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IS TRANSPARENT INTENSIONAL LOGIC A NON-CLASSICAL LOGIC?

Abstract. It is shown that:
(a) classicality is connected with various criteria some of which are fulfilled by TIL while some other are not;
(b) some more general characteristic of classicality connects it with philosophical realism whereas (radical) anti-realism is connected with non-classical logics;
(c) TIL is highly expressive due to its hyperintensionality, which makes it possible to handle procedures as objects *sui generis*.
Thus TIL is classical in obeying principles of realism and non-classical in transcending some principles taught by textbooks of classical logic.

Keywords: TIL, bivalence, classicality, construction, expressivity, extensions, intensions, hyperintensionality, procedure, realism, anti-realism

1. Criteria

Principle of bivalence. The classical logic evidently obeys this principle but only one of its formulations:

- The formulation **A**:

Every sentence/proposition is either true or false.

- The formulation **B**:

There are just two truth-values: **T** (*true*), **F** (*false*).

Evidently **A** implies **B** but not *vice versa*.

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For **T**(ransparent) **I**(ntensional) **L**(ogic) **B** holds: TIL is *not* a many-valued logic. **A** cannot hold because there are not only total functions in TIL. Sentences that express (a) *improper constructions* or (b) denote propositions whose Strawsonian[1] presuppositions do not hold cannot accept any truth-value (which does not mean that they accept a 'third value').

Examples. *Ad* (a) The sentence *The greatest prime is odd* expresses a construction that does not construct anything: it is an improper construction.

Ad (b) The sentence *Charles knows that the Moon is greater than the Earth* denotes a proposition one of whose presuppositions is not true (viz. that what is *known* is true). It cannot be true but it cannot be false either.

Thus TIL accepts truth-gaps (*but does not accept truth-gluts*).

Extensionalism. Classical logic(s?) is extensionalist in that it obeys principles of extensionality, in particular the principle of compositionality. In this sense we can say that

> "Transparent Intensional Logic flouts none of the principles of extensional logic and is, insofar, an extensional logic."
>
> Duží *et al.* (2010, p. 2)

The term "extensionalism" is however connected with a more radical version: extensionalist in this second sense are such logical systems whose notions are not sufficient to tell intensions from extensions, in other words: the specific character of intensions cannot be evaluated. TIL has shown that one can define intensions and, at the same time, continue obeying principles of extensionality. Being not able to deal with logical problems induced by specific character of intensions means limitation of expressivity, leads to a form of reductionism, which treats properties as classes, propositions as truth-values, magnitudes as numbers etc.

TIL is classical in that it obeys principles of extensionality, it is not classical due to treating intensions as not reducible to extensions.

First-order paradigm. Formal systems connected with classical logic are first-order. They can be sound and complete. Only propositional (truth-functional) logic is however decidable.

[1] See Strawson (1950).

Observe: with increasing expressivity some "good property" is lost: first-order predicate logic is more expressive than propositional logic but is no more decidable. Second-order predicate logic is more expressive than first-order predicate logic but is no more complete. Note: we cannot prefer decidability: otherwise we would be happy with propositional logic only.

TIL is not based on first-order logic of relations. It is a *functional* logic using (interpreted) typed λ-calculus instead. Its first stage based on *using* procedures ('constructions') works with first-order hierarchy of types, which is however more expressive than first-order predicate logic. (Classes of any order can be treated within this first-order TIL.) The second stage based on *mentioning* constructions becomes a *hyperintensional* logic proceeding top-down and distinguishing three levels of context: *hyperintensional* (procedures are mentioned), *intensional* (procedures are used to construct functions) and *extensional* (procedures are used to construct values of functions applied to arguments). The degree of expressivity essentially exceeds that one of first-order systems. What is lost is just the relative simplicity (including computational simplicity) of those systems.

Applied to logical analysis of natural language TIL defines a 'neofregean' semantics, where Frege's *Sinn* (we will use *meaning* rather than *sense*) is explicated as a construction, his *Bedeutung* (*denotation*) as the result (if any) of the meaning, i.e. as what the construction constructs. The term *reference* of the expression E is used as the name of the contingent value of the intension that is the denotation of E (when E is an empirical expression) in the actual world-time[2]. A desirable feature of the resulting semantics is that *the meaning of an expression is totally independent of any context.*

TIL abandons the 1st order paradigm, being in this sense not a classical logic.

Total functions only. Using just total functions might be a feature of classical logic. This feature essentially limits a potential expressivity of logic.[3] Not only classical logic avoids treating partiality (intuitionists

[2] To adduce an example: while the *denotation* of an empirical sentence is a proposition, its (contingent) *reference* is its truth-value in the actual world. (And the *meaning* of the sentence is the procedure that constructs that proposition.)

