

$B)) = 1$. This implies:

$$\begin{aligned} V(w_0, A \vee B) = 0 & \text{ iff } V(w_0, A) = 0 \text{ and } V(w_0, B) = 0 \\ & \text{ iff } V(w_0, \sim A) = 1 \text{ and } V(w_0, \sim B) = 1. \end{aligned}$$

Therefore, $S, T(\sim A), T(\sim B)$ is shown to be refutable.

Next, consider the rule ($F \sim \rightarrow$). By the assumption, there is a strong negation model (W, \leq, w_0, val, D) , in which w_0 refutes S and $V(w_0, \sim(A \rightarrow B)) \neq 1$. This implies:

$$V(w_0, A \rightarrow B) \neq 0 \text{ iff } V(w_0, A) \neq 1 \text{ and } V(w_0, \sim B) \neq 1.$$

Therefore, S, FA and $S, F(\sim B)$ are refutable.

Next, consider the rule ($T \sim \forall$). By the assumption, we have a strong negation model (W, \leq, w_0, val, D) , in which w_0 refutes S and $V(w_0, \sim \forall x A(x)) = 1$. This implies:

$$\begin{aligned} V(w_0, \sim \forall x A(x)) = 1 & \text{ iff } V(w_0, \forall x A(x)) = 0 \\ & \text{ iff } V(w_0, A(a)) = 0 \text{ for some } a \in D(w_0) \\ & \text{ iff } V(w_0, \sim A(a)) = 1 \text{ for some } a \in D(w_0). \end{aligned}$$

Here, a is subject to the variable restriction. Then, $S, T(\sim A(a))$ is also refutable. We can similarly handle other cases. \square

We are now in a position to prove completeness of TN^- , which is the converse of theorem 1. The proof below is similar to the Henkin proof described in Akama [3, 5].

A finite set of signed formulas Γ is *non-trivial* if no tableau for it is closed. An infinite set of signed formulas is non-trivial if every finite subset is non-trivial. If a set of formulas is not non-trivial, it is trivial. Every formula is provable from a trivial set.

DEFINITION 1. Let \mathbf{P} be a set of parameters and Γ a set of signed formulas. We say that Γ is *maximally non-trivial with respect to \mathbf{P}* if

- (1) every signed formula in Γ uses only parameters of \mathbf{P} ,
- (2) Γ is non-trivial,
- (3) for every formula A with parameters in \mathbf{P} , either $TA \in \Gamma$ or $FA \in \Gamma$.

We write $L(C)$ to denote the new language extending the set of constants C_L of the original language L with a set of constants $C = \{c_1, \dots, c_n\}$.

DEFINITION 2. We say that a non-trivial set of signed formulas Γ is *C-saturated* if

- (1) Γ is maximally non-trivial with respect to $C_L \cup C$,
- (2) if $T(\exists xA(x)) \in \Gamma$, then $T(A(c)) \in \Gamma$ for some $c \in C_L \cup C$,
- (3) if $T(\sim \forall xA(x)) \in \Gamma$, then $T(\sim A(c)) \in \Gamma$ for some $c \in C_L \cup C$.

LEMMA 1. A non-trivial set of signed formulas Γ_0 can be extended to a maximally non-trivial set of signed formulas Γ .

PROOF. Since the language L has a countably infinite set of sentences, we can enumerate sentences A_1, A_2, \dots . Now, we define for a non-trivial set of signed formulas Γ_0 a sequence of non-trivial sets of signed formulas $\Gamma_0, \Gamma_1, \Gamma_2, \dots$ in the following way:

$$\Gamma_{n+1} = \begin{cases} \Gamma_n \cup \{TA_{n+1}\} & \text{if } \Gamma_n \cup \{TA_{n+1}\} \text{ is non-trivial,} \\ \Gamma_n \cup \{FA_{n+1}\} & \text{if } \Gamma_n \cup \{FA_{n+1}\} \text{ is non-trivial,} \\ \Gamma_n & \text{otherwise.} \end{cases}$$

Then, we set

$$\Gamma = \bigcup \Gamma_i$$

It is obvious that Γ satisfies the desired properties of a maximally non-trivial set. \square

LEMMA 2. A non-trivial set Γ of signed formulas in L can be extended to a *C-saturated non-trivial set* Δ of signed formulas in $L(C)$.

