subvaluationists have to repeat supervaluationists' argument to the effect that although there exists such  $n$ , the actual witness

cannot be produced. Hence, they are faced with the counterintuitiveness objection immediately.

In the above reasoning the notion of truth is barely used. It is “determinate truth” that plays the major role. Notice, moreover, that subvaluationists’ defence of LNC also involves determinate truth-values. LNC is valid, because there is no such precisification in which “ $A \ \& \ \neg A$ ” is true. The claim that it is not true is tantamount to the claim that it is determinately false. It suggests that subvaluationists should take this latter notion, rather than the former one, to be *the* notion of truth. If they do so however, they will have to abandon paraconsistent logic and use the incomplete logic instead, for borderline statements are neither determinately true nor determinately false. But then, subvaluationism would collapse into supervaluationism.

## 5. Conclusion

The differences between the supervaluation and subvaluation theories speak to the disadvantage of the latter. There is nothing in subvaluationism that would help in replying to the objections raised against supervaluationism. As has been mentioned the same objections can be raised against the subvaluationism itself. Moreover, both theories employ notions of truth that are by no means common notions. However, supervaluationists’ (super)truth seems to have more in common with the ordinary notion of truth than the notion of “truth in a precisification”. The use of paraconsistent logic has been motivated by this latter notion. It seems though, that subvaluationists are not able to solve the sorites paradox unless they appeal to determinate truth, which is a counterpart of (super)truth. Moreover, paraconsistent logic can be applied to vagueness only if we make vagueness the same phenomenon as ambiguity. As has been argued there are strong arguments against such an identification.

What is even more important, subvaluationism, as well as supervaluationism, proposes to analyse vagueness in a precise metalanguage. Applying paraconsistent logic to precisifications results in the claim that although there is no sharp boundary between determinately true statements and determinately false statements, there are *two* boundaries instead: one separates determinately true statements from those that are both true and false, and the other separates true and false statements from determinately false ones. Someone who finds this implausible had better look for some other theory of vagueness.



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