[3] See Duží (2003) for some convincing arguments.

do not like it as well). Definability of partial functions is however a brute fact.[4] TIL presupposes partial functions and defines the way of propagating partiality of a constituent of a complex construction C over C. Thus the construction (concept) that is expressed by the term *the greatest prime* is *improper*, i.e., does not construct anything: the class of the (natural) numbers that are the greatest prime is empty, so the singularizer ("the only x such that") is not defined. In the construction expressed by the sentence *The greatest prime is odd* the oddity cannot be predicated because its constituent is improper, so that the whole sentence expresses an improper construction and it does not denote any truth-value.

If being classical presupposes dealing just with total functions, then TIL is not a classical logic.

Excluded middle. This Principle of Excluded Middle (PEM) is characteristic of classical logic especially as in contrast to intuitionism. Do not forget however that intuitionists have criticized PEM having redefined connectives so that the intuitionist sentences have not denoted propositions in classical sense. Otherwise we could be a little upset, as Kolmogorov has stated in his (1932):

> "So entsteht diese ganz besondere Art von Aussagen, welche zwar einen mit der Zeit nicht veränderlichen Inhalt haben sollen und doch nur unter speziellen Bedingungen ausgesprochen werden können."
>
> Kolmogorov (1932, p. 64)

Kolmogorov's interpretation is well-known: Heyting's formalization can be considered to be a logic of *problems*.

Thus it seems that since classical logic deals (due to "classical interpretation") with propositions rather than with constructions PEM should be classically a principle whose tautological counterpart would be

$$A \vee \neg A \tag{1}$$

where A is any sentence that is true or false.

Yet when we want to really comprehend PEM as an old principle very important for distinguishing classical vs. non-classical exposition of logic then we have to adequately interpret the scheme (1).

[4] Strawson's presuppositions are also a brute linguistic fact.

Actually, we can distinguish two interpretations. According to *Int 1*, A is just a variable whose range is $\{\mathbf{T}, \mathbf{F}\}$. Then there can be no doubt that our disjunction *is* a tautology so that PEM holds trivially.

Consider however *Int 2*, according to which A is just a scheme of any (declarative) sentence. Then — see above *Principle of bivalence* — we can see that on this interpretation PEM does not hold as soon as we admit sentences that are neither true nor false. Then , of course, PEM is not universally valid.

Thus *Int 2* shows a way how to escape the "jail of classicality". We know two kinds of this escape:

- admitting more than two truth-values ("Łukasiewicz way") or
- admitting partial functions ("Boczwar way").

As we have already stated, TIL is a Boczwar way, i.e., no third, fourth etc. truth-value is admitted but truth-gaps arise due to partiality.

If PEM is interpreted in the Int 2 sense then TIL is not a classical logic.

2. Classicality as a vague positive idea

P. Banks (allegedly J. M. Bocheński) in 1950 develops (as a fictive dialogue of a conservative, a modern and an "Aristotelian" logician) an idea of logic which is based on Aristotelism, which is simply a scientific approach starting with common sense, believing in reason and experience and accepting the possibility of an unlimited progress based on tradition.

This idea of compatibility of progress with tradition can be perceived (vaguely, of course) as the idea that the only logic which can be called 'logic' is just what is called *classical logic*. Indeed, 'classical' means 'based on tradition', and we surely cannot refuse the possibility of developing (classical) logic. Further, Banks believes that the *genuine logic is just a collection of metalogical rules* and that such a collection contains only evident, Aristotelian rules. He states that *one cannot find any such logical system whose metalogical rules would contain non-evident, non-aristotelian rules not obeying, e.g., the principle of forbidden contradiction.*

In my opinion, this is a strong argument against the trendy spirit of relativism that misuses (or exploits?) the fact that there are unbelievably many particular logical systems. Banks-Bocheński admits that these systems may contain interesting results, he only wants to distinguish

between Logic and these systems. The latter can be interesting and useful but they remind us of something like counter. Language of the genuine logic can be found, e.g., in *Principia Mathematica*.

Clearly, some 'non-classical logics' could not be representatives of Logic in Bocheński's sense, for example many-valued or intuitionist logics, surely paraconsistent logics. But to say that e.g. Łukasiewicz did not accept PEM is not convincing because neither he would admit that there could be a third possibility when saying that a proposition can have or not have the value **T**.

It seems that the choice of "the adequate logic" is dependent on some philosophical standpoint, or — if you like — on some "metaphysical" assumptions. Thus our choice of that "degree of classicality" which characterizes TIL should be founded on the choice of some such "metaphysics". But as Dummett in (1991) rightly stresses,

> "We must not try to resolve the metaphysical questions first, and then construct a meaning-theory in the light of the answers."
>
> Dummett (1991, p. 338)

Thus what is primary here and will eventually "determine the answers to the metaphysical ones" (*ibidem*) is the choice of "a Correct Meaning-Theory" (p. 339).