PROOF. Let $\Gamma_0 = \Gamma$. Extend Γ_0 to a set Γ_1 maximally non-trivial with respect to C_L . Since $C = \{c_1, c_2, \dots\}$ is a countable set of constants not in L , we can enumerate sentences of the form $\exists xA(x)$ in $L(C)$ as $\exists xA_1(x), \exists xA_2(x), \dots$. By definition $\forall xA(x) \leftrightarrow \sim \exists x \sim A(x)$, we here only consider the case of $\exists xA(x)$. Γ_i can be then defined for any $n \geq 1$ as follows:

($i = 2n$) Take the first formula of the form $\exists xA_m(x)$. If $T(\exists xA(x)) \in \Gamma_{n-1}$ but $T(A(c)) \notin \Gamma_{n-1}$ for all $c \in C_L \cup \{c_1, \dots, c_n\}$, then set $\Gamma_{2n} = \Gamma_{2n-1} \cup \{T(A(c_1))\}$. We need a similar construction on the formula of the form $\sim \forall xA_m(x)$.

($i = 2n + 1$) By Lemma 1, we extend Γ_{2n} to Γ_{2n+1} , which is maximally non-trivial with respect to $C_L \cup C$.

Then, we define $\Delta = \bigcup \Gamma_i$. It suffices to check that Δ is *C-saturated*. Because each Γ_n is non-trivial, Δ is also non-trivial. Let $A(c)$ be any sentence

of $L(C)$ with $c \in C$. Due to the maximality of Γ_{2n+1} , one of the conditions $TA \in \Gamma_{2n+1}$, $FA \in \Gamma_{2n+1}$, or $\Gamma_{2n+1} \cup \{TA\}$ and $\Gamma_{2n+1} \cup \{FA\}$ are provable, holds. Thus, Δ is shown to be maximally in $L(C)$. Finally, $T(\exists xA(x)) \in \Delta$ (also $T(\sim \forall xA(x)) \in \Delta$). We suppose $\exists xA_n(x)$ (also $\sim \forall xA(x)$), i.e. the n -th enumeration. From the above construction, $T(A(c)) \in \Delta$ (also $T(\sim A(c)) \in \Delta$) for some $c \in C_L \cup \{c_1, \dots, c_n\}$ must be satisfied. This implies that Δ is C -saturated. \square

We here define a canonical strong negation model with respect to the tableau:

DEFINITION 3. Let C_0, C_1, C_2, \dots be a countable sequence of disjoint countable sets of constants not occurring in L . We denote $C_0 \cup C_1 \cup \dots \cup C_n$ by C_n^* . Then, we define a canonical strong negation model (W, \leq, w_0, val, D) as follows:

- (1) $W = \{\Gamma \mid \Gamma \text{ is } C_n^*\text{-saturated in } L(C_n^*) \text{ for some } n\}$.
- (2) If Γ is C_n^* -saturated and $L(\Gamma) = L \cup C_n^*$, then $D(\Gamma) = C_n^*$.
- (3) Define \leq in the following way:

$$\Gamma \leq \Delta \text{ iff } TA \in \Gamma \Rightarrow TA \in \Delta \text{ and } D(\Gamma) \subseteq D(\Delta).$$

- (4) For any $\Gamma \in W$ and atomic formula $p(c)$ with $c \in D(\Gamma)$, we define the three-valued valuation val as:

$$val(\Gamma, p(c)) = \begin{cases} 1 & \text{if } T(p(c)) \in \Gamma, \\ 0 & \text{if } T(\sim p(c)) \in \Gamma, \\ -1 & \text{otherwise.} \end{cases}$$

We can easily verify that a canonical strong negation model satisfies the desired properties of a strong negation model.

LEMMA 3. For any $\Gamma \in W$ in a canonical strong negation model (W, \leq, w_0, val, D) , we have the following properties:

- (1) if $T(A \& B) \in \Gamma$, then $TA \in \Gamma$ and $TB \in \Gamma$,
- (2) if $F(A \& B) \in \Gamma$, then $FA \in \Gamma$ or $FB \in \Gamma$,
- (3) if $T(A \vee B) \in \Gamma$, then $TA \in \Gamma$ or $TB \in \Gamma$,
- (4) if $F(A \vee B) \in \Gamma$, then $FA \in \Gamma$ and $FB \in \Gamma$,
- (5) if $T(A \rightarrow B) \in \Gamma$, then $FA \in \Gamma$ or $TA \in \Gamma$,
- (6) if $F(A \rightarrow B) \in \Gamma$, then for some Δ such that $\Gamma \leq \Delta$, $TA \in \Delta$ and $FB \in \Delta$,

