

Furthermore some postulates can be stated in \mathcal{ST} again, for instance Post. 2 to Post. 5 in [10], that allow us to prove, e.g., the uniqueness of the actual space-time metric — see Theorem 2 of \mathcal{ST} in [10, sect. 3]. The existence of the same metric is asserted just by Post. 1 of the object theory \mathcal{T} in [10, sect. 2]. Incidentally the auxiliary theory \mathcal{ST} , which contains more notions than \mathcal{T} , is also supposed to include all postulates of \mathcal{T} (and some additional postulates).

For the present aims it is not necessary to consider the details of the afore-mentioned definitions and postulates. It suffices to note that in order to state, for instance, the existence and uniqueness of the space-time metric, one can in effect use a necessity assumption and a possibility one. In more details, in [10] the following pattern is implemented in this connection; and incidentally the same pattern is used for defining, e.g., mass in classical particle mechanics — see footnote 1 in [10].¹⁰ First, in the afore-mentioned Definition 1 of space-time metric one requires that, if certain experiments take place, then necessarily certain results are obtained.

Second, the existence of this metric is postulated (Post. 1 in [10]) and thus the afore-mentioned necessity assumption is in effect stated.

Third, the afore-mentioned possibility assumption is stated just as a postulate — see Post. 5 in [10] — and incidentally it is essential, as well as the necessity assumption, in order to prove the afore-mentioned Theorem 2 (of uniqueness) in [10].

3. Some preliminaries for \mathcal{ST} 's 2nd formalization

Since \mathcal{T} 's 2nd formalization is based on [2], let us note that within the logical modal calculus MC^ν introduced there one defines an analogue for the set of possible worlds: the set $\mathcal{E}\ell$ of *absolute elementary ranges* — see [2, Definition 48.2], p. 204 — as well as the assertion $|_u$ which contains only the variable „ u ” free and in effect means *the possible world $u(\in \mathcal{E}\ell)$ occurs*. Then $\diamond \cdot |_u \wedge p$ says that p occurs in u . Let us remember the logical theorems

$$(3.1) \quad \begin{array}{ll} \vdash \mathcal{E}\ell \in \text{Abs}, & \vdash \diamond |_{u} \supset u \in \mathcal{E}\ell, \\ \vdash |_{u} \wedge^{\cup} p \equiv |_{u} \supset^{\cap} p, & \vdash \Box p \equiv (\forall u \in \mathcal{E}\ell) \cdot |_{u} \wedge^{\cup} p. \end{array}$$

where, e.g.

$$(3.2) \quad \begin{array}{ll} p \supset^{\cap} q \equiv_D \Box \cdot p \supset q, & x =^{\cap} y \equiv_D \Box x = y, \\ p \wedge^{\cup} q \equiv_D \diamond \cdot p \wedge q, & x \in^{\cap} F \equiv_D \Box x \in F. \end{array}$$

¹⁰ The same pattern is used to define force, besides mass, within a non-mandatory part of my notes for students of Rational Mechanics.

In case $\mathcal{W} \subseteq \mathcal{E}\ell$, it is useful to define p is $\begin{cases} \text{possible} \\ \text{necessary} \end{cases}$ on \mathcal{W} and, e.g., F holds for x on \mathcal{W} by

$$(3.3) \quad \begin{cases} \diamond_{\mathcal{W}} p \\ \square_{\mathcal{W}} p \end{cases} \equiv_D \mathcal{W} \subseteq \mathcal{E}\ell \wedge \begin{cases} (\exists u \in \mathcal{W}) \\ (\forall u \in \mathcal{W}) \end{cases} \cdot |_u \wedge^{\cup} p,$$

$$x \in {}_{\mathcal{W}}F \equiv_D \square_{\mathcal{W}} x \in F.$$

Obviously, e.g., $=_{\mathcal{W}}$ (equal on \mathcal{W}), $\supset_{\mathcal{W}}$, and $\subset_{\mathcal{W}}$ are defined like $\in_{\mathcal{W}}$. By (3.3) we have the logical theorems:

$$(3.4) \quad \begin{aligned} \mathcal{W} \subseteq \mathcal{E}\ell \supset \cdot \square_{\mathcal{W}} p &\equiv \sim \diamond_{\mathcal{W}} \sim p, \\ \diamond_{\mathcal{E}\ell} p &\equiv \diamond p, \\ \vdash p \supset_{\mathcal{E}\ell} q &\equiv \cdot p \supset^{\cap} q. \end{aligned}$$