Two extreme positions determine two incompatible ways of choosing criteria for deciding which theory should be recognized as a *correct meaning theory*: One of them is called (a radical) *anti-realism* well-known just from Dummett's works, while the other one could be any form of *realism*. As for *classicality* or a "degree of classicality" radical anti-realism unambiguously demands a non-classical logic.[5] We have shown, however, that being realistic (logic) is not a very determinate criterion. In the broadest sense the minimum of classicality means that the main thesis of anti-realism — i.e. the claim that e.g. the principle of bivalence is not justified and that the meaning of mathematical / logical terms attached to them by classical logic(s) is incoherent — is rejected.

Concerning just the incompatibility of *classicality* vs. *anti-realism* we should not acquiesce in stating that *there are more logics*: we should find "some neutral manner of formulating the rival conceptions of meaning so as to be able to argue their merits without prejudging the issue in favor of one or the other" (p. 17).

[5] See Dummett (1991, p. 340)

This problem of finding "some neutral manner" can be formulated in another way, as the problem of adequate *explication*.

3. TIL epistemic framework as a most adequate explication. Hyperintensionality

The necessity of first explicating notions used in a claim or a theory before judging whether the claim / the theory is at least rational or even true has been recognized by Carnap (see his 1950[6]). Carnap has mentioned some general principles that a good explication should obey (simplicity, exactness, fruitfulness, similarity to the initial intuitions) and has applied them in his works. Now we will quote from Tichý's (1988):

> "The purpose of theoretical explication is to represent intuitions in terms of rigorously defined entities. It is to Frege that we owe the insight that the mathematical notion of function is a universal medium of explication not just in mathematics but in general. To explicate a system of intuitive, pre-theoretical, notions is to assign to them, as surrogates, members of the functional hierarchy over a definite objectual base. Relations between the intuitive notions are then represented by the mathematically rigorous relationships between the functional surrogates." Tichý (1988, pp. 194–195)

Well, TIL has been built up in harmony with this characterization[7]. To adduce an example, properties of individuals are explicated as functions that map possible worlds to chronologies of classes of individuals (type $(((o\iota)\tau)\omega)$, abbr. $(o\iota)_{\tau\omega}$). We could expect an objection formulated, e.g. by G. Bealer in his (1982), where he objects in the sense that the aroma of coffee is a property (an intension), but certainly not a mapping (mappings having no aroma); hence, properties are not mappings. The objections of this kind are misunderstandings. Surely no property is a mapping but the above mentioned mapping is just a functional surrogate, which can be logically handled. This explication makes it possible to logically capture the fact that properties (e.g., of individuals) "produce classes" dependently on the current state of the world.

Let us try to accept this characteristic of explication. (Observe however that explication in this sense is not an *interpretation* defined for

[6] Already in his (1947) Carnap briefly characterizes explication.

[7] For details see Duží, Jespersen, Materna (2010).

formal systems.) A question arises: Can such an explication be 'neutral' w.r.t. the opposition *realism* (or *classicality in a broadest sense*) vs. *anti-realism*?

For example: As soon as I decide that there are such entities that are intuitively addressed as *properties* and seek some functional surrogate for them I have already voted for realism, or not?

Perhaps yes: whether some individual has a property is determined — according to our explication — by the current state of the world, which means that any individual has or has not a given property independently of our knowledge. This realistic spirit of our explication transfers to handling mathematical objects. The type of entities associated with mathematical sentences as their meanings is such that these entities construct a truth-value. If no truth-value is associated with such meanings then only for objective reasons. Thus the meaning of the Goldbach conjecture constructs a truth-value independently of the fact that no proof has been found till now, while no truth-value is associated with the sentence *The greatest prime is odd* because there is no greatest prime, not because we do not know which prime is the greatest one.

But now can we imagine an explication that would support anti-realism? Reading Brouwer, Heyting, Kolmogorov, Per Martin-Löf etc. we can imagine an explication where there would be explicated logical constants in a proof-theoretic manner (but frankly, explication of empirical entities would be extraordinarily difficult). Then of course, neither such an explication would be *neutral*.

The classical realistic systems had in principle underestimated one kind of entity that had been treated only in meta-languages: abstract procedures, complexes, constructions that are not simple like set-theoretic entities.[8] Anti-realists, I mean intuitionists, have recognized the essential role played by procedures, mainly proofs, and have reinterpreted entities that the realists of the classical kind have defined as set-theoretical objects.

The significance of TIL in this connection consists in the fact that TIL is a *hyperintensional* logic, which makes it possible not only to *use* but also *mention* the abstract procedures (*constructions*). Constructions are legitimate logical objects and as possible meanings (senses) of expressions they fulfill the ideal of *structured meaning*. The expressivity

[8] See Tichý (1995, 2004).

of the resulting semantics essentially dwarfs the expressivity of extensional/intensional logics.

Thus TIL is classical in following the principles of realism and non-classical in transcending some principles standardly used by classical logic (where *classical logic* is what is taught by textbooks of classical logic).

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