- (7) if $T(\sim(A \& B)) \in \Gamma$, then $T(\sim A) \in \Gamma$ or $T(\sim B) \in \Gamma$,
- (8) if $F(\sim(A \& B)) \in \Gamma$, then $F(\sim A) \in \Gamma$ and $F(\sim B) \in \Gamma$,
- (9) if $T(\sim(A \vee B)) \in \Gamma$, then $T(\sim A) \in \Gamma$ and $T(\sim B) \in \Gamma$,
- (10) if $F(\sim(A \vee B)) \in \Gamma$, then $F(\sim A) \in \Gamma$ or $F(\sim B) \in \Gamma$,
- (11) if $T(\sim(A \rightarrow B)) \in \Gamma$, then $TA \in \Gamma$ and $T(\sim B) \in \Gamma$,
- (12) if $F(\sim(A \rightarrow B)) \in \Gamma$, then $FA \in \Gamma$ or $F(\sim B) \in \Gamma$,
- (13) if $T(\sim\sim A) \in \Gamma$, then $TA \in \Gamma$,
- (14) if $F(\sim\sim A) \in \Gamma$, then $FA \in \Gamma$,
- (15) if $T(\forall xA(x)) \in \Gamma$, then $T(A(a)) \in \Gamma$ for all $a \in D(\Gamma)$,
- (16) if $F(\forall xA(x)) \in \Gamma$, then for some Δ such that $\Gamma \leq \Delta$, $F(A(a)) \in \Delta$ for some $a \in D(\Delta)$,
- (17) if $T(\exists xA(x)) \in \Gamma$, then $T(A(a)) \in \Gamma$ for some $a \in D(\Gamma)$,
- (18) if $F(\exists xA(x)) \in \Gamma$, then $F(A(a)) \in \Gamma$ for all $a \in D(\Gamma)$,
- (19) if $T(\sim \forall xA(x)) \in \Gamma$, then $T(\sim A(a)) \in \Gamma$ for some $a \in D(\Gamma)$,
- (20) if $F(\sim \forall xA(x)) \in \Gamma$, then $F(\sim A(a)) \in \Gamma$ for all $a \in D(\Gamma)$,
- (21) if $T(\sim \exists xA(x)) \in \Gamma$, then $T(\sim A(a)) \in \Gamma$ for all $a \in D(\Gamma)$,
- (22) if $F(\sim \exists xA(x)) \in \Gamma$, then for some Δ such that $\Gamma \leq \Delta$, $F(\sim A(a)) \in \Delta$ for some $a \in D(\Delta)$.

PROOF. For (1), assume that $T(A \& B) \in \Gamma$ but $TA \notin \Gamma$. If $\Gamma, T(A \& B)$ is non-trivial, then $\Gamma, T(A \& B), TA$ is also non-trivial. But, it contradicts the maximality of Γ . Thus, $TA \in \Gamma$. Similarly, $TB \in \Gamma$. Similar reasoning can be applied to (4), (8), (9), (11), (13) and (14).

For (3), assume that $TA \notin \Gamma$ and $TB \notin \Gamma$. Since Γ is maximal, both Γ, TA and Γ, TB are inconsistent (i.e. provable). For a finite subset S of Γ , both S, TA and S, TB are inconsistent. By $(T\vee)$, $S, T(A \vee B)$ is also provable. This implies that $\Gamma, T(A \vee B)$ is provable and $T(A \vee B) \notin \Gamma$. We can justify (2), (5), (7), (10) and (12) in the similar way.

For (6), by $F(A \rightarrow B) \in \Gamma$ we have Γ_T, TA, FB is not provable. By Lemma 1, Γ_T, TA, FB can be extended to a maximally non-trivial set Δ such that $TA \in \Delta$ and $FB \in \Delta$.

For (15), assume $T(\forall xA(x)) \in \Gamma$ but $T(A(a)) \notin \Gamma$ for all $a \in D(\Gamma)$. If $\Gamma, T(\forall xA(x))$ is consistent, so is $\Gamma, T(\forall xA(x)), T(A(a))$. So Γ is not maximal. Thus, $T(A(a)) \in \Gamma$. We can similarly deal with (17), (19) and (21).

For (16), assume $F(\forall xA(x)) \in \Gamma$. If a does not occur in Γ , then $\Gamma_T, F(A(a))$ is consistent. By lemma 2, we can extend $\Gamma_T, F(A(a))$ to Δ such that $F(A(a)) \in \Delta$ for some $a \in D(\Delta)$. Similarly, for (18), (20) and (22). \square

THEOREM 2. *For any $\Gamma \in W$ in a canonical strong negation model and any formula A ,*

$$\begin{aligned} TA \in \Gamma & \text{ iff } V(\Gamma, A) = 1, \\ FA \in \Gamma & \text{ iff } V(\Gamma, A) \neq 1. \end{aligned}$$

PROOF. By induction on A . The case A is an atomic formula is immediate.

