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BOOK REVIEWS

Some chapters in the history of logic turn out to be less visible than others, due to several reasons which include author's influence, orientation of reviewers, among others. Although *LLP* intends to publish reviews of recent books, reviews of not so new books can be also published, if they would contribute to the connections between logic and philosophy and to the history of logic. This is the case of the *From Peirce to Skolem. A Neglected Chapter in the History of Logic* by Geraldine Brady, as reviewed by Davide Bondoni which we publish in the present issue.

Walter Carnielli

GERALDINE BRADY, **From Peirce to Skolem. A Neglected Chapter in the History of Logic**, Studies in the History and Philosophy of Mathematics, vol. 4, Elsevier (imprint: North-Holland), 2000, ISBN-13: 978-0444503343, ISBN-10: 0-444-50334-X, 625 pp.

In the 2000, Geraldine Brady of the department of Computer Science in Chicago has published by Elsevier a book, entitled *From Peirce to Skolem, A Neglected Chapter in the History of Logic* [Bra00]. Her main effort is to fill a gap in the seminal handbook by van Heijenoort [vH67]. According to her opinion, Heijenoort would have ignored the algebraic tradition in logic, not inserting in his work any paper belonging to this tradition.

This is not at all correct; in Heijenoort's book it would be contained a work of Tarski, but this last refused his approval. Apart this small inaccuracy, Brady's book presents itself as a valuable resource for whom interested and trained in the algebra of logic. She considers both the work of Peirce and

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Schröder, the Löwenheim-Skolem's theorem and in an appendix translates much material from Schröder's *Lectures on the Algebra of Logic*. Despite the fact that in all the book only 30 pages are devoted to Schröder, this is the most appealing section of the book, given the lack of studies on Schröder's logic.

1. Here, Brady occupies herself chiefly with the schröderian calculus of relations as the most significant facet of Schröder's endeavours on logic. In particular, there are analyzed accurately the solution problem and the chain theory in terms of relatives. Unfortunately, Brady accepts Lewis' statement, according to that Schröder would have merely translated Dedekind's theory of chains in his theory of relations. For example:

> Schröder translates Dedekind's set-theoretic treatment of chains line-by-line into the second-intentional calculus of relatives.
> [Bra00, p. 158]

This is a great misunderstanding. What Brady doesn't see is that Schröder puts in evidence the equivalence between the concept of chain and that of the smallest reflexive transitive closure of a relation. This is an important result obtained by Schröder. It's sufficient bringing to mind the centrality of such closure in many logical and mathematical areas to grasp the meaning and the scope of the concept of reflexive transitive closure. I think about Kleene's algebras, to graph theory, to computer science, etc.
It's true that Schröder doesn't use the expression 'reflexive transitive closure'; that will be introduced later in topology, but it's manifest that he has in mind such concept from the equivalences he lays down about the chains. I.e, he shows, firstly, that a chain, seen as a relation R, is a relative fulfilling the following conditions:

1. crescence
2. idempotence
3. monotony

and, after, that R is a reflexive and transitive relation. It's also true that Schröder emphasizes the importance of his work on chains from a linguistic point of view:

> [...] trotz allem unsre Darstellung der Kettentheorie an *Übersichtlichkeit* keinen andern [...] nachstellen wird. [Sch66, p. 353]
>
> [...] our presentation of the chain theory is second to none with respect to clarity [...]. [Bra00, p. 302]

This focus on the linguistic features of the calculus of relatives is maintained by Schröder also in a short paper published in the *Mathematische Annalen* [Sch95], where he condenses the main matter of the third volume of the *Lectures*. But, an historian of logic must read beyond Schröder's lines casting light on the *not-said*. There's certainly a linguistic dimension in the calculus of relatives; in fact, the last Schröder saw in it a language, a pasigraphy, to translate accurately the concepts of science. But we must not forget that in this place Schröder shows that the chain theory is independent from the concept of function. Schröder doesn't limit himself to translate and generalize the chain theory, as maintained from Brady. It's a pity that Brady doesn't appreciate this fact.

2. Even in treating the solution problem, Brady makes a great mistake. She states that Schröder foresaw in this context Skolem functions.

> This is where he [Schröder] introduces a precursor of Skolem functions, replacing existential quantifiers by function symbols that witness them.
> [Bra00, p. 258]

It's not true. Schröder exploits something like Skolem functions in another occasion, but not here. In the fifth lecture, that devoted to the *Auflösungsproblem*, Schröder aimed only to find that relation obtaining among these relations which are *not* solutions of a given equation of the form $F(x) = 0$. Said better. According Brady's reading, Schröder would have said something like:

$$\forall x F(x, f(x)) = 0 \leftrightarrow \forall x \exists y F(x, y) = 0 \tag{1}$$

Taking for granted (1), Brady's conclusion is straightforward. However Schröder doesn't state (1), but

$$\forall x F(x, f(x)) = 0 \leftrightarrow \neg\forall x \exists y F(x, y) = 0 \tag{2}$$

What Schröder is searching for, is the relation obtaining among every value *not* satisfying the given equation. It's only in the eleventh lecture that Schröder tries to eliminate the existential quantifiers, producing something as the Skolem functions. In fact, Löwenheim will refers himself to this lecture, proving his theorem.

3. As said before, Brady translates much material from the third volume of the *Lectures* but, unfortunately, this text cannot substitute the original; first, because Brady's translation is incomplete and obviously reflects her thought; second, because it is non indipendent. In this sense: Schröder, sometimes, refers himself to a given page, for example x; it happens that this page is not translated; so, the reference is unknowable. I think that Brady could have put a note, in this case, observing something as *here Schröder refers himself to....*

Content of the book *From Peirce to Skolem. A Neglected Chapter in the History of Logic*

1. Introduction
2. The Early Work of Charles S. Peirce
3. Peirce's Calculus of Relatives: 1870
4. Peirce on the Algebra of Logic: 1880
5. Peirce on the Algebra of Relatives: 1883
6. Peirce's Logic of Quantifiers: 1885
7. Schröder's Calculus of Relatives
8. Löwenheim's Contribution
9. Skolem's Recasting
10. Appendices
    - Schröder's Lecture I
    - Schröder's Lecture II (until page 68)
    - Schröder's Lecture III (page 76 and from page 97 to 101)
    - Schröder's Lecture V (until page 190)
    - Schröder's Lecture IX
    - Schröder's Lecture XI (page 491 and from page 497 to end)
    - Schröder's Lecture XII (from page 596 to end)
    - Norbert Wiener's Thesis (it contains some excerpt from Wiener's thesis)
11. Bibliography
12. Index

References

[Bra00] Geraldine Brady, *From Peirce to Skolem. A Neglected Chapter in the History of Logic*, Elsevier, 2000.

[Mad01] Roger D. Maddux, "Relation algebras", Draft version: 7 May 2001.

[Pec91] Volker Peckhaus, “Ernst Schröder und die pasigraphischen Systeme von Peano und Peirce”, *Mod. Logic* 1 (1990/1991), 34–35.

[Sch95] Ernst Schröder, *Note über die Algebra der binären Relative*, Math. Ann. (1895), 144–158.

[Sch66] ———, *Vorlesungen über die Algebra der Logik, Exakte Logik*, vol. 3, Algebra und Logik der Relative, Chelsea Publishing Company, 1966.

[vH67] Jean van Heijenoort, *From Frege to Gödel, A Source Book in Mathematical Logic, 1879–1931*, Harvard University Press, 1967.

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