To prove (3.4)₁ assume q , where $q \equiv_D \mathcal{W} \subseteq \mathcal{E}\ell$. By (3.3)₁ and (then) (3.1)₃ it easily implies the equivalences $\sim \diamond_{\mathcal{W}} \sim p \equiv \sim q \vee \sim (\exists u \in \mathcal{W}) (|_u \wedge^{\cup} \sim p) \equiv q \supset (\forall u \in \mathcal{W}) (|_u \supset^{\cap} p) \equiv (\forall u \in \mathcal{W}) (|_u \wedge^{\cup} p) \equiv \square_{\mathcal{W}} p$ by (3.3)₁. We conclude that (3.4)₁ holds.

For $\mathcal{W} \subseteq \mathcal{E}\ell$, let us also define F is *modally constant*, *modally separated*, or (*modally*) *absolute on \mathcal{W}* , \mathcal{W} holds (briefly $|\mathcal{W}$), and F is *extensional at \mathcal{W}* by

$$(3.5) \quad F \in \begin{cases} M\text{Const}_{\mathcal{W}} \\ \text{Sep}_{\mathcal{W}} \end{cases} \equiv_D \mathcal{W} \subseteq^{\cap} \mathcal{E}\ell \wedge \begin{cases} (\forall x) \cdot \diamond_{\mathcal{W}} x \in F \supset x \in {}_{\mathcal{W}}F, \\ (\forall x, y) \cdot \diamond_{\mathcal{W}} (x \in F \wedge y \in F \wedge x = y) \supset x =_{\mathcal{W}} y, \end{cases}$$

$$(3.6) \quad \begin{aligned} \text{Abs}_{\mathcal{W}} &=_{\text{D}} M\text{Const}_{\mathcal{W}} \cap M\text{Sep}_{\mathcal{W}}, \\ |\mathcal{W} &\equiv_D \mathcal{W} \subseteq \mathcal{E}\ell \wedge (\exists u \in \mathcal{W}) |_u, \end{aligned}$$

and

$$(3.7) \quad F \in \text{Ext}_{(\mathcal{W})} \equiv_D (\forall x, y) \cdot x =_{\mathcal{W}} y \wedge x \in F \supset_{\mathcal{W}} y \in F.$$

It is natural to define \mathbb{P} is an (*absolute*) *partition* of $\mathcal{E}\ell$ by — see footnote 5

$$(3.8) \quad \begin{aligned} \mathbb{P} \in \text{AbPrt}_{\mathcal{E}\ell} &\equiv_D \mathbb{P} \in \text{Abs} \wedge \cup \mathbb{P} = \mathcal{E}\ell \wedge \\ &(\forall \mathcal{W}, \mathcal{W}' \in \mathbb{P}) \cdot \mathcal{W} \in \text{Abs} \wedge \cdot \mathcal{W} \cap \mathcal{W}' \neq \emptyset \equiv \mathcal{W} = \mathcal{W}'. \end{aligned}$$

In connection with such \mathbb{P} it is useful to say that F is *extensional w.r.t. \mathbb{P}* , if F is both absolute on and extensional at every member \mathcal{W} of \mathbb{P} :

$$(3.9) \quad F \in \text{Ext}^{(\mathbb{P})} \equiv_D \mathbb{P} \in \text{AbPrt}_{\mathcal{E}\ell} \wedge \\ (\forall \mathcal{W} \in \mathbb{P}). F \in \text{Abs}_{\mathcal{W}} \wedge F \in \text{Ext}_{(\mathcal{W})} .^{11}$$

Briefly it is obvious that

(A) if \mathbb{P} and Q are partitions of $\mathcal{E}\ell$, Q is finer than \mathbb{P} , and the property F is absolute on \mathbb{P} 's members, then it is certainly such also on Q 's members.

However, under the reasonable assumption

$$(3.10) \quad \Box(\exists x, y)x \neq y,$$

(B) if in addition Q is strictly finer than \mathbb{P} and F is extensional w.r.t. Q , then F cannot be absolute on \mathbb{P} 's members.