(1) $A = B \& C$ (also $\sim(B \vee C)$):

$$\begin{aligned} T(B \& C) \in \Gamma & \text{ iff } TB \in \Gamma \text{ and } TC \in \Gamma \\ & \text{ iff } V(\Gamma, B) = 1 \text{ and } V(\Gamma, C) = 1 \\ & \text{ iff } V(\Gamma, B \& C) = 1. \\ F(B \& C) \in \Gamma & \text{ iff } FB \in \Gamma \text{ or } FC \in \Gamma \\ & \text{ iff } V(\Gamma, B) \neq 1 \text{ or } V(\Gamma, C) \neq 1 \\ & \text{ iff } V(\Gamma, B \& C) \neq 1. \end{aligned}$$

(2) $A = B \vee C$ (also $\sim(B \& C)$):

$$\begin{aligned} T(B \vee C) \in \Gamma & \text{ iff } TB \in \Gamma \text{ or } TC \in \Gamma \\ & \text{ iff } V(\Gamma, B) = 1 \text{ or } V(\Gamma, C) = 1 \\ & \text{ iff } V(\Gamma, B \vee C) = 1. \\ F(B \vee C) \in \Gamma & \text{ iff } FB \in \Gamma \text{ and } FC \in \Gamma \\ & \text{ iff } V(\Gamma, B) \neq 1 \text{ and } V(\Gamma, C) \neq 1 \\ & \text{ iff } V(\Gamma, B \vee C) \neq 1. \end{aligned}$$

(3) $A = B \rightarrow C$:

$$\begin{aligned} T(B \rightarrow C) \in \Gamma & \text{ iff } \forall \Delta \geq \Gamma (T(B \rightarrow C) \in \Delta) \\ & \text{ iff } \forall \Delta \geq \Gamma (FB \in \Delta \text{ or } TC \in \Delta) \\ & \text{ iff } \forall \Delta \geq \Gamma (V(\Delta, B) \neq 1 \text{ or } V(\Delta, C) = 1) \\ & \text{ iff } V(\Gamma, B \rightarrow C) = 1. \\ F(B \rightarrow C) \in \Gamma & \text{ iff } \exists \Delta \geq \Gamma (TB \in \Delta \text{ and } TC \in \Delta) \\ & \text{ iff } \exists \Delta \geq \Gamma (V(\Delta, B) = 1 \text{ and } V(\Delta, C) \neq 1) \\ & \text{ iff } V(\Gamma, B \rightarrow C) \neq 1. \end{aligned}$$

(4) $A = \sim \sim B$:

$$\begin{aligned}
 T(\sim \sim B) \in \Gamma & \text{ iff } TB \in \Gamma \\
 & \text{ iff } V(\Gamma, B) = 1 \\
 & \text{ iff } V(\Gamma, \sim B) = 0 \\
 & \text{ iff } V(\Gamma, \sim \sim B) = 1. \\
 F(\sim \sim B) \in \Gamma & \text{ iff } FB \in \Gamma \\
 & \text{ iff } V(\Gamma, B) \neq 1 \\
 & \text{ iff } V(\Gamma, \sim B) \neq 0 \\
 & \text{ iff } V(\Gamma, \sim \sim B) \neq 1.
 \end{aligned}$$

(5) $A = \exists xB(x)$ (also $\sim \forall xA(x)$):

$$\begin{aligned}
 T(\exists xB(x)) \in \Gamma & \text{ iff } T(B(a)) \in \Gamma \text{ for some } a \in D(\Gamma) \\
 & \text{ iff } V(\Gamma, B(a)) = 1 \text{ for some } a \in D(\Gamma) \\
 & \text{ iff } V(\Gamma, \exists xB(x)) = 1. \\
 F(\exists xB(x)) \in \Gamma & \text{ iff } F(B(a)) \in \Gamma \text{ for all } a \in D(\Gamma) \\
 & \text{ iff } V(\Gamma, B(a)) \neq 1 \text{ for all } a \in D(\Gamma) \\
 & \text{ iff } V(\Gamma, \exists xB(x)) \neq 1.
 \end{aligned}$$