More thoroughly, it is easy to check that by (3.10)

(C) if the property F is extensional w.r.t. a partition \mathbb{P} of $\mathcal{E}\ell$, then this partition is uniquely determined:

$$(3.11) \quad \vdash F \in \text{Ext}^{(\mathbb{P})} \wedge F \in \text{Ext}^{(Q)} \supset \mathbb{P} = Q, \\ \text{i.e. } (\exists \mathbb{P})F \in \text{Ext}^{(\mathbb{P})} \supset (\exists_1 \mathbb{P})F \in \text{Ext}^{(\mathbb{P})} .$$

By (c) and (d) in sect. 1 the notion $\mathcal{E}\mathcal{P}$ of event points, reasonably used by most scientists in connection with the semantics of the theory \mathcal{T} (or with \mathcal{ST}), appears to be extensional just w.r.t. the partition OPW of the SPW s, determined by the objective possible worlds. Thus by (3.11) this partition can be defined naturally within \mathcal{ST} :

$$(3.12) \quad OPW =_D (i\mathbb{P})\mathcal{E}\mathcal{P} \in \text{Ext}^{(\mathbb{P})} \\ (\Vdash OPW \neq a^* =_D (iF)F \neq F, \text{ the non-existing object}).$$

4. Second formalization of the auxiliary theory \mathcal{ST} , regarded to embody the object theory \mathcal{T}

We regard the auxiliary theory \mathcal{ST} to have, as primitives both \mathcal{T} 's primitives except *admissible space-time frame* and *space-time metric*, and the notions (1) to (7) below (2.3) in [10]. In addition let \mathcal{ST} include the nonlogical axiom (3.10) and the postulates

¹¹ Note that the replacement of „ $F \in \text{Ext}_{(\mathcal{W})}$ ” in (3.9) with „ $F \in \text{Ext}_{\mathcal{W}}$ ”, which expresses *F is extensional on \mathcal{W}* — i.e. $(\forall x, y).x = y \wedge x \in F \supset_{\mathcal{W}} y \in F$ — would cause \mathcal{W} to be a singleton under the assumption (3.10).

$$(4.1) \quad (\exists \mathbb{P} \in \text{AbPrt}_{\mathcal{E}\ell}) \mathcal{E}\mathcal{P} \in \text{Ext}^{(\mathbb{P})} \quad \text{--- see (3.8)–(3.9)}$$

$$\quad \square. \mathcal{E}\mathcal{P} \notin (\text{Ext} \cup \text{Abs}) .$$

By (4.1)₁ and (3.11)₂ the condition of exact uniqueness for definition (3.12)₁ is satisfied. Hence — see (3.11) and (3.6)₂ —

$$(4.2) \quad \vdash OPW \in \text{AbPrt}_{\mathcal{E}\ell}, \quad \vdash (\exists_1 \mathcal{W} \in OPW) |^{\mathcal{W}} .$$

By (4.1)₂ the nontriviality of $\mathcal{E}\ell$'s partition $\mathcal{E}\mathcal{P}$ is in effect stated.

Of course we regard \mathcal{ST} to contain \mathcal{T} 's postulates and the additional postulates Post. 2 to Post. 5 considered in [10, sects. 2–3]. In order to express these postulates easily, it is convenient to introduce the *actual objective possible world* \mathcal{A} by

$$(4.3) \quad \mathcal{A} =_D (\iota \mathcal{W} \in OPW) |^{\mathcal{W}}, \quad \text{hence } \vdash \square |^{\mathcal{A}} \text{ by (4.2)}_2,$$

as well as the *extensionalization* $F^{e,\mathcal{A}}$ of the property F with respect to \mathcal{A} :

$$(4.4) \quad F^{e,\mathcal{A}} =_D (\lambda x) (\exists y \in F) x =_{\mathcal{A}} y \quad \text{--- see below (3.3).}$$

By (4.3)₁, (4.2)₁, (3.8), (3.6)₁, and (3.5),

$$(4.5) \quad \vdash \mathcal{A} \notin \text{MSep}, \quad \vdash \mathcal{A} \notin \text{MConst}, \quad \vdash \mathcal{A} \notin OPW$$

— see footnote 5 — and

$$(4.6) \quad \vdash \mathcal{A} \subseteq^{\cap} \mathcal{E}\ell, \quad \vdash \mathcal{A} \in OPW^{(e)},$$

where $F^{(e)} =_D (\lambda x) (\exists y \in F) x = y$.