(6) $A = \forall xB(x)$ (also $\sim \exists xB(x)$):

$$\begin{aligned}
 T(\forall xB(x)) \in \Gamma & \text{ iff } \forall \Delta \geq \Gamma (T(B(a)) \in \Delta \text{ for all } a \in D(\Delta)) \\
 & \text{ iff } \forall \Delta \geq \Gamma (V(\Delta, B(a)) = 1 \text{ for all } a \in D(\Delta)). \\
 & \text{ iff } V(\Gamma, \forall xB(x)) = 1 \\
 F(\forall xB(x)) \in \Gamma & \text{ iff } \exists \Delta \geq \Gamma (F(B(a)) \in \Delta \text{ for some } a \in D(\Delta)) \\
 & \text{ iff } \exists \Delta \geq \Gamma (V(\Delta, B(a)) \neq 1 \text{ for some } a \in D(\Delta)) \\
 & \text{ iff } V(\Gamma, \forall xB(x)) \neq 1. \quad \square
 \end{aligned}$$

As a consequence, we obtain the completeness of TN :

THEOREM 3 (Completeness Theorem). $\vdash_{TN^-} A$ iff $\models_{N^-} A$.

PROOF. The soundness was already proved in theorem 1. For the completeness, it suffices to show that an open tableau is refutable by a counter strong negation model by theorem 2. By contraposition, we reach the completeness theorem. \square



We can also see that N^- is a paraconsistent logic.

THEOREM 4. $A \& \sim A \rightarrow B$ is not provable in N^- .

PROOF. Due to the lack of (AX3) in TN^- , we cannot prove the formula. This means that in N^- both A and $\sim A$ can be true at some world w for some formula A as in other paraconsistent logics. \square

4. Discussion

We have addressed the logical aspects of Almkvad and Nelson's paraconsistent constructive logic N^- . The Nelson systems strengthen the concept of negation in intuitionistic logic in that strong negation is *constructible*. In addition, strong negation has rather surprising features concerning a contradiction. Namely, all contradictions are equivalent in both classical and intuitionistic logic, but they may not be equivalent in N^- . We also observe that $A \leftrightarrow \sim A$ can be consistently added as an axiom for some formula A in N^- . Like all paraconsistent logics, N^- can form an inconsistent, but non-trivial, theory.

Now, we compare N^- with da Costa's projects on paraconsistent logics initiated in 1974. First, da Costa used classical logic as the starting point, whereas Nelson used intuitionistic logic as the basis. Secondly, the da Costa constructions of non-trivial theories in C_ω or related systems rely on pedagogical ideas to tolerate contradictions. Thirdly, the semantics for the da Costa systems are not always intuitive. The Kripke semantics for N^- has the intuitive appeal. In fact, strong negation can be interpreted as the refutability. On these grounds, Nelson should be counted as a forerunner of paraconsistent logics.

Why did not Nelson explicitly develop "paraconsistent" logics? Indeed Nelson did formulate a paraconsistent logic in Nelson [17], but some discussions may be in order. For instance, we can find the following remarks (p. 224); "The system has been constructed, of course, to show that the logical operations may be interpreted in such a way that a mathematical system for arithmetic may be inconsistent without being overcomplete." A similar observation also appeared in Almkvad and Nelson [9]. The constructive system S in Nelson [17] is slightly different from N^- . Technically, S is a variant of N^- by deleting contraction and adding contraposition. Similar systems were also investigated by Zaslavskii [28] and Akama [6, 7]. These works reveal that paraconsistency is closely related to structural rules in the sense of Gentzen systems (cf. Wansing [26]). In this regard, further work is

needed to address the connection of subsystems of N^- and substructural logics. We should also point out that the da Costa constructions using the formula of the form A° can also be made in the Nelson systems. But, Nelson's logics can dispense with da Costa's ideas to construct a paraconsistent system.

Nelson [17] continued, "Does the system have any practical interest? I should not want to claim much in this direction." For mathematicians, Nelson's logics are alternative to Heyting's intuitionistic logic. It is thus clear that paraconsistent mathematics is possible in the arithmetic based on N^- . Philosophers appear to be interested in paraconsistent constructive logics. For example, Fitch [11] proposed a similar system with strong negation to overcome paradoxes. Prawitz [20] firstly noted the usefulness of N^- for naive set theory in the context of natural deduction. The line of investigation was recently explored by Akama [4, 7]. The contractionless fragment of N^- is capable of giving a formalization of naive set theory. The paraconsistent constructive logics are also attracted to those who are working on computer science, in particular, artificial intelligence. Pearce and his students showed that Nelson's logics can serve as the foundation for logic programming in Pearce and Wagner [18, 19] and Wagner [24, 25]; also see Akama [6]. Wagner [25] and Wansing [27] also tried to employ strong negation systems for non-monotonic reasoning. Akama [8] recently sketched another tableau formulation of N^- , providing a wider fragment of logic programming with strong negation. We believe that Nelson's paraconsistent constructive logics deserve further investigations from both foundational and practical point of view.

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