Now one can easily check that the modal operators (restricted to the actual OPW), in effect used within \mathcal{ST} 's additional Posts. 2–5 written in [10, sects. 2–3], can be identified with $\square_{\mathcal{A}}$ and $\diamond_{\mathcal{A}}$ (in \mathcal{ST} 's 2nd formalization). In Post. 2.1 the space-time frame ϕ is in effect said to be a function from event points to \mathbb{R}^4 . In order to specify the properties of this notion — also used in [10] within Post. 4, Definition 1, and Post. 5 — let us now consider the natural absolute notion \mathcal{R} of the real numbers (constructed in the usual way on the basis of the analogous notion \mathbb{N} of natural numbers defined in [2]); furthermore set

$$(4.7) \quad \mathbb{R} =_D \mathcal{R}^{e,\mathcal{A}}, \quad \text{hence } \vdash \mathbb{R} \in \text{Abs}_{\mathcal{A}} \text{ and } \vdash \mathbb{R} \notin \text{Abs} .$$

Now we can say that we must have

$$(4.8) \quad \phi \in (\mathcal{E}\mathcal{P} \rightarrow \mathbb{R}^4) \quad (\text{obviously } \vdash \square \mathcal{E}\mathcal{P} \in \text{Abs}_{\mathcal{A}} \cap \text{Ext}_{(\mathcal{A})})$$

and that *frame on* S_4 can be regarded as the (extensional) class of these functions. Since \mathcal{EP} is not absolute, one cannot replace \mathbb{R} in (4.8) with \mathcal{R} . In fact, for some \mathcal{E}_1 and \mathcal{E}_2 , we have $\diamond.\mathcal{E}_1 = a^* = \mathcal{E}_2$ where $a^* =_D (\iota x)x \neq x$ (the nonexisting object), $\mathcal{E}_1 \neq \mathcal{E}_2$, and $\mathcal{E}_1, \mathcal{E}_2 \in \mathcal{EP}$. Hence from $\phi \in (S_4 \rightarrow \mathcal{R}^4)$ and $\mathcal{R} \in \text{Abs}$ one deduces that $\diamond.\phi(\mathcal{E}_1) = \phi(\mathcal{E}_2)(= a^*)$, $\phi(\mathcal{E}_1) \neq \phi(\mathcal{E}_2)$, and $\phi(\mathcal{E}_1), \phi(\mathcal{E}_2) \in \mathcal{R}^4$; and this is absurd — see footnote 5.

We can identify S_4 with \mathcal{EP} added, if preferred, with the space-time topology — see Post. 3 in [10]. Now we can define (*continuous*) *line of* S_4 and can regard this notion as satisfying condition (4.8)₂ in \mathcal{EP} . Thus we can mean ℓ is a *possible world-line of the test particle* \bar{P} as follows:

$$(4.9) \quad \ell \in (\text{line of } S_4) \wedge \diamond_{\mathcal{A}}\ell = \text{the world line of } \bar{P}$$

— see (4) in [10, sect. 1] — where $S_4 \in \text{Abs}_{\mathcal{A}}$ and $(\text{line of } \bar{P}) \in \text{Abs}_{\mathcal{A}}$. Modalities often occur through such a notion in sects. 2 to 3 — see e.g. Post. 4 (11), Definition 1 (α) to (γ), and the consequent of Post. 5 in [10].

Part II

On the influence of relativistic theories on the semantics of pragmatic languages in Carnap's sense

5. On the relativistic semantics of „now”, „past”, and „future” for the present human community

Briefly speaking, in working on e.g. the semantics of pragmatic languages in Carnap's sense (so that, e.g., „now” and „here” are involved) it may appear natural to interpret the „now” uttered by a speaker \mathcal{A} (Adam) at the event point \mathcal{E} , as „in \mathcal{E} 's casual present”, i.e. as „in the complement of \mathcal{E} 's causal past $\{\mathcal{E}' : \mathcal{E}' \prec \mathcal{E}\}$ joint with \mathcal{E} 's causal future $\{\mathcal{E}' : \mathcal{E} \prec \mathcal{E}'\}$. This interpretation is quite possible in speaking of facts happening in a region \mathcal{R} near \mathcal{E} , e.g. on the earth, so that the causal present is a 4-dimensional space-time region intersecting \mathcal{R} very near some hypersurface. Otherwise it seems to me important to note the following. Adam has, at \mathcal{E} , a 4-velocity α , which determines the 3-dimensional manifold $\mathcal{M}_{\mathcal{E},u}$ formed by the spatial geodesics through \mathcal{E} , orthogonal to u there; in special relativity $\mathcal{M}_{\mathcal{E},u}$ is \mathcal{A} 's (locally) rest inertial-space at \mathcal{E} ; and one can show that

(a) *in some cases, e.g. in speaking of a star three light years far, say Σ , it may be incorrect to regard „now” as an equivalent of „in the causal present”*; and that

(b) *it is better to interpret „now” as „on $\mathcal{M}_{\mathcal{E},u}$ ” or as „near $\mathcal{M}_{\mathcal{E},u}$ ” for some \mathcal{E} in the space-time region of utterance.*

In fact, roughly speaking, assume that (i) \mathcal{A} is correct in asserting (on the earth), at his proper time s when he occupies \mathcal{E} :

(α) *tomorrow (at s_1) I shall observe the value of the magnitude \mathbf{m} — e.g. radio-activity — on the star Σ .*

Then \mathcal{A} can correctly assert, at s ,

(β) *thus I shall know a causally present value of \mathbf{m} on Σ ;*

and in case \mathbf{m} 's value is practically constant for some days, the same holds with \mathcal{A} 's possible assertion at s :

(γ) *thus I shall not know the value that \mathbf{m} now has on Σ , but the value taken there by \mathbf{m} three years (minus one day) ago.*

From (β) and (γ) we deduce (α). Let us add that (α) also holds because, in connection with (γ), \mathbf{m} 's value on Σ now is intuitively unique, while \mathbf{m} has many values on Σ in the causal present; in fact, if Σ 's distance from the earth is regarded as constant, Σ 's causal present for \mathcal{A} lasts six years instead of being instantaneous.

Now we note that, since the utterance time of an assertion is appreciably longer than an instant (even for every day life), all assertions involving „now” have to be treated in an approximate way. Furthermore (b) appears reasonable, e.g., when the every-day life assertion (γ) is considered in special relativity.

Note that *past* („*ago*”) is in effect used within (γ) as (*causally*) *before* $\mathcal{M}_{\mathcal{E},u}$; the analogue holds in every-day speech also for *future*, within special or general relativity.

Remark that the non-constancy of the speaker's velocity w.r.t. the earth may be troublesome in (γ) especially in connection with the past assertion involved by (γ). Roughly speaking, this defect is not avoided by replacing u with the earth's 4-velocity u_e , because neither u_e is constant; and for the present human community it is better to replace u with the sun's 4-velocity u_σ : as far as u_σ (unlike u_e) can be regarded as (nearly) constant, *the replacement of u with u_σ in (b)* renders the semantical rules for „now” independent of the month of utterance.

Being now interested in assertions referring to stars — like (γ) — or to galaxies, we cannot regard space-time as stationary. Instead, also looking forward to more precise conventions for the relativistic semantics of „now”, it is convenient to associate to \mathcal{E} (within general relativity) the point \mathcal{E}_σ of σ 's world line such that $\mathcal{E} \in \mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}$, and to set

$$\mathcal{M}_{\mathcal{E}}^\sigma =_D \mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}, \quad \text{where } \sigma \text{ denotes the sun.}$$

Now (a) can be improved as follows.

(c) *For the present human community, within general relativity, „now” can be (satisfactorily) interpreted, in any case, as „on $\mathcal{M}_{\mathcal{E}}^{\sigma}$ ” or as „near $\mathcal{M}_{\mathcal{E}}^{\sigma}$ ” for some \mathcal{E} in the space-time region of utterance.*

Some analogues of the above examples on „now” hold for „past” (or „future”). In fact assume that no other observations of \mathbf{m} on Σ were performed before the one mentioned in (a); and that, one year before s , \mathcal{A} uttered the 1st [2nd] of the sentences below.

(δ) *Some values taken by \mathbf{m} on Σ in the causal past will be observed by me.*

(ε) *Some of the values that \mathbf{m} has taken on Σ will be observed by me.*

Then \mathcal{A} was wrong [correct] in this utterance. We conclude that the phrase „in the present” (or „now”) [„in the past”] in effect involved by (γ) [(ε)] cannot be (equivalently) replaced by „in the causal present” [„in the causal past”] (which in effect gives rise to (β) [(δ)]).

6. On the relativistic semantics for „now” and „here” possibly used by special human communities

Now let us consider a (special) human community that is travelling on a rocket \mathcal{R} ; and assume that (i) they left Σ six years ago, (ii) they were always travelling along a geodesic of S_4 at about the speed $c/2$ w.r.t. the earth (or σ), where c the speed of light in vacuum, (iii) now they are at an event point \mathcal{E} , near the earth, and (iv) they are not interested in stopping at or communicating with the earth. Then

(d) *in order to interpret „now” in any pragmatic sentence uttered at \mathcal{E} by a member \mathcal{B} (Bernard) of the community travelling in \mathcal{R} , it is convenient to use the semantical rule proposed in (c) with the manifold $\mathcal{M}_{u_{\sigma}}^{\sigma}$ relative to the sun σ , replaced by its analogue $\mathcal{M}_{\mathcal{E}_{\mathcal{R}}, u_{\mathcal{R}}}$ for \mathcal{R} (so that practically $\mathcal{E}_{\mathcal{R}} = \mathcal{E}$).*

Thus, in particular, \mathcal{B} is practically correct in uttering at \mathcal{E} :

(η) *now we are $3\sqrt{3}/2$ light years far from Σ , and the rocket \mathcal{R} was there $3\sqrt{3}$ years ago,*

while \mathcal{A} , with 4-velocity $u = u_{\sigma}$, would obviously be correct in uttering at \mathcal{E} :

(ι) *now we are 3 light years far from Σ , and the rocket \mathcal{R} was there 6 years ago.*

In fact, since we are interested in avoiding (only) big mistakes, we can consider Σ and σ as steady in an inertial space \mathcal{I} of special relativity. Then

lengths [times] have the contraction [dilatation] factor $\sqrt{1 - \beta^2} = \sqrt{3}/2$ [$2/\sqrt{3}$] ($\beta = 1/2$). Furthermore, by using Römer units ($c = 1$) and the year as time unit, the proper length of the segment (Σ, σ) is 3, so that $\sqrt{1 - \beta^2} 3$ is its length for \mathcal{B} ; and the time τ that \mathcal{R} 's trip from Σ to σ lasted according to \mathcal{B} is \mathcal{R} 's proper time elapsed during this trip, so that $\tau 2/\sqrt{3} = 6$. Now (η) too appears true.

Note that, by e.g. interchanging u_σ and $u_{\mathcal{R}}$ in connection with (η) and (ι) , we would obtain assertions including big mistakes about both times and — unlike what happens with the preceding examples — also lengths.

Note that (d), (η) , and (ι) refer to a special (human) community supposed not to be feeling itself as a part of the earth community; furthermore \mathcal{R} 's intrinsic acceleration $a_{\mathcal{R}}$ is supposed to vanish. Otherwise, and especially if $a_{\mathcal{R}} \neq 0$ appreciably, I think people travelling in \mathcal{R} (and passing through \mathcal{E}) would naturally refer to $\mathcal{M}_{\mathcal{E}}^\sigma$ when they are using „now”, „past”, „future”, and „here”, as well as when they are evaluating times or distances.¹²

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¹² One can try and define geometrical points of $\mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}$ (and hence those of $\mathcal{M}_{\mathcal{E}}^\sigma =_D \mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}$) in effect as follows. Let $\mathcal{F}_{\mathcal{E}_\sigma}$ be a Fermi (-Walker) frame with the origin at $\mathcal{E}_\sigma \in \ell_\sigma$, where ℓ_σ is σ 's world line. For $(\vartheta, \varphi, s) \in [0, \pi] \times [0, 2\pi) \times \mathbb{R}^+$ call $\mathcal{E}_{\vartheta, \varphi, s}(\mathcal{E}_\sigma)$ the event point \mathcal{E} such that (i) the (spatial) geodesics ℓ of end points \mathcal{E}_σ and \mathcal{E} (supposed unique) has the length s and (ii) ℓ 's unit vector at \mathcal{E}_σ has the spherical coordinates ϑ and φ w.r.t. $\mathcal{F}_{\mathcal{E}_\sigma}$. We say that P is a *geometric point* of $\mathcal{M}_{\mathcal{E}_\sigma, u_\sigma}$ if $P = \mathcal{E}_{\vartheta, \varphi, s}(\mathcal{E}_\sigma)$ for some $(\vartheta, \varphi, s) \in [0, \pi] \times [0, 2\pi) \times \mathbb{R}^+